Young Physicists Tournament

Problem 4: soap film

Explain the appearance and development of colors in a soap film, arranged in different geometrical ways.

Studies

Tuomas Pylkkänen
Finland

We studied soap films made of different solutions by attaching them to different frames and observing the light reflected from them. Based on our surveys, we divide the possible geometries for soap films in two categories. As a generalization, soap films assume either a flat or a curved position when they attach to different frames. Furthermore, these films can be hung up either vertically, horizontally, or in various inclined ways.

In each case, we observe on the surface of the film a pattern, which consists of stripes of different colors. The color series seen are similar in all cases. The vertical soap film has assumed a wedge-like shape as a result of draining. The horizontal soap film has curved upwards, and draining has made it thinnest at the top. In both cases, the colors observed change as the films' thickness changes, forming a characteristic series. The series is well known (see Table 1).

ries

These kind of films do not last long; a simple solution made of a common detergent and water produced films lasting from a few seconds up to half a minute. (*See Picture 1)* Glycerin, on the other hand, prolongs soap films' lifespans but also makes them less homogenic. This destroys the characteristic pattern but gives life to new ones as the films thickness changing randomly. *(See Picture 2)*

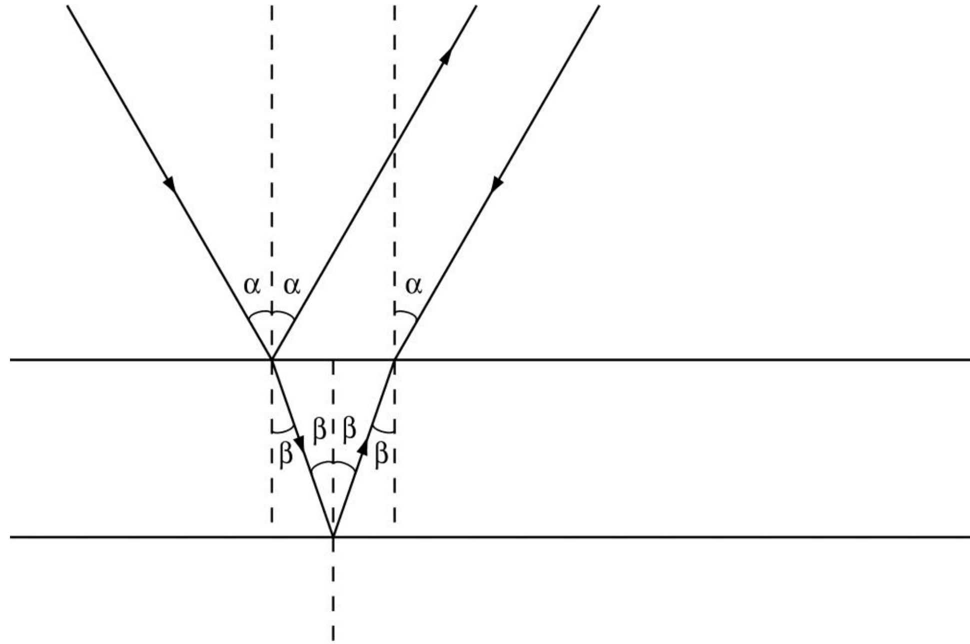
By observing the colors one can see how soap films drain. A horizontal film gets thinner at the top, and if one observes the borders, one can see a flux, in which the thinner regions move up. A vertical film gets thin in the middle as a result of surface tension and draining.

On a more theoretical level, the color patterns observed are due to thin film interference: the light reflected from the surface of the film and the light reflected from the bottom of the film interfere constructively or destructively, giving a color tint to the mirror image. These rays are always coherent, as can be seen from Illustration 1.

I	Film thickness (μm):
black	0
light gray	0.080
brown-yellow	0.115
red	0.170
II	
violet	0.190
blue	0.210
green	0.270
yellow	0.305
red	0.340
III	
violet	0.385
green	0.455
yellow	0.505
red (incarnadine)	0.525
IV	
gray-blue	0.595
green	0.655
red (incarnadine)	0.695
V	
blue-green	0.820
From here on alternating green and magenta stripes are observed, up to the point when no color tint is observed.	(M. Minnaert: Licht en kleur in het landschap, 1968)

r
i
e
s

Illustration 1



A simple equation can be formulated to describe this. Consider a light wave of wavelength λ that meets a perpendicular film of thickness d and an index of refraction n (for soap films, the indices of refraction are close to that of water (1,33), and the index of refraction of air is, for all practical purposes, one). At an interface, part of a wave is reflected and part of it refracted and transmitted through the other media. In this limited view, there is no refraction, but two reflections: one at the surface (W1), and one at the bottom (W2). There is a phase shift of half a wavelength for (W1), for $n > 1$, but not with (W2), as $1 < n$.

We neglect the interference of the incident and the first reflected ray by approximating the angle of incident to be nearly 90°. For that interference the incident angle would have to be exactly 90 degrees, which is in practice impossible. However, the two reflected rays are always coherent, so our approximation is valid.

r i e s

(W2) interferes constructively with (W1), if during its longer path it comes back in phase with (W1), that is if the extra distance is half the wavelength of (W2) in the soap solution (λ_2). Of course, the same happens if the distance is a whole number plus half the wavelength.

$2d = (k+½)\lambda_2$

The wavelength in the soap solution can be expressed with the original wavelength and the index of refraction (let c_2 be the speed of light in the solution, frequency f is a constant in all media):

$c / c_2 = n$ and $v = f\lambda$, $c = f\lambda$ and $c_2 = f\lambda_2$

thus $c / c_2 = \lambda / \lambda_2$, which equals $\lambda / \lambda_2 = n$, or $\lambda_2 = \lambda / n$. We get

$2nd = (k+½)\lambda$, where k = 1,2,3,... (Eq.1)

Similarly, the interference is destructive if the waves meet with a phase difference of half a wavelength:

$2nd = k\lambda$, where k = 1,2,3,... (Eq.2)

This consideration can be extended to include the angle of incident for the light wave, which produces the following equation, but it gives no further insight.

$2nd \cos\theta = (k+½)\lambda$

For a given d, there can be many wavelengths in the visible spectrum (380 - 760 nanometers) which satisfy Eq.1 and others which satisfy Eq.2.

r
i
e *These interference maxima give the light* reflected a color tint. We modeled these maxima s with a computer, using Eq.1 and Eq.2. (*picture: Eq.1 and Eq.2 solved for values of k from 1 to 8, combined*).

This model only includes the maxima, whereas in reality, there are various amounts of every color. Interfering sine waves of same the wavelength produce sine waves of the same wavelength but of different amplitudes. Using Table 1, and solving the phase differences between (W1) and (W2) for those film thickness, we have Table 2.

Table 2

In fact, if one mixes colors with those relations (considering the amount of phase difference and its effect on amplitude), one obtains the very colors from Table 1.

When the film is thin enough, there is not enough distance for (W2) to catch up the phase difference with (W1), and we observe only destructive interference, or a black film, which does not reflect any light. At this stage the film is only from 50 to 300 Å thick. However, the first colors to appear are those

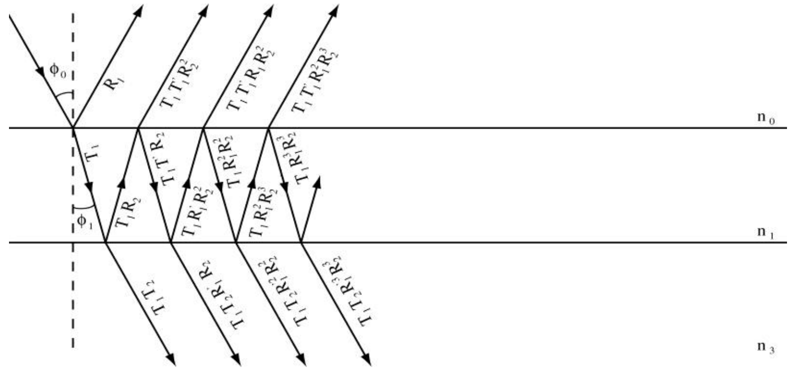
r
i
e
s

	Phase differences			1,33				
thickness	violet	blue	green	yellow	orange	red	nanometres	
micrometers	380...450 (415)	450...490 (470)	490...560 (525)	560...590 (575)	590...630 (610)	630...760 (705)	**The color series**	
0,08	1%	5%	9%	13%	15%	20%	light gray	
0,12	24%	15%	8%	3%	0%	7%	brown-yellow	
0,17	41%	46%	36%	29%	24%	14%	red	
0,19	28%	42%	46%	38%	33%	22%	violet	
0,21	15%	31%	44%	47%	42%	29%	blue	
0,27	23%	3%	13%	25%	32%	48%	green	
0,31	45%	23%	5%	9%	17%	35%	yellow	
0,34	32%	42%	22%	7%	2%	22%	red	
0,39	3%	32%	45%	28%	18%	5%	violet	
0,46	42%	8%	19%	40%	48%	22%	green	
0,51	26%	36%	6%	16%	30%	41%	yellow	
0,53	13%	47%	16%	7%	21%	48%	red (incarnadine)	
0,60	31%	13%	49%	25%	9%	26%	gray-blue	
0,66	30%	21%	18%	47%	36%	3%	green	

r
i
e
s

The phase difference between two successive rays is

$\delta = \frac{4\pi a n_1 \cos\phi_1}{n_0 \lambda} + \pi$. Look at Illustration 2, where the amplitudes for the successive rays are given:



This results in the equation:

$$R = r_1 + t_1 t_1' r_2 e^{-2i\delta_1} - t_1 t_1' r_1 r_2 e^{-4i\delta_1} + ... = r_1 + \frac{t_1 t_1' r_2 e^{-2i\delta_1}}{1 + r_1 r_2 e^{-2i\delta_1}} \quad \text{(Eq.3)}$$

As this is a non-absorbing media we can write the Fresnel transmission coefficients in terms of r_1, r_2. From conservation of energy we have $t_1 t_1' = 1 - r_1^2$.

$$R = \frac{r_1 + r_2 e^{-2i\delta_1}}{1 + r_1 r_2 e^{-2i\delta_1}} \quad \text{(Eq.4)}$$

The film is non-absorbing, so the energies of the beams are given by

$$n_0 RR^* = \frac{n_0 (r_1^2 + 2 r_1 r_2 \cos 2\delta_1 + r_2^2)}{(1 + 2 r_1 r_2 \cos\delta_1 + r_1^2 r_2^2)} \quad \text{(Eq.5)}$$

Reflectance and transmittance are defined as ratios of reflected and transmitted energy to the incident energy. Since we have considered a wave of unit amplitude, the reflectance is given by:

$$\mathrm{R} = \frac{r_1^2 + 2 r_1 r_2 \cos 2\delta_1 + r_2^2}{1 + 2 r_1 r_2 \cos\delta_1 + r_1^2 r_2^2} \quad \text{(Eq.6)}$$

r
i
e
s

We express this in terms of refractive indices:

Since $r_2 = \frac{n_1 - n_2}{n_1 + n_2}$ and $r_1 = \frac{n_0 - n_1}{n_0 + n_1}$, Eq.4 becomes

$$R = \frac{(n_0 - n_1)(n_1 + n_2)e^{i\delta_1} + (n_0 + n_1)(n_1 - n_2)e^{-i\delta_1}}{(n_0 + n_1)(n_1 + n_2)e^{i\delta_1} + (n_0 - n_1)(n_1 - n_2)e^{-i\delta_1}}$$

and Eq.6:

$$\mathrm{R} = \frac{(n_0^2 + n_1^2)(n_1^2 + n_2^2) - 4n_0 n_1^2 n_2 + (n_0^2 - n_1^2)(n_1^2 - n_2^2)\cos 2\delta_1}{(n_0^2 + n_1^2)(n_1^2 + n_2^2) + 4n_0 n_1^2 n_2 + (n_0^2 - n_1^2)(n_1^2 - n_2^2)\cos 2\delta_1}$$

In our case $n_0 = 1$ and $n_2 = 1$, we mark $n_1 = n$ and get

$$\mathrm{R} = \frac{(n^2 + 1)^2 - 4n^2 - (n^2 - 1)^2 \cos 2\delta_1}{(n^2 + 1)^2 + 4n^2 - (n^2 - 1)^2 \cos 2\delta_1}$$

If we mark n=1.33, we get

$\mathrm{R} = \frac{1 - \cos 2\delta_1}{24{,}936 - \cos 2\delta_1}$. This completes our report.