7. HEATED NEEDLE

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Experimental device

Experimental regimes

\[ \alpha_0 = 40^\circ \quad l_n = 40 \text{ mm} \]
\[ \tau_1 = t_2 - t_1 \approx 0.43 \text{ sec} \]
\[ \tau_2 = t_4 - t_3 \approx 1 \text{ sec} \]

\[ F = mg \tan \alpha_0 \]
\[ \omega = \sqrt{\frac{F^2 + (mg)^2}{ml}} \]
\[ \tau = \frac{2\pi}{\omega} \approx 0.53 \]

where \( l_n \) is length of needle, \( F \) is a magnet force, \( m \) is mass of needle and \( \omega \) - frequency of oscillations near magnet.

\[ \alpha_0 = 37^\circ \quad \alpha_1 = 25^\circ \quad l_n = 40 \text{ mm} \]
IN CLASSICAL THEORY

\[ \bar{\mu} = I\bar{s} \]

\[ u = -\mu B \cos \theta \quad \theta = (\bar{\mu}, \bar{B}) \]

\[ F_x \Delta x = -\Delta U \Rightarrow F_x = -\frac{\Delta U}{\Delta x} \]

\[ F_x = \mu \cos \theta \frac{\Delta B}{\Delta x} \]

\[ B(x) = \text{const} \Rightarrow F_x = 0 \]

where \( \mu \) is a magnet moment of circuit, \( I \) – current in the circuit, \( F_x \) – interaction force between circuit and magnet and \( S \) is square of the circuit.

From pictures and expression for \( F_x \) is clear that we have repulsion or attraction when magnetic field is not uniform.
QUANTUM THEORY

\[
\begin{align*}
M &= N < \mu > \\
N_{\uparrow \uparrow} &= a e^{\beta / kT} \\
N_\downarrow &= a \\
N_{\uparrow \downarrow} &= e^{\beta / kT} \\
N &= N_{\uparrow \uparrow} + N_{\uparrow \downarrow} + N_\downarrow \\
< \mu > &= \frac{N_{\uparrow \downarrow} (+\mu) + N_{\uparrow \uparrow} (-\mu) + 0 \cdot N_\downarrow}{N} \\
M &= \mu N \frac{e^{\beta \mu / kT} - e^{\beta \mu / kT}}{e^{\beta \mu / kT} + e^{\beta \mu / kT} + 1}
\end{align*}
\]

where \( M \) is magnetization of iron, \( N \) - concentration of atoms, \( \mu \) - magnetic moment of an atom, \( N_{\uparrow \uparrow}, N_{\uparrow \downarrow}, N_\downarrow \) - are respectively concentrations of atoms whose magnetic moments are directed to, opposite and perpendicularly to the outer magnetic field,

\[
p \sim e^{-\frac{(\text{State energy})}{kT}}
\]

\[B = \mu_0 \cdot (H_{\text{ext}} + \omega \cdot M)\]

Dependence of magnetization on temperature

Dependence of coil inductivity on temperature of an iron core.
Thermodynamical calculations

For balance law we will get following expression:

\[-cm \, dT = \xi S (T - T_0) \, dt + \varepsilon \sigma S T^4 \, dt\]

\[\xi \approx 20 \, \text{J/Ksm}^2 \quad \varepsilon \approx 0.5\]

\[T_0 \approx 290 \, \text{K} \quad m \approx 0.15 \, \text{g}\]

\[S \approx 1.25 \times 10^{-4} \, \text{m}^2 \quad c = 460 \, \text{J/kg} \cdot \text{K}\]

where \(T_0\) is temperature of environment, \(m\) – mass of needle and \(S\) – square of needle.

\[-t \frac{S \varepsilon \sigma}{cm} = \int_{T_2}^{T_1} \frac{dT}{T^4} + \frac{\xi}{\varepsilon \sigma} T - \frac{\xi T_0}{\varepsilon \sigma}\]

\[t = 3 \, \text{s}\]

\[T_1 \approx 990 \, \text{K} \quad T_2 \approx 820 \, \text{K}\]

\(T_1\) and \(T_2\) are respectively initial and after \(t\) time (time in which needle comes to magnet) temperatures.