WEAK SIGNALS

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Problem 16: Weak signals

Controlling **signal-to-noise ratio** is important in many measurements to **distinguish** a **meaningful signal** from **statistical fluke.** Propose a problem requiring experiments to **detect very weak signals.**

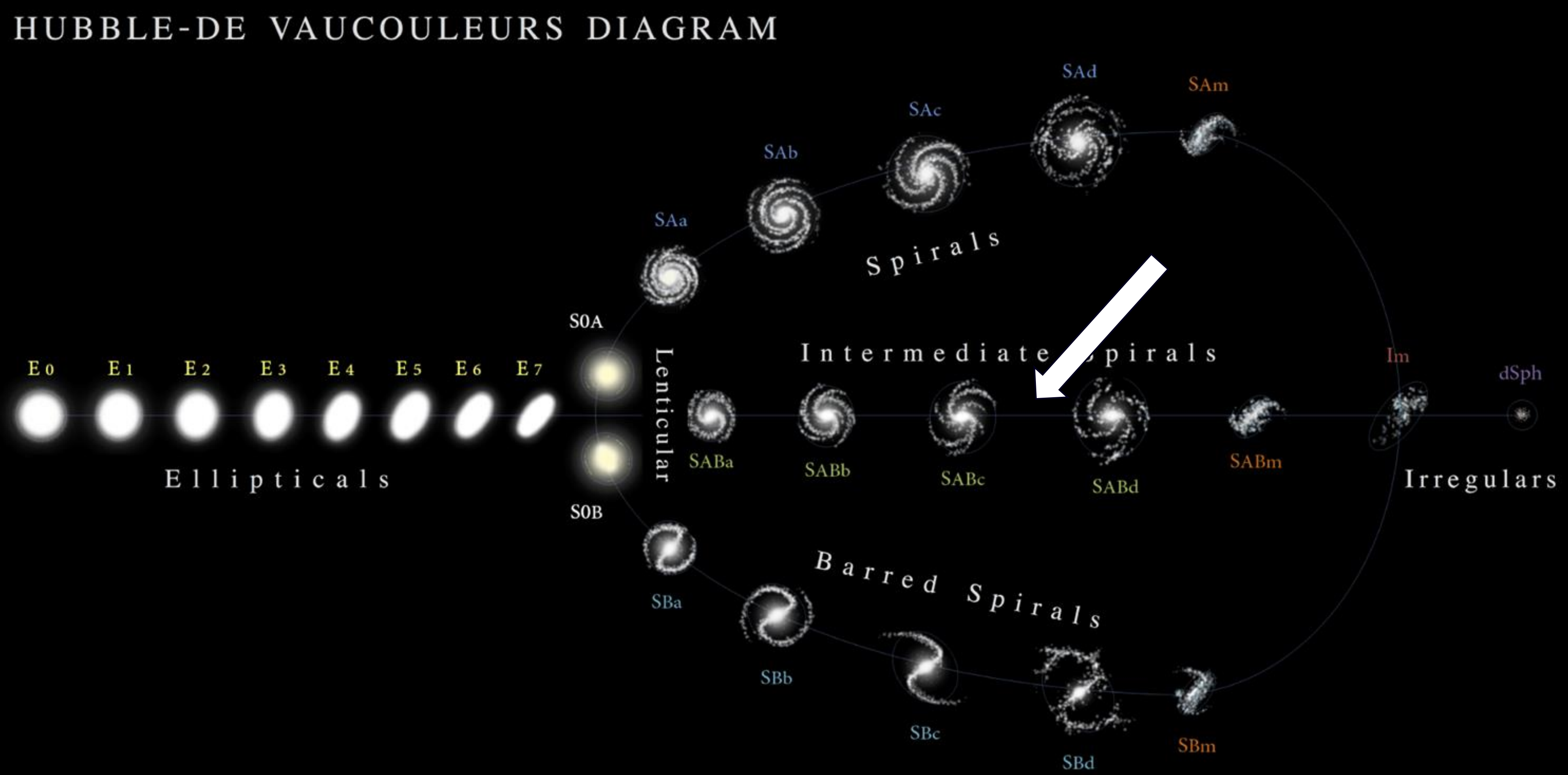
Problem statement

Make an investigation, based on data with **low signal-to-noise ratio** to explore a **distant Space object**.

M101: Pinwheel Galaxy

- Spiral galaxy type SAB(rs)cd, located in the constellation Ursa Major
- +54° 20′ 57″ DE, 14h 03m 12.6s RA
- Distance: 6.4 Mpc

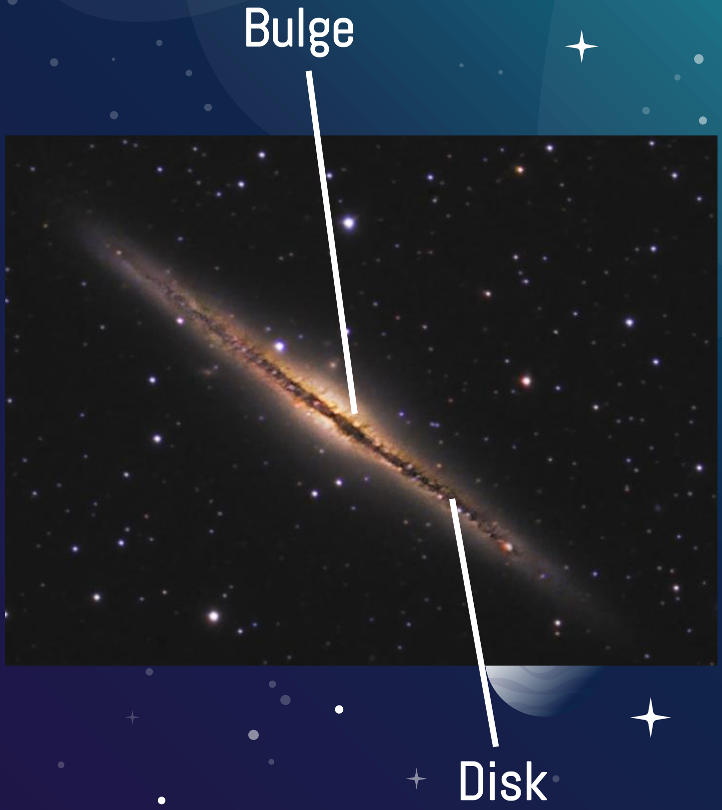
https://en.wikipedia.org/wiki/Pinwheel_Galaxy



https://commons.wikimedia.org/wiki/File:Hubble_-_de_Vaucouleurs_Galaxy_Morphology_Diagram.png

Components

- Bulge - central cluster of stars in most spiral galaxies
- Bulge consists of older and more red stars
- Disk - disk of stellar and gaseous components
- Intensive stellar formation and HII regions



https://en.wikipedia.org/wiki/NGC_891

GOAL

1. Achieve a photometric decomposition of the galaxy M101 from observations in B, V and R filter

2. Determine the bulge-to-disk ratio for the luminosities of both components

3. **Hypothesis:** very low signal-to-noise ratio near the edge of the disk

https://www.astrobin.com/405529/?nc=all

Observations

- Celestron CGE Pro 14 Inch Schmidt-Cassegrain Telescope - 11089

Observations

- CCD camera SBIG STL-11000M
  Matrix: Kodak KAI-11000M
  Dimensions: 4008 x 2672 px
  1 px = 9 x 9 µm
- Filters B, V, R, I
  (diameter 45 mm)

Theoretical basis:

Illuminance - total luminous flux incident on a surface, per unit area

Apparent magnitude - relative brightness; an *increase* in 5 magnitudes is a 100 times *decrease* in brightness

Vega - 0.03 mag in all filters

$$E = \frac{L}{4\pi r^2}$$

Pogson's law: $$\frac{E_1}{E_2} = 10^{0.4(m_2 - m_1)}$$

Theoretical basis:

- Surface brightness - apparent brightness per unit angular area of a spatial object (galaxy or nebula)

Measured in units *mag/arcsec²*

$$\mu = \frac{\Delta E}{\Delta t \Delta S \Delta \Omega} \qquad I \propto \frac{1}{r^2} \qquad \Omega = \frac{S}{r^2}$$

Surface brightness **does not** depend on the distance to the object.

Theoretical basis:

Different change in surface brightness in disk and bulge of galaxy (as a function of the mean radius)

Disk law: $\mu(r) = \mu_0 + 1.0857\frac{r}{h}$

h – scale factor

De Vaucouleurs law: $\mu(r) = \mu_e + 8.32678\left[\left(\frac{r}{r_e}\right)^{\frac{1}{4}} - 1\right]$

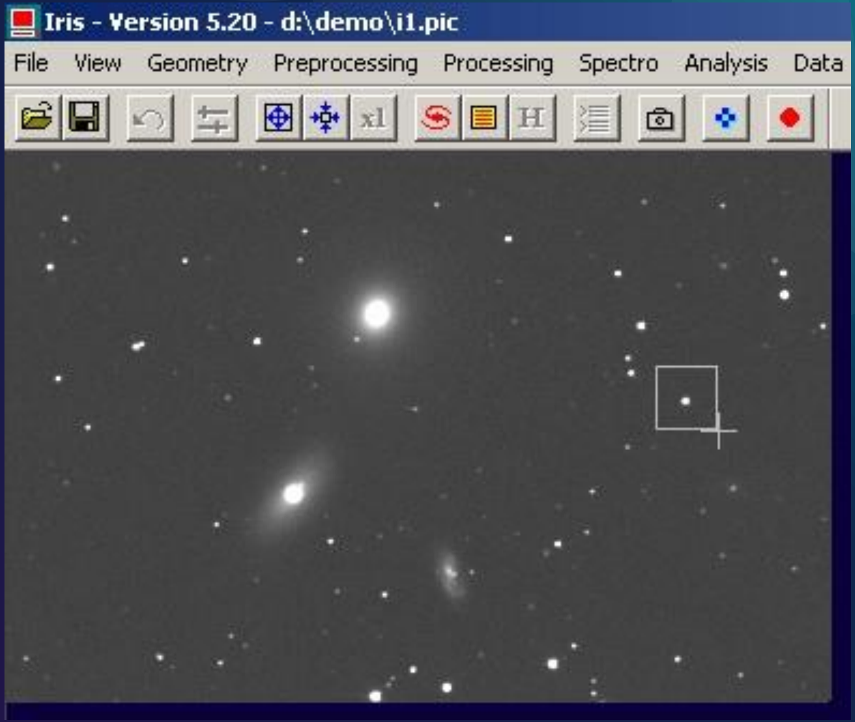
r_e – effective radius

μ_e – surface brightness at $r = r_e$

http://www.astro.spbu.ru/staff/resh/Lectures/lec1.pdf

Software and data used

- Images of the galaxy from observation in B, V and R filters
- IRIS software for processing images
- Microsoft Excel for data analysis

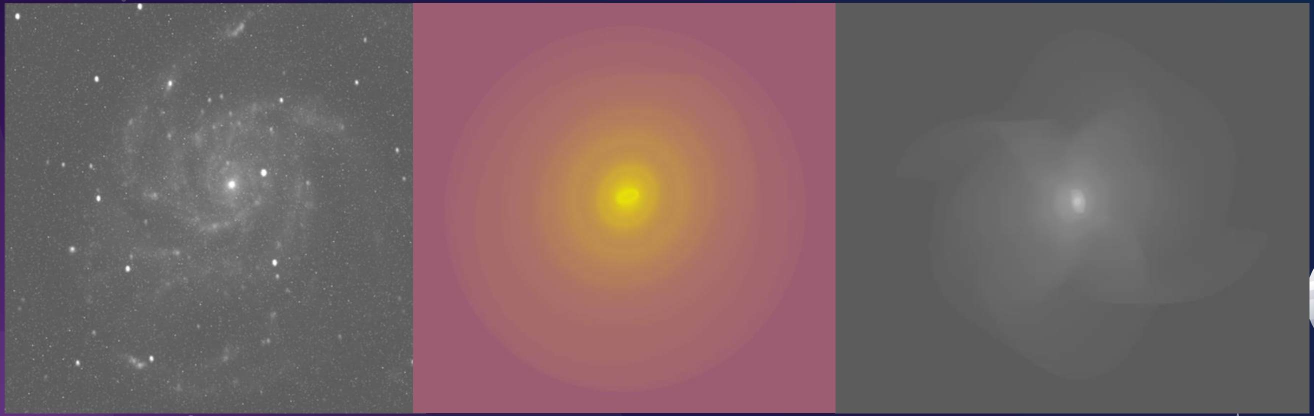


http://www.astrosurf.com/buil/iris-software.html
https://www.versionmuseum.com/history-of/microsoft-excel

Isophotes and elliptical fitting

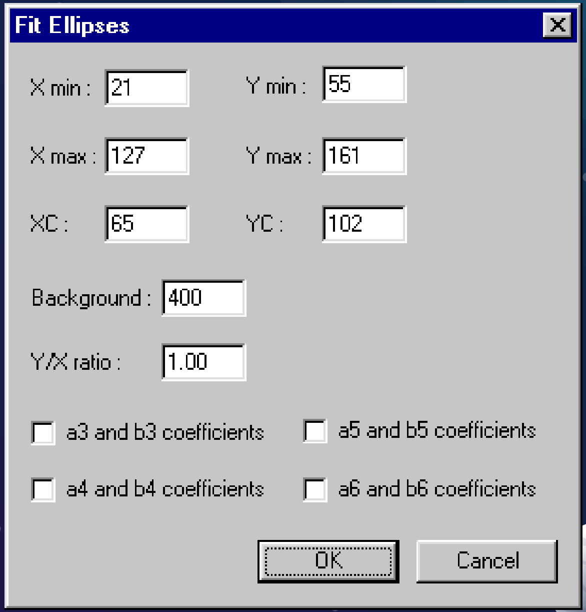
Isophote - curve connecting points of equal brightness

Isophotes are defined by the Fourier expansion of deviations from the elliptical contour



Isophotes and elliptical fitting

- Each ellipse is fitted onto the galaxy and centred
- A Fourier series decomposition is applied with 3rd, 4th, 5th and 6th Fourier coefficients



Experiment

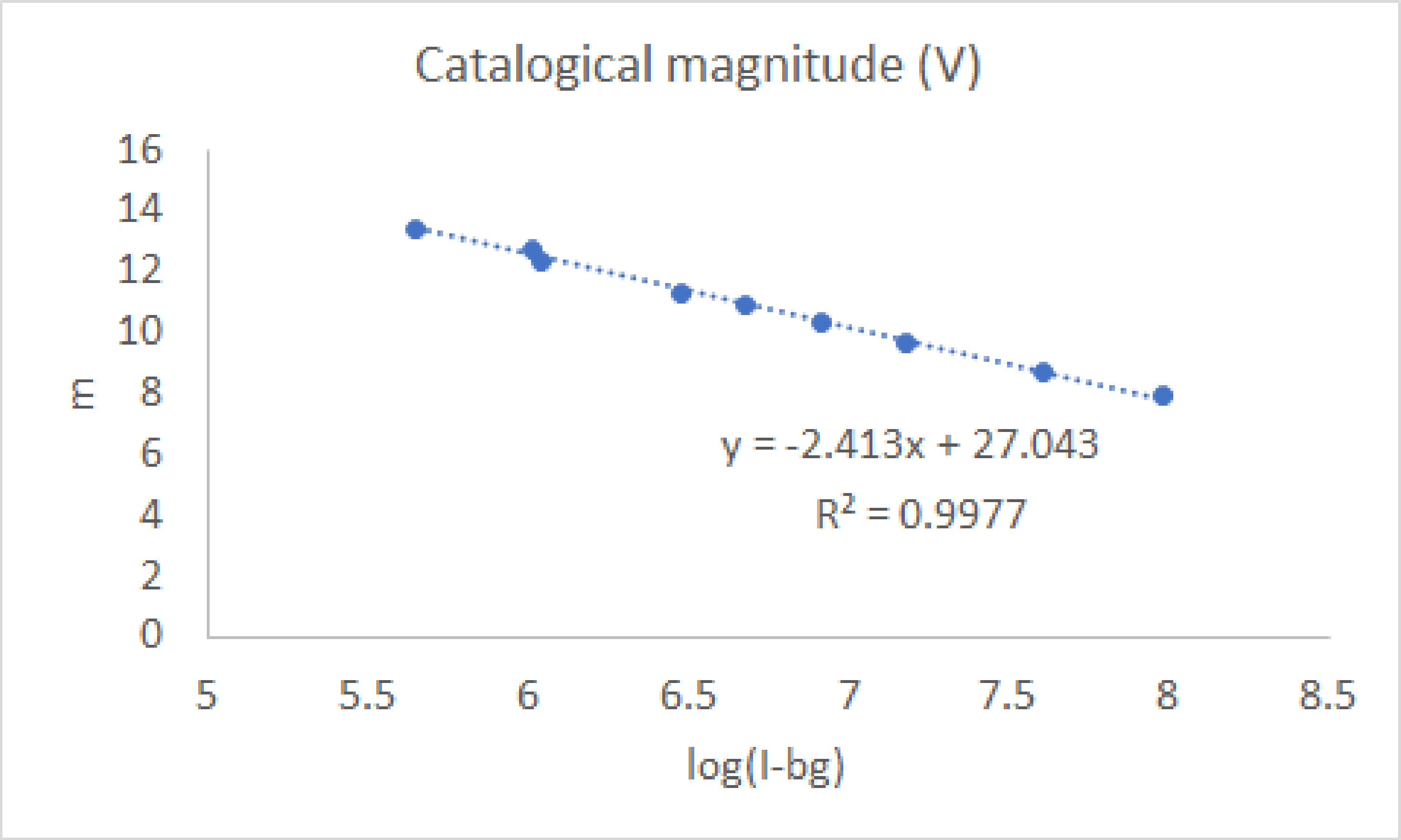
1. Estimating background intensity (per pixel)
2. Taking the mean intensity of the background (this intensity needs to be subtracted to separate the galaxy from the rest of the image)
3. Standard deviation of the intensity - *noise* in the background

	B	V	R
Standard deviation	6.01	5.45	6.12

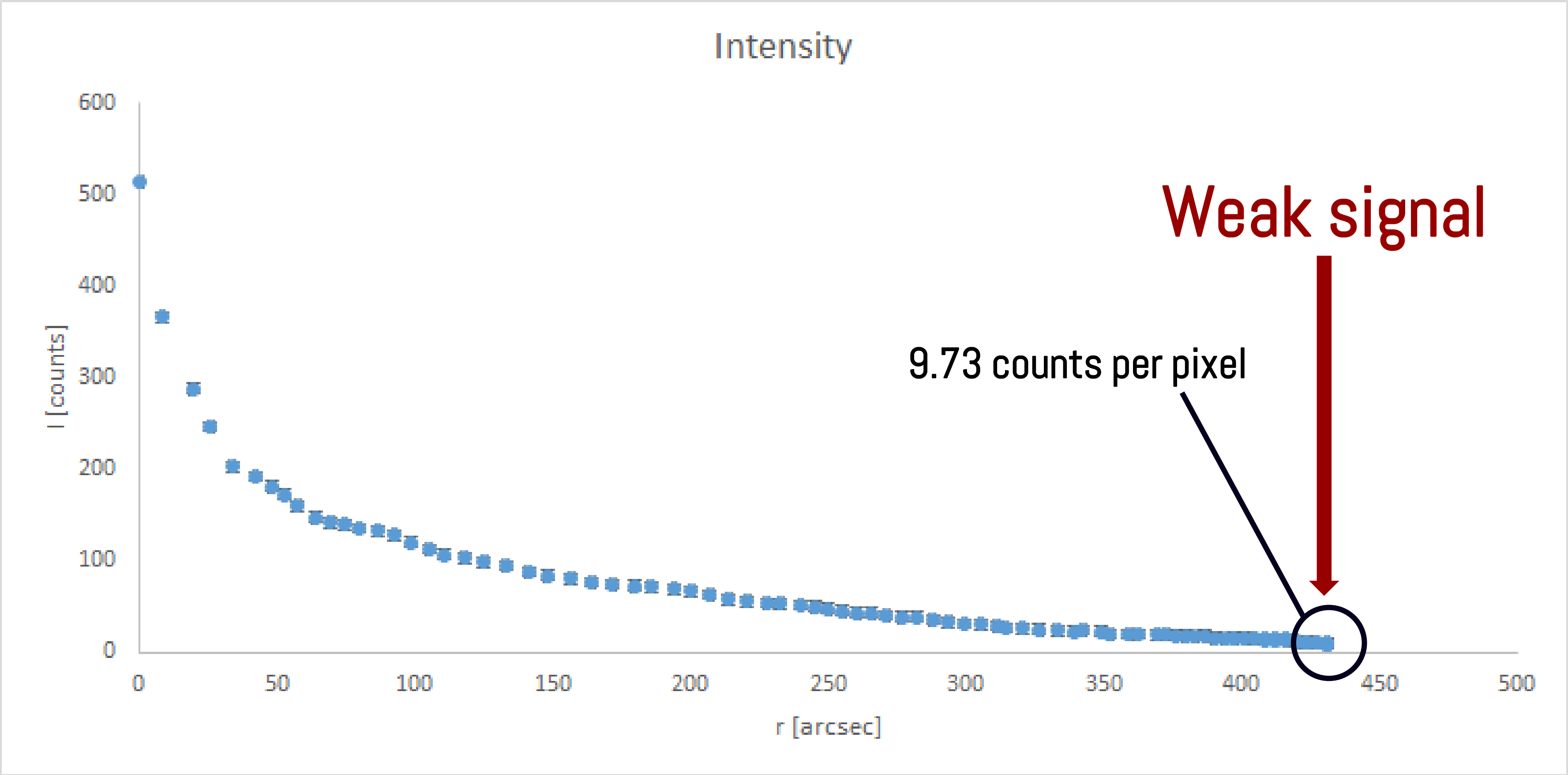
Experiment

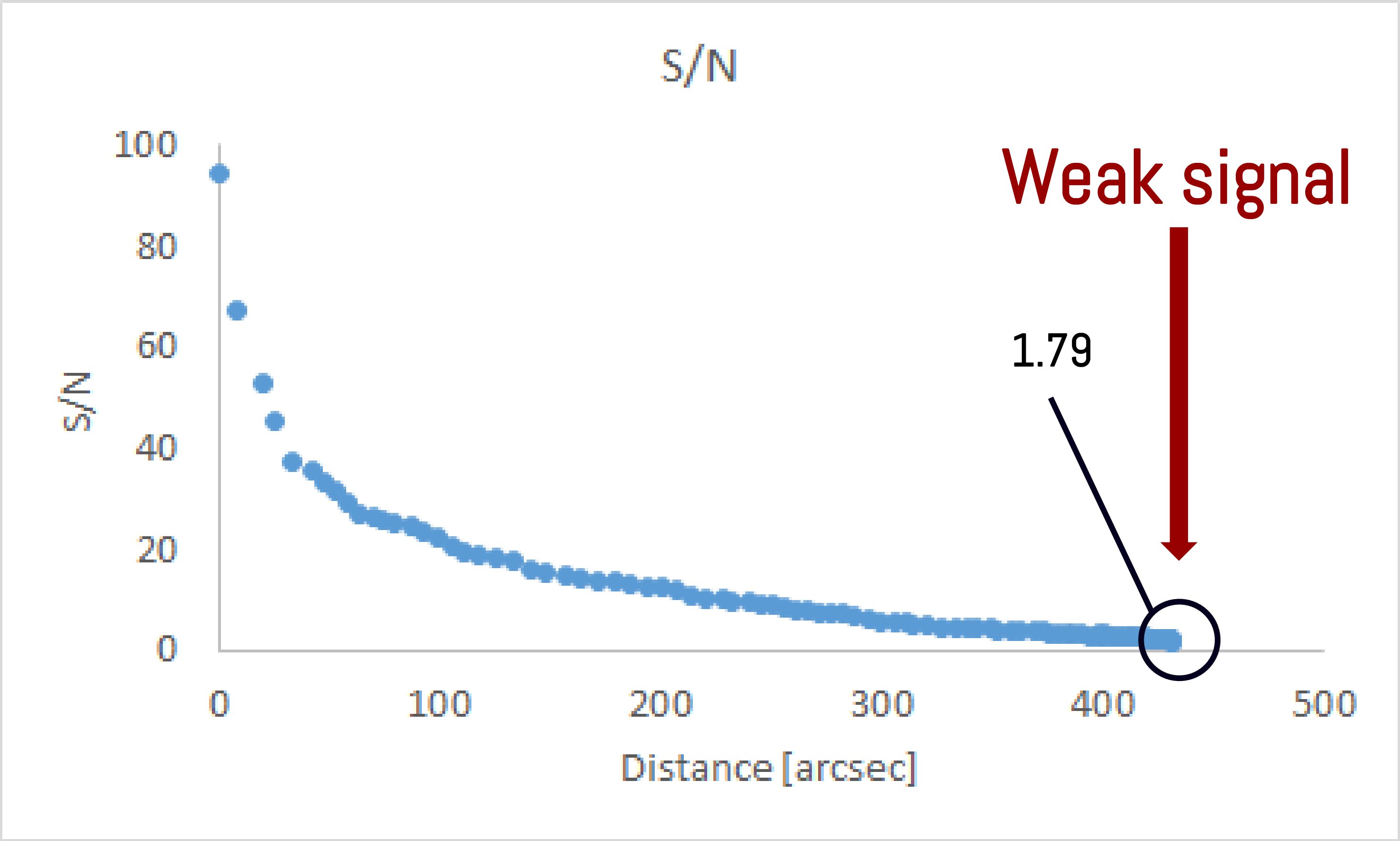
- Fitting isophotes (ellipses with the same surface brightness) to the image of the galaxy with IRIS
- Calibrating the instrumental surface brightness to the catalogue brightness
- The scale of the image is 1px = $(1.07)^2$ arcsec^2

$$\mu_{\text{instr}} = -2.5\log(I) + 5\log(1.07)$$
$$m_{\text{cat}} = k(-2.5\log(I)) + D$$
$$\mu_{\text{cat}} = k\mu_{\text{instr}} + D$$



Catalogical magnitude (V)
m
16
14
12
10
8
6
4
2
0
5
5.5
6
6.5
7
7.5
8
8.5
log(I-bg)
y = -2.413x + 27.043
R² = 0.9977


Intensity
Weak signal
9.73 counts per pixel
600
500
400
300
200
100
0
I [counts]
0
50
100
150
200
250
300
350
400
450
500
r [arcsec]

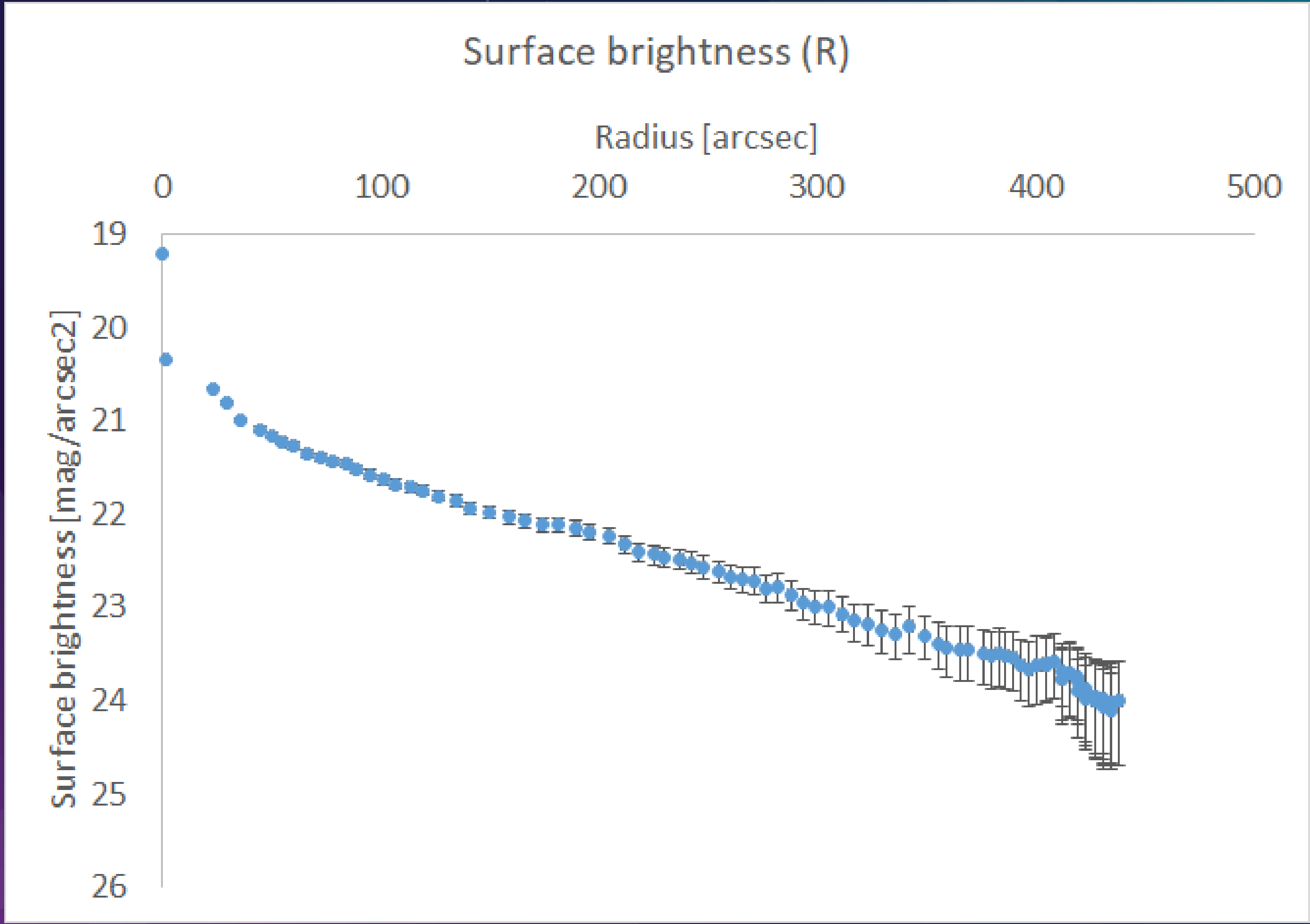

S/N
100
80
60
40
20
0
S/N
0
100
200
300
400
500
Distance [arcsec]
Weak signal
1.79

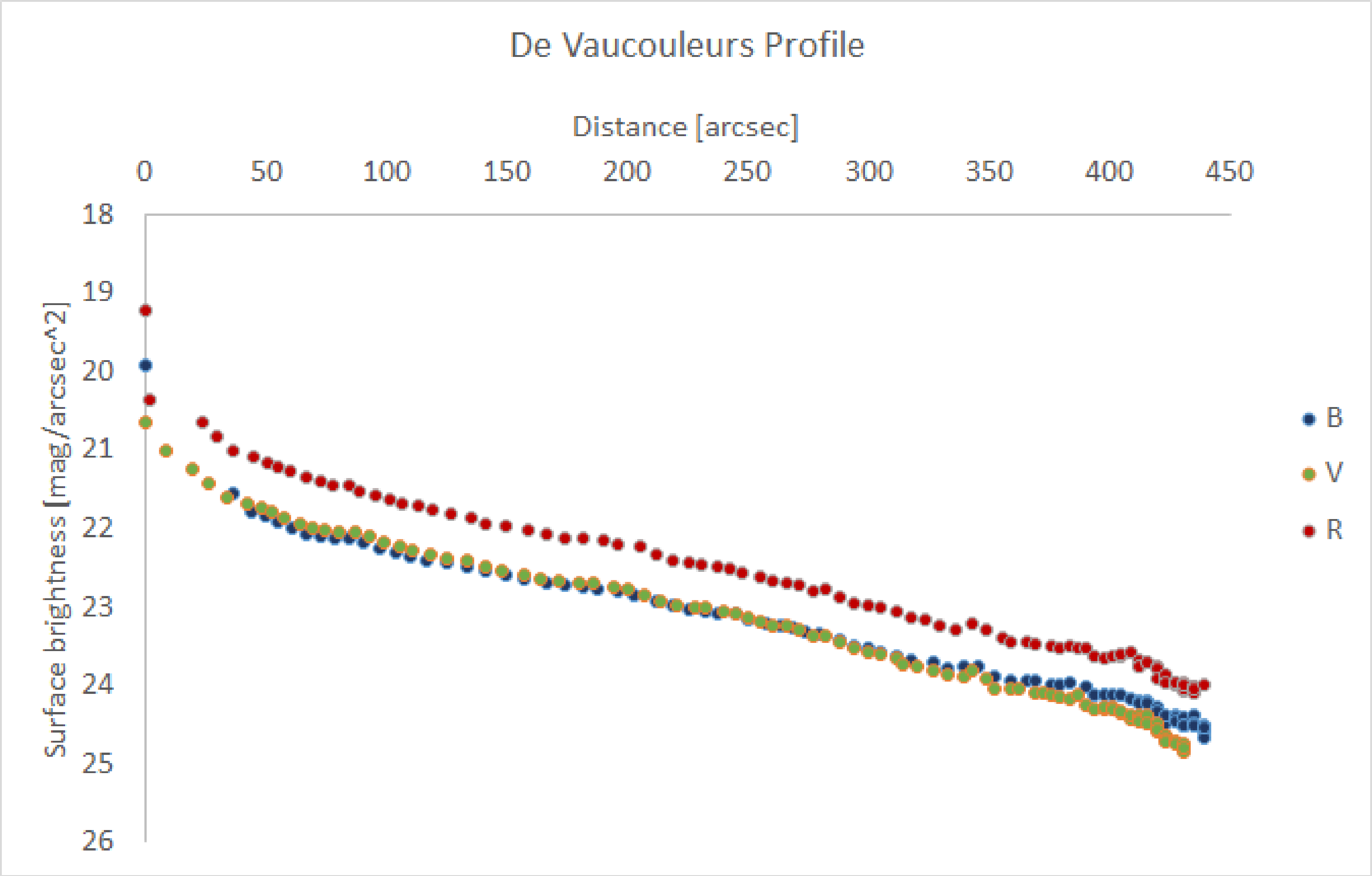
Experiment

- De Vaucouleurs profile of the galaxy - graph of surface brightness as a function of distance from the centre

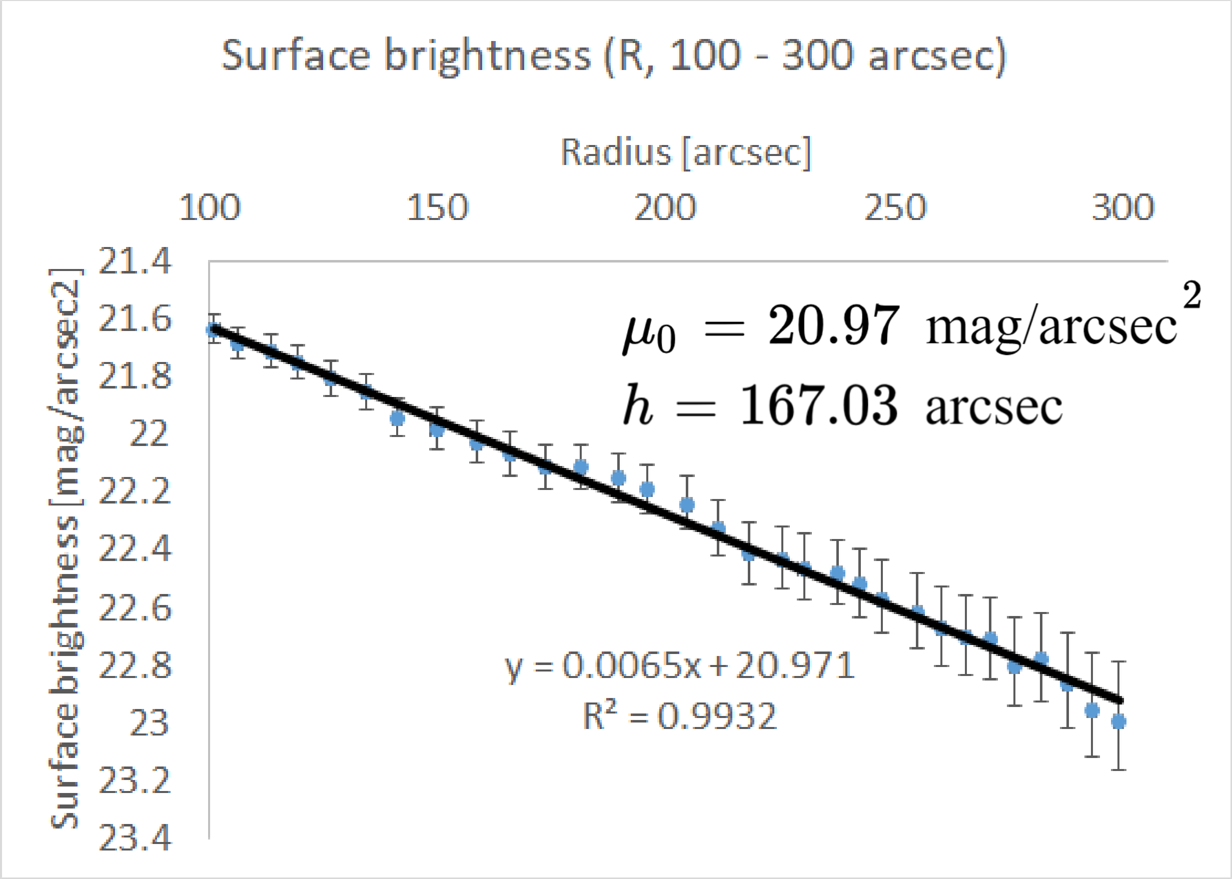
- Separating bulge from disk - magnitude between 100 - 300 arcsec distance from the centre (disk law)

- Interpolating the surface brightness of the bulge (de Vaucouleurs law)

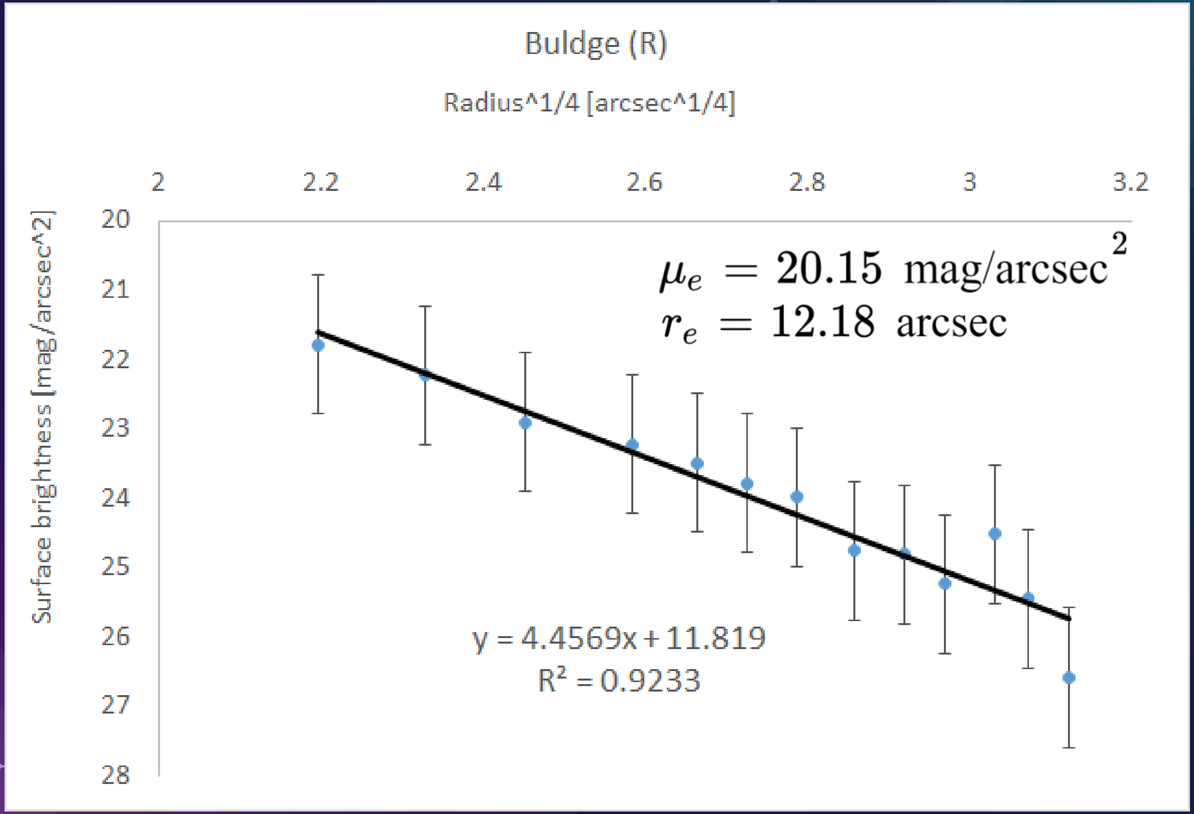

Surface brightness (R)
Radius [arcsec]
0
100
200
300
400
500
Surface brightness [mag/arcsec2]
19
20
21
22
23
24
25
26


De Vaucouleurs Profile
Distance [arcsec]
0
50
100
150
200
250
300
350
400
450
18
19
20
21
22
23
24
25
26
Surface brightness [mag/arcsec^2]
B
V
R

Disk law: $\mu(r) = \mu_0 + 1.0857\frac{r}{h}$



De Vaucouleurs law: $\mu(r) = \mu_e + 8.32678\left[\left(\frac{r}{r_e}\right)^{\frac{1}{4}} - 1\right]$



Integral parameters

- Calculating the absolute magnitude of the bulge and disk

$$M_{\text{bulge}} = \mu_e - 5\log(r_e)[\text{kpc}] - 2.5\log(1-\varepsilon) - 39.961$$
$$M_{\text{disk}} = \mu_0 - 5\log(h)[\text{kpc}] - 38.57$$

- Calculating the bulge-to-disk ratio using Pogson's law

$$\frac{B}{D} = 10^{0.4\left(M_{\text{disk}} - M_{\text{bulge}}\right)}$$

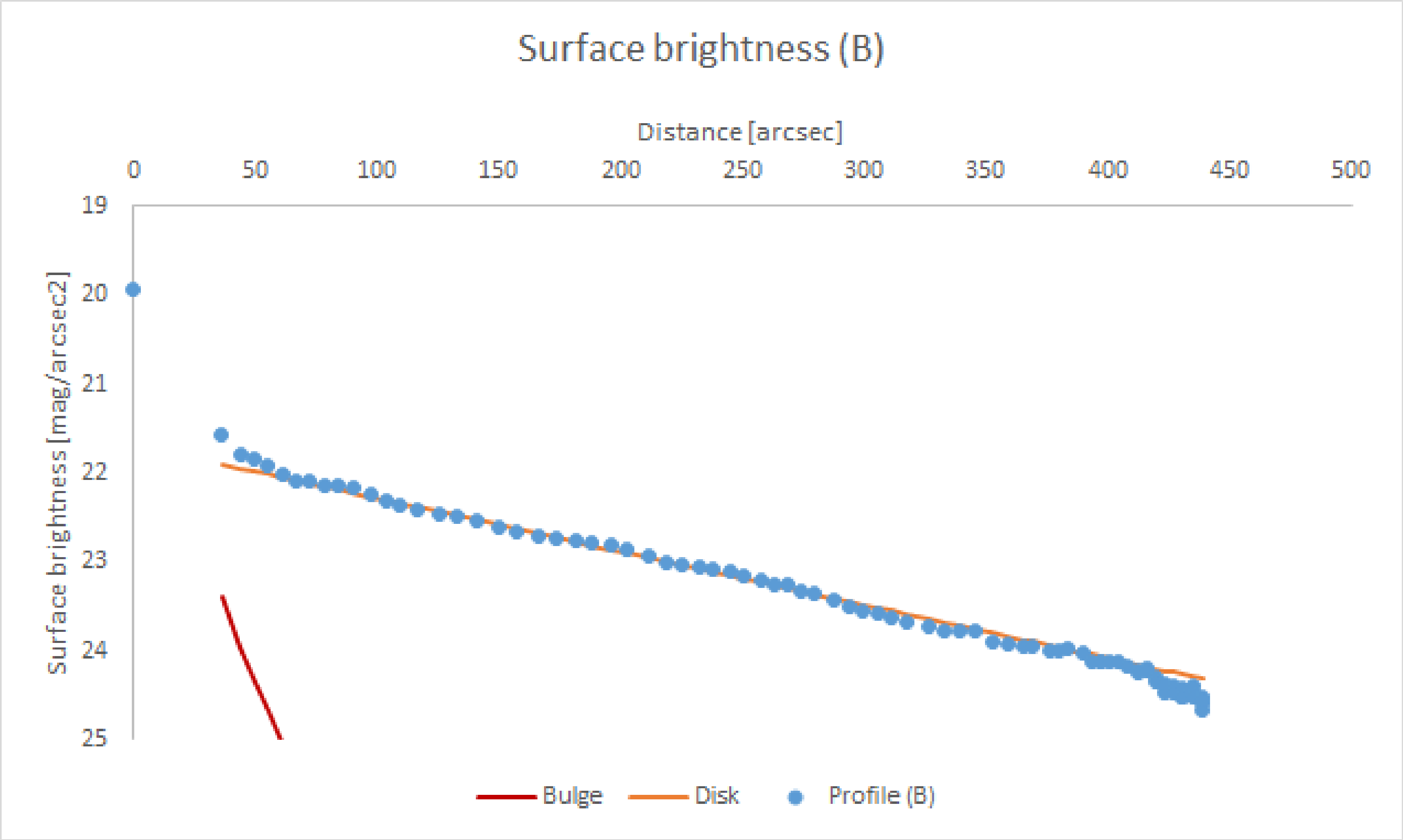
Results

Filter	B	V	R
Bulge [mag]	-17.14	-16.31	-17.62
Disk [mag]	-20.64	-20.61	-21.17
B/D	0.039	0.019	0.038

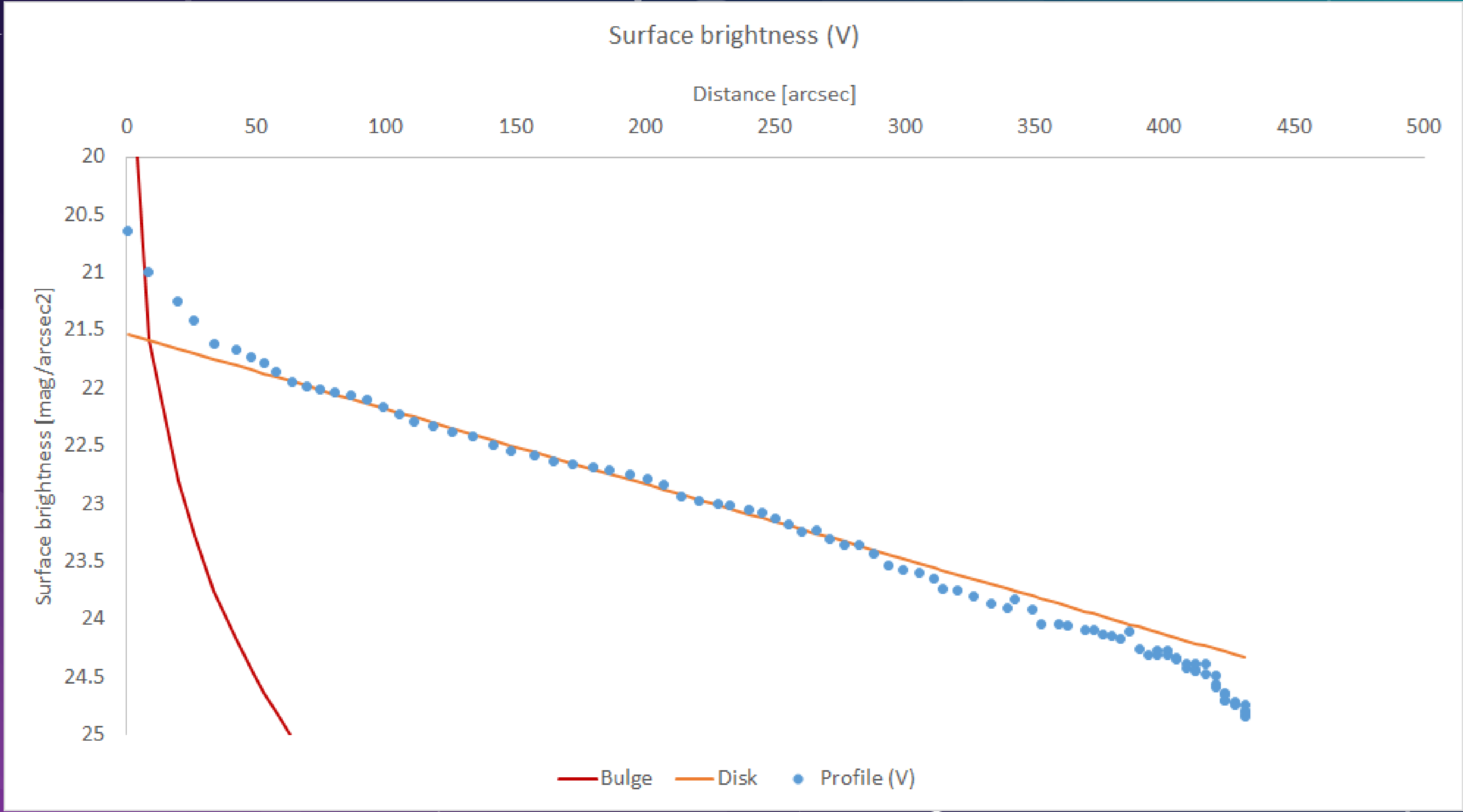
Signal-to-noise ratio

- The intensity per pixel - signal
- The noise - standard deviation of the background

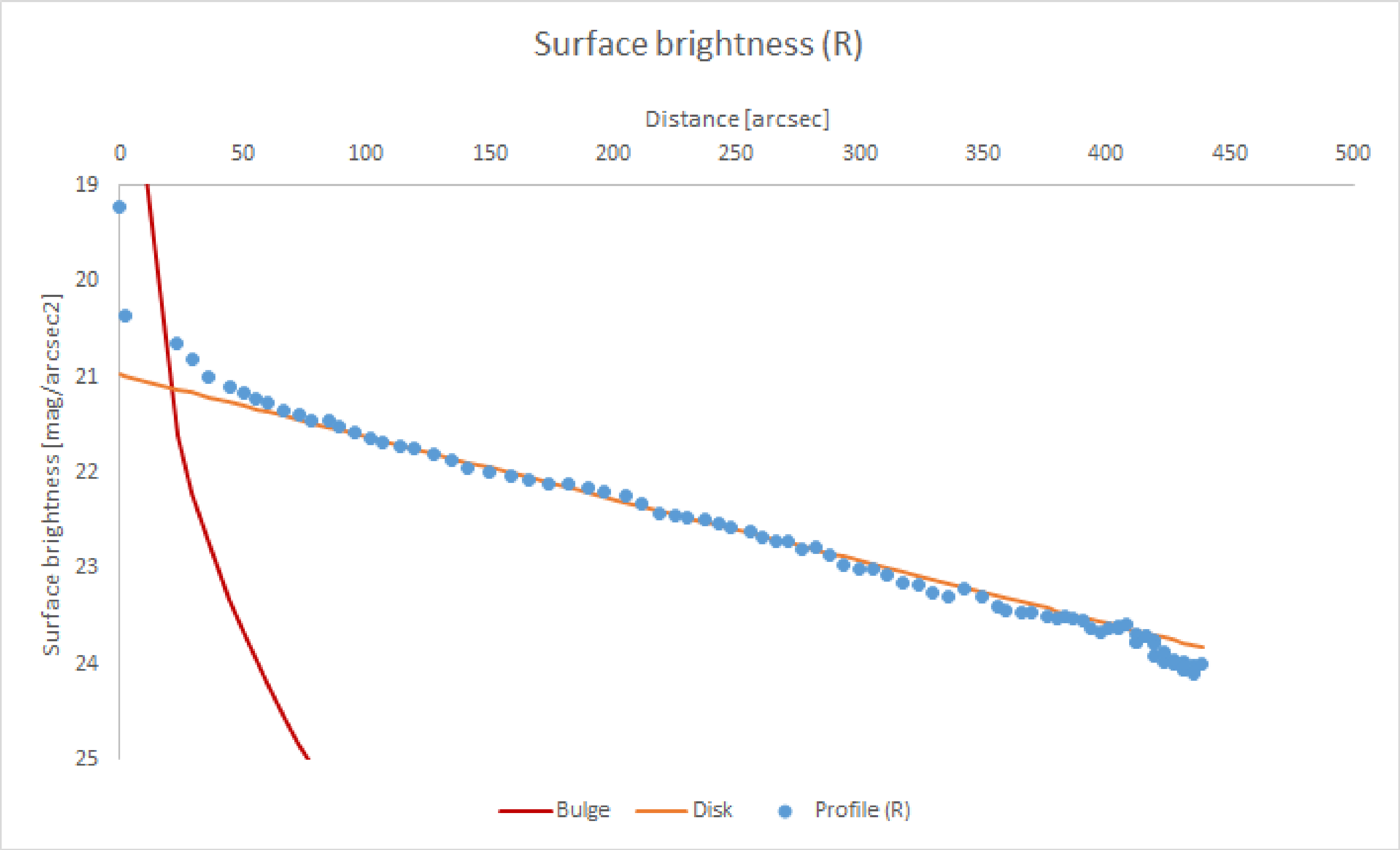
S/N	B	V	R
Centre of bulge	2114.08	94.57	219.30
Edge of disk	2.30	1.79	2.23



Surface brightness (B)
Distance [arcsec]
0
50
100
150
200
250
300
350
400
450
500
19
20
21
22
23
24
25
Surface brightness [mag/arcsec2]
Bulge
Disk
Profile (B)



Surface brightness (V)
Distance [arcsec]
0
50
100
150
200
250
300
350
400
450
500
20
20.5
21
21.5
22
22.5
23
23.5
24
24.5
25
Surface brightness [mag/arcsec2]
Bulge
Disk
Profile (V)

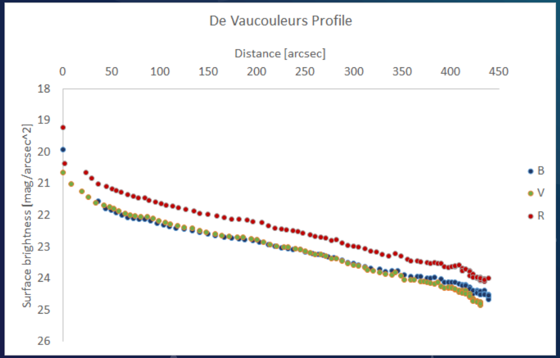

Surface brightness (R)
Distance [arcsec]
0
50
100
150
200
250
300
350
400
450
500
Surface brightness [mag/arcsec2]
19
20
21
22
23
24
25
Bulge
Disk
Profile (R)

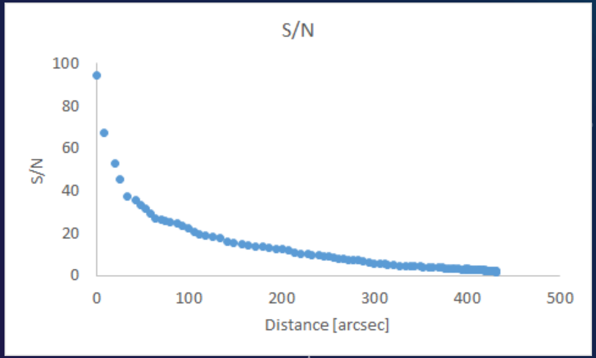
Agreement with theory

- The surface brightness drops linearly near the middle of the disk, consistent with the theory

- The surface brightness of the bulge drops linearly with respect to $r^{1/4}$

- The signal-to-noise ratio drops rapidly near the edge of the galaxy, where it is significantly low





Conclusion

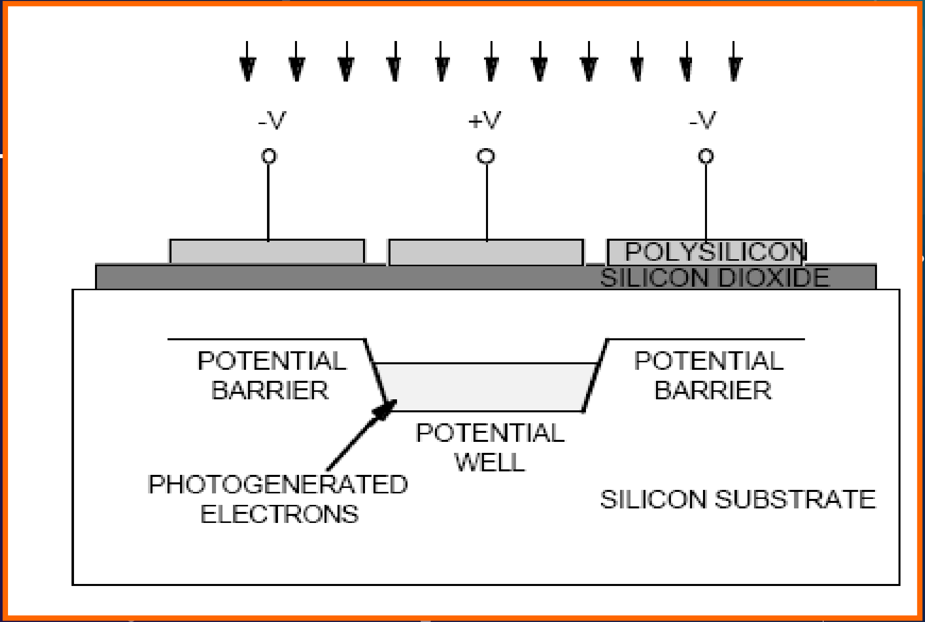
The surface brightness of the bulge and disk are well-described by the laws of the disk and de Vaucouleurs law

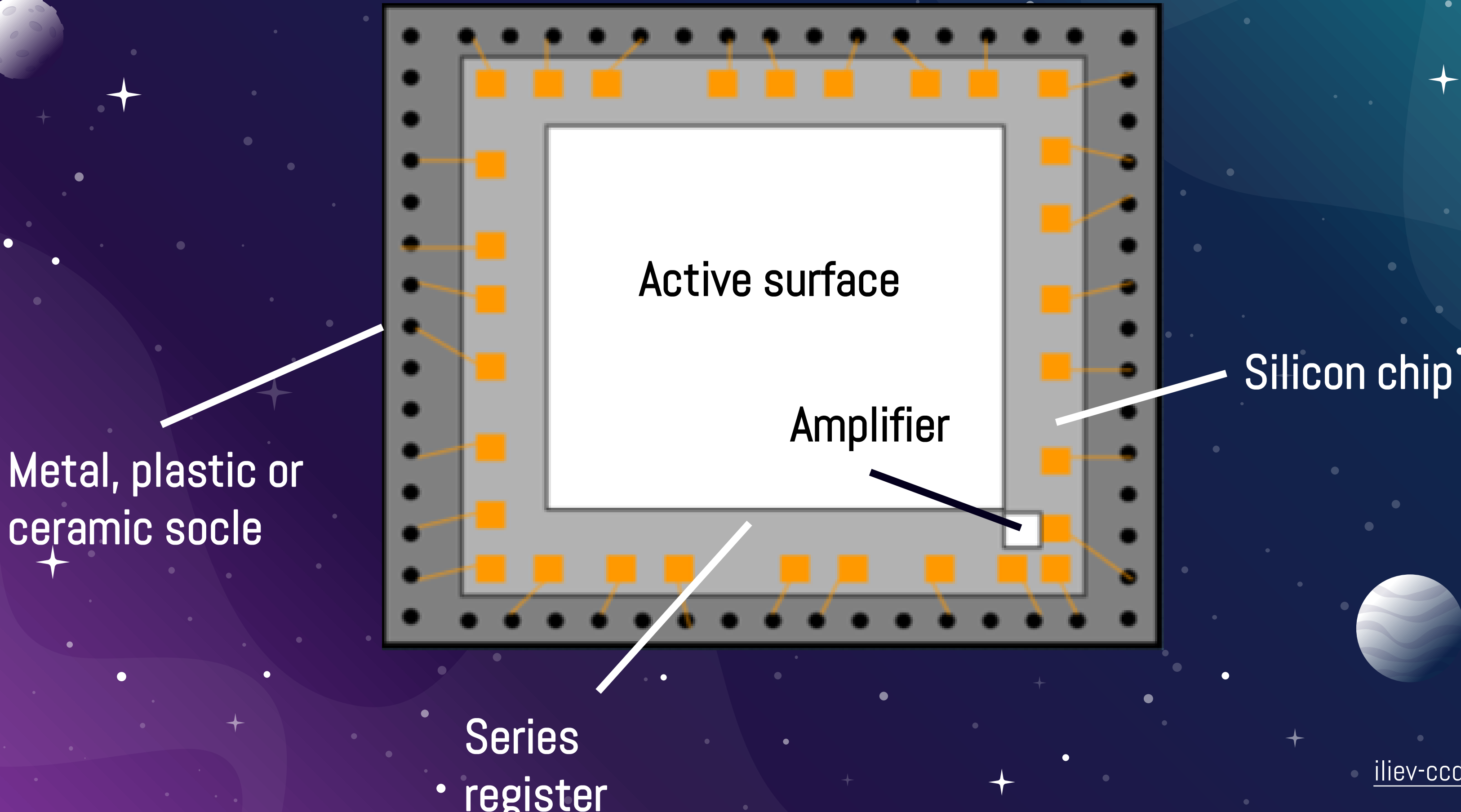
The experiment demonstrates discerning very weak signals, especially near the edge of the galactic disk

Thank You!

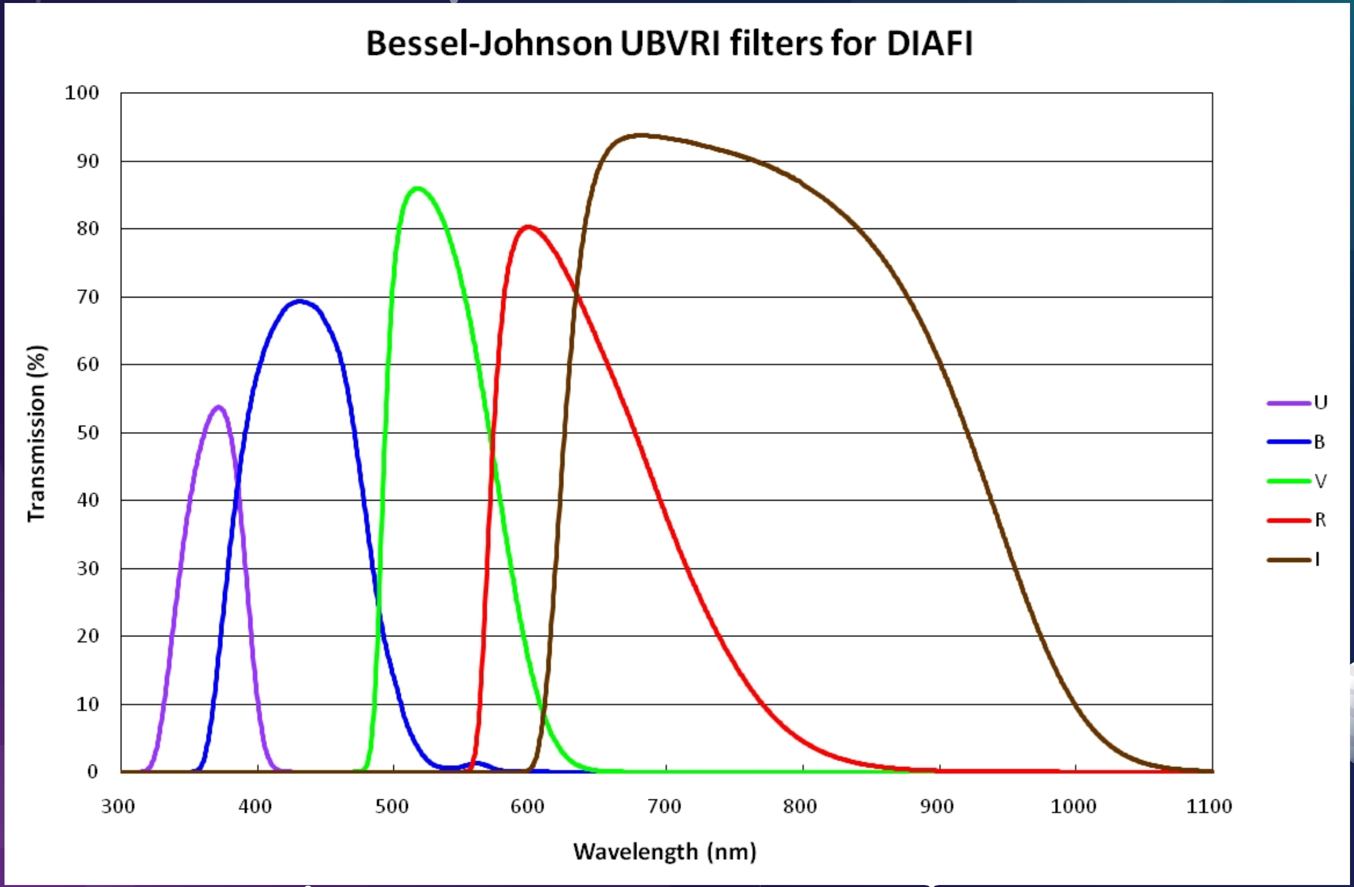
CCD Camera

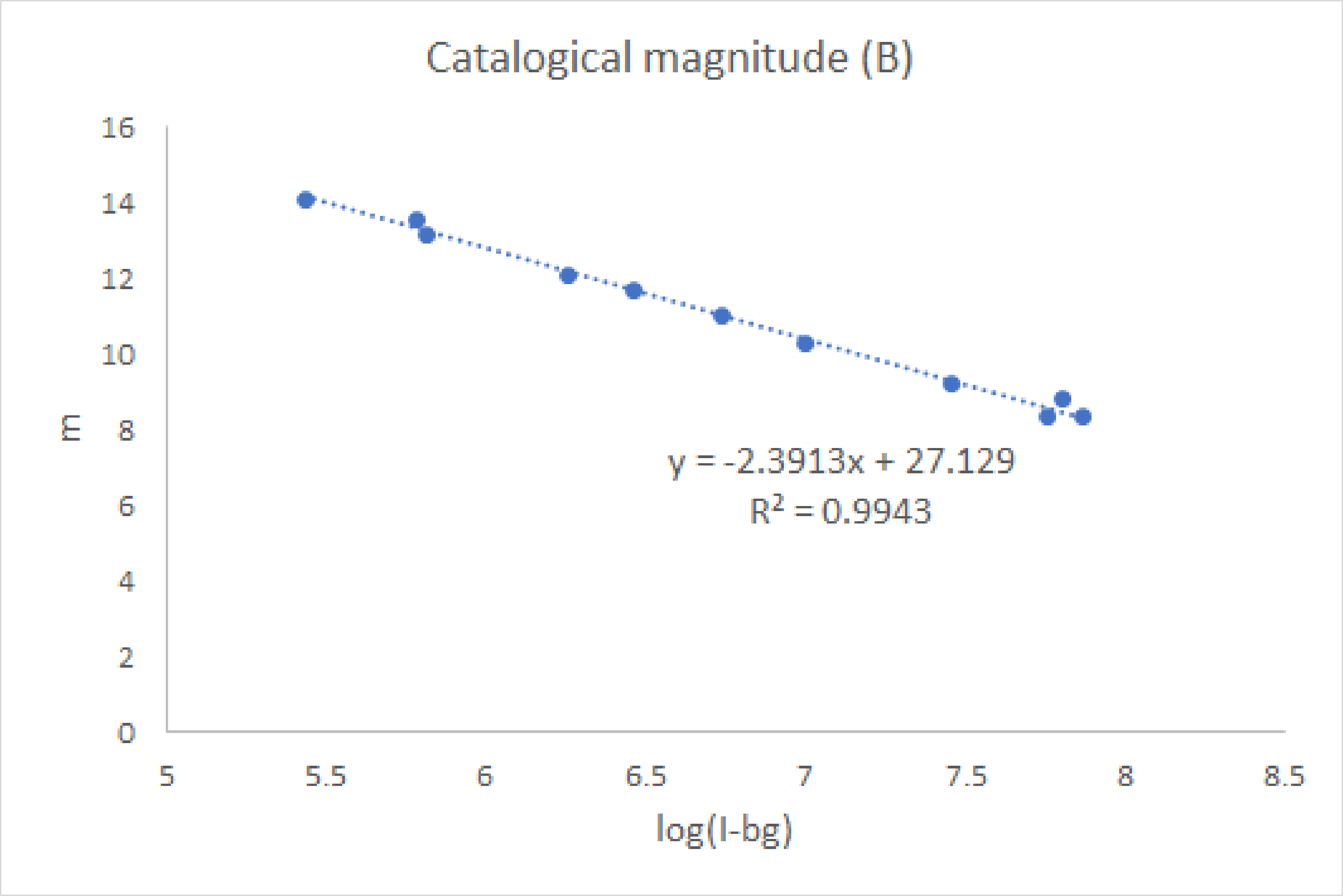
- Charged-Coupled Device
- Registered photones remove electrons (charges) from the CCD-matrix with a certain probability (quantum efficiency)
- Each pixel contains
  - 2^{16} -1 = 65 535 counts
- Images of CCD-cameras are monochromatic



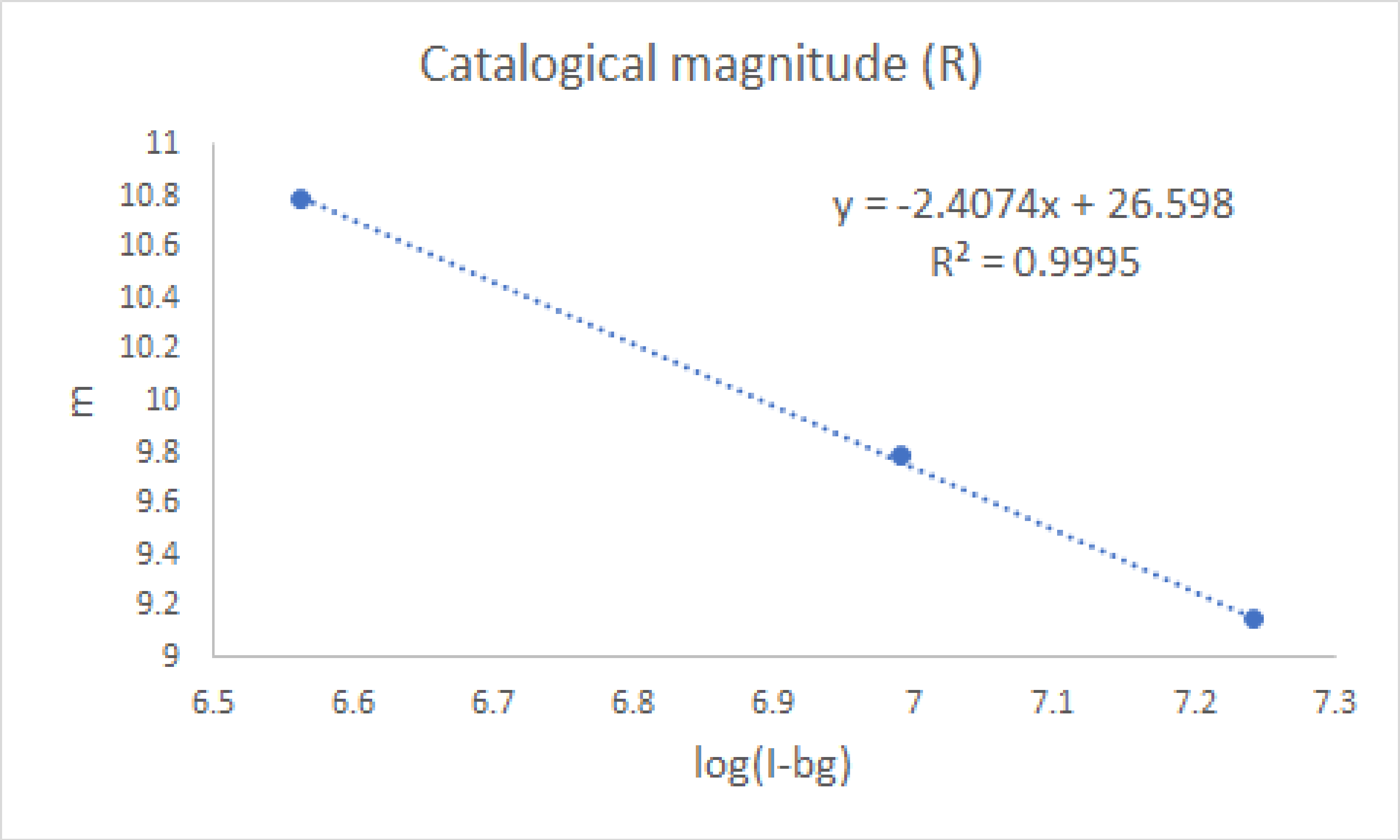

Active surface
Amplifier
Silicon chip
Metal, plastic or ceramic socle
Series register

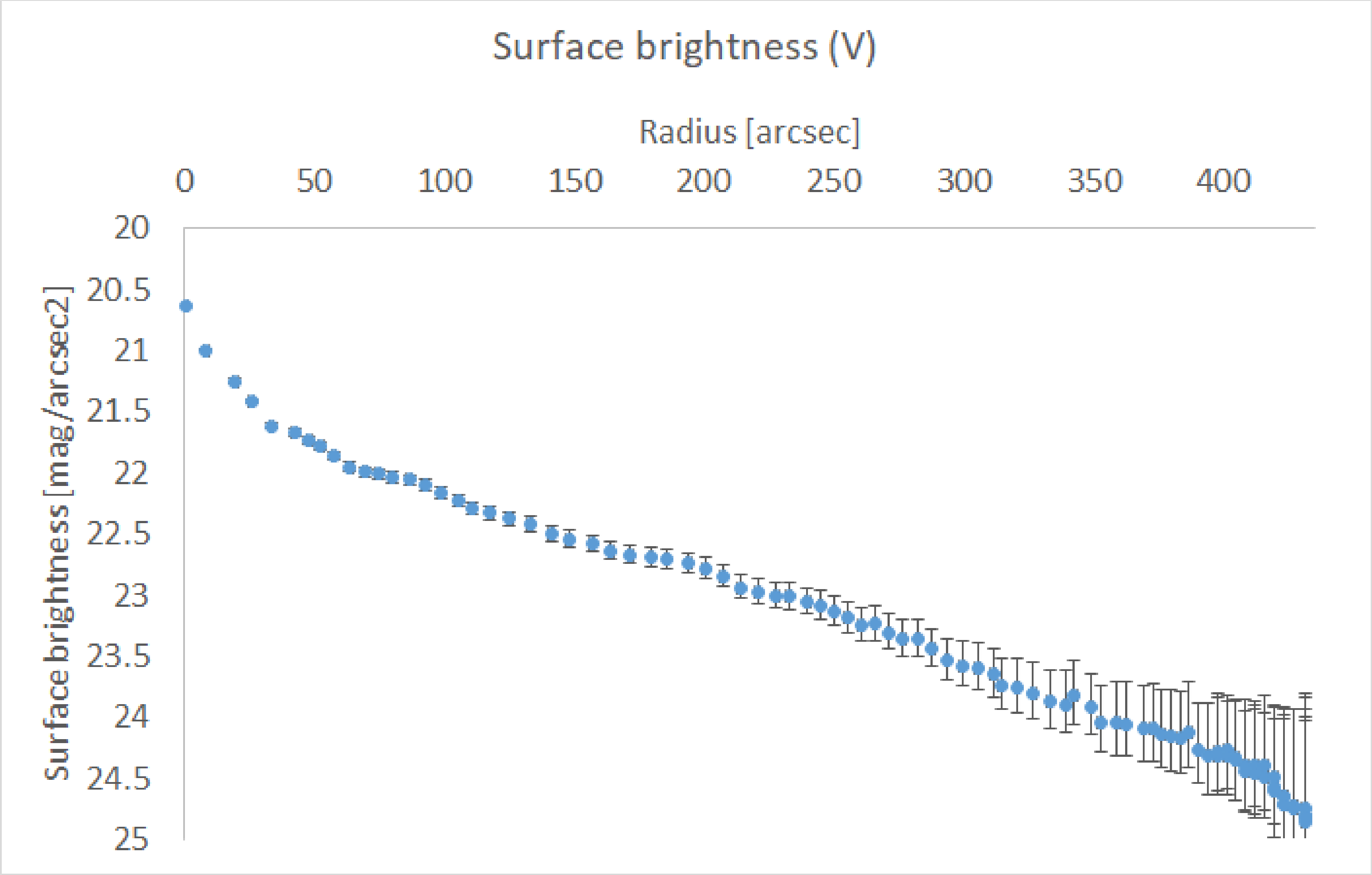
UBVRI transmissivity curves



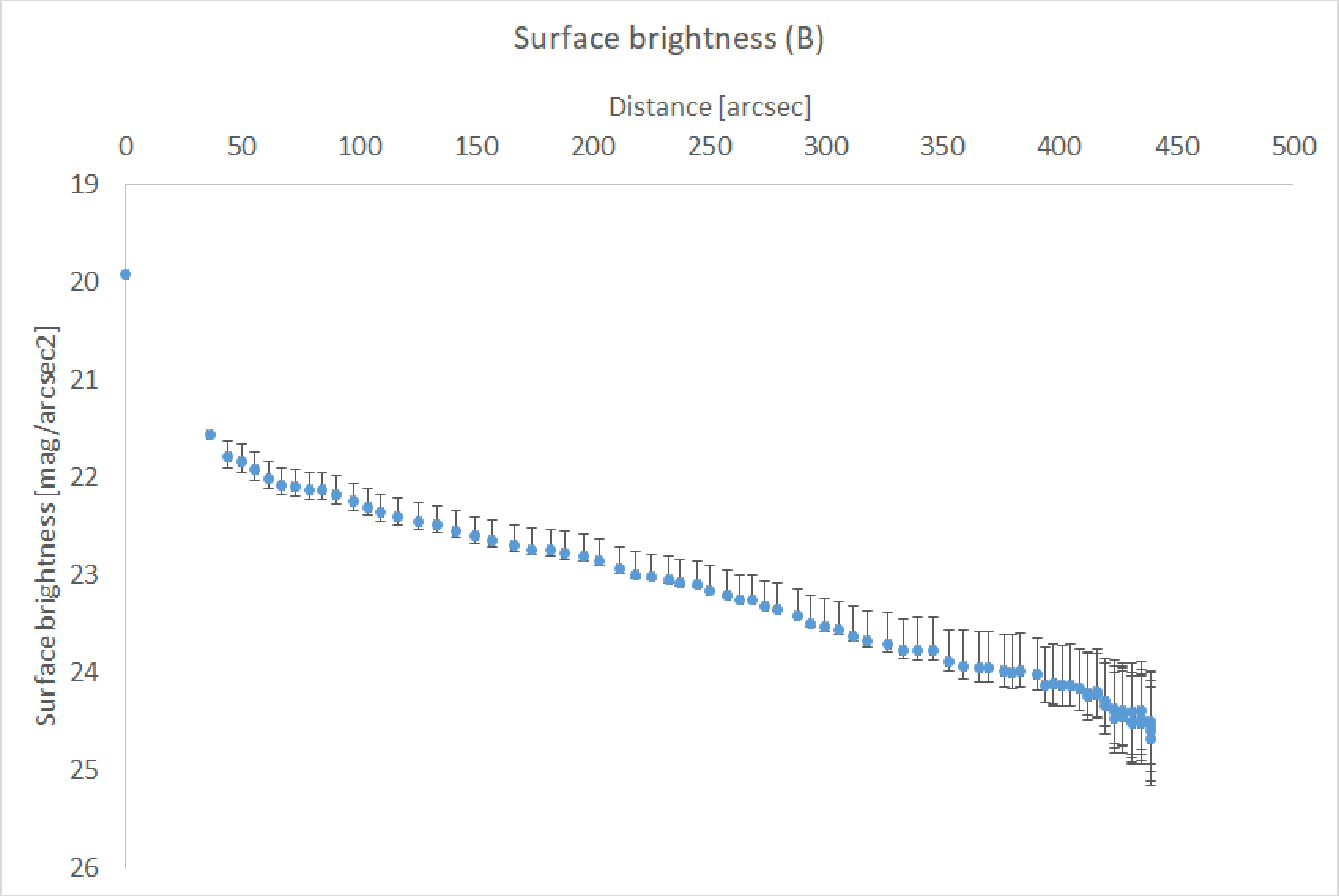


Catalogical magnitude (B)
16
14
12
10
8
6
4
2
0
m
5
5.5
6
6.5
7
7.5
8
8.5
log(I-bg)
y = -2.3913x + 27.129
R² = 0.9943


Catalogical magnitude (R)
y = -2.4074x + 26.598
R² = 0.9995
m
11
10.8
10.6
10.4
10.2
10
9.8
9.6
9.4
9.2
9
6.5
6.6
6.7
6.8
6.9
7
7.1
7.2
7.3
log(I-bg)



Surface brightness (V)
Radius [arcsec]
0
50
100
150
200
250
300
350
400
20
20.5
21
21.5
22
22.5
23
23.5
24
24.5
25
Surface brightness [mag/arcsec2]



Surface brightness (B)
Distance [arcsec]
0
50
100
150
200
250
300
350
400
450
500
19
20
21
22
23
24
25
26
Surface brightness [mag/arcsec2]

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\left(\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}\right)$$

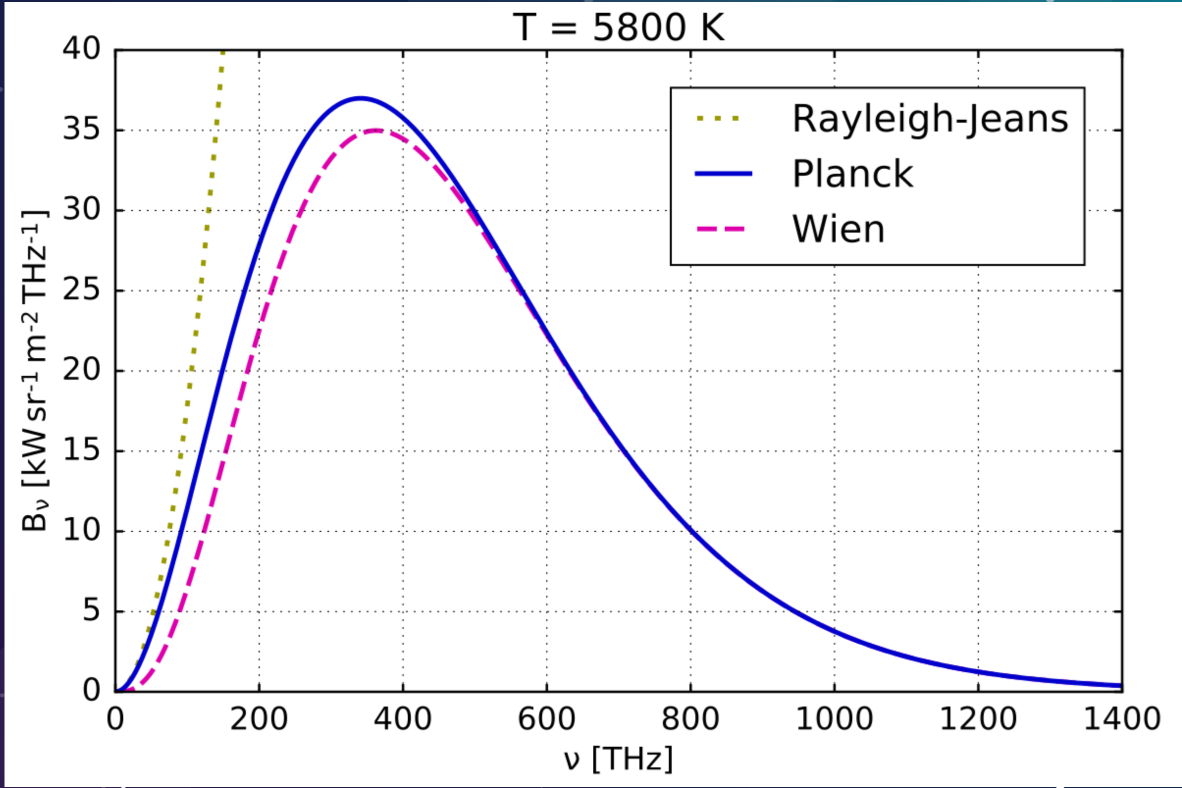
$$L = S\sigma T^4$$

$$L = 4\pi R^2 \sigma T^4$$

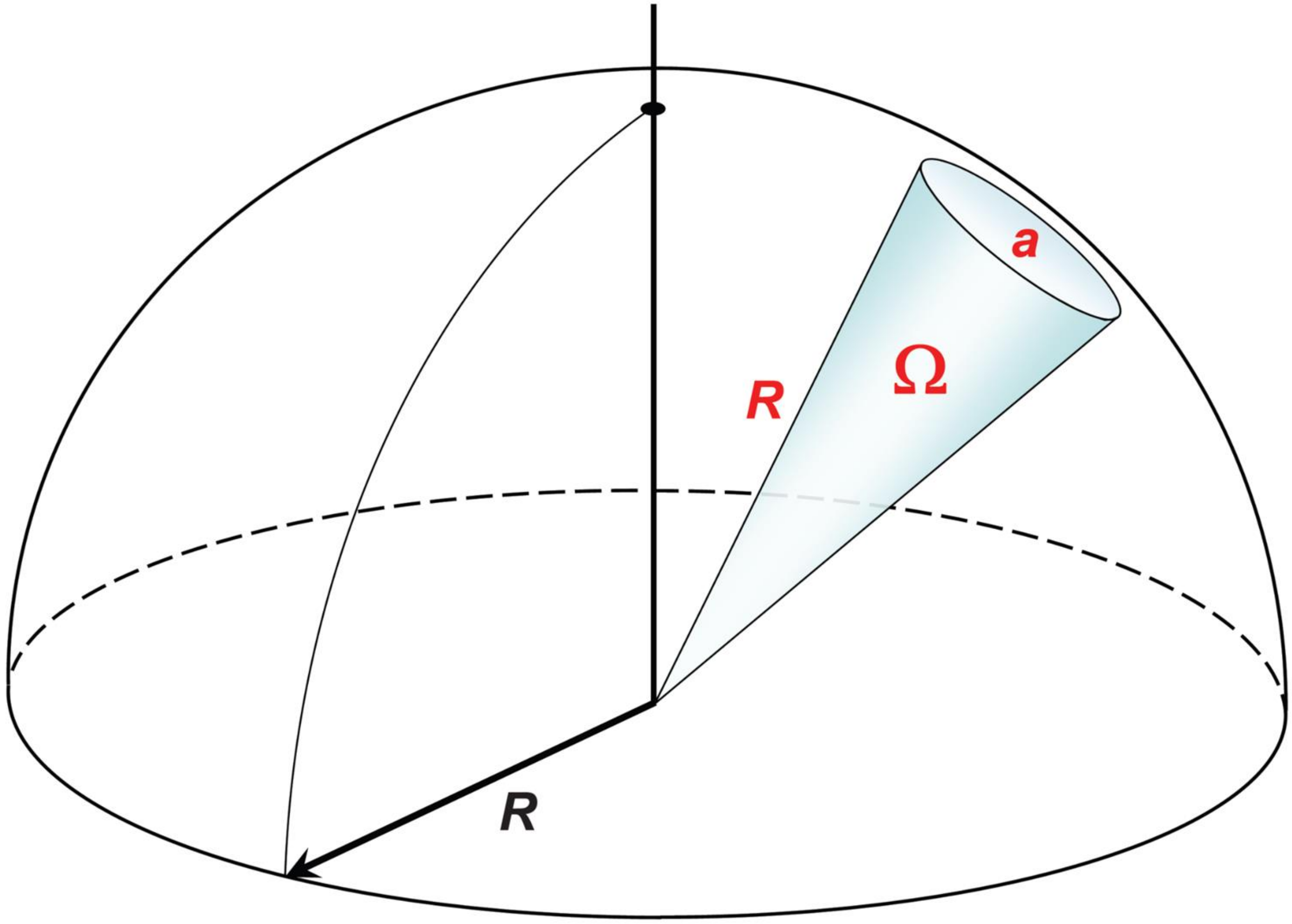
$$\sigma = 5.67 \cdot 10^{-8} \ \mathrm{W/m^2K^4}$$

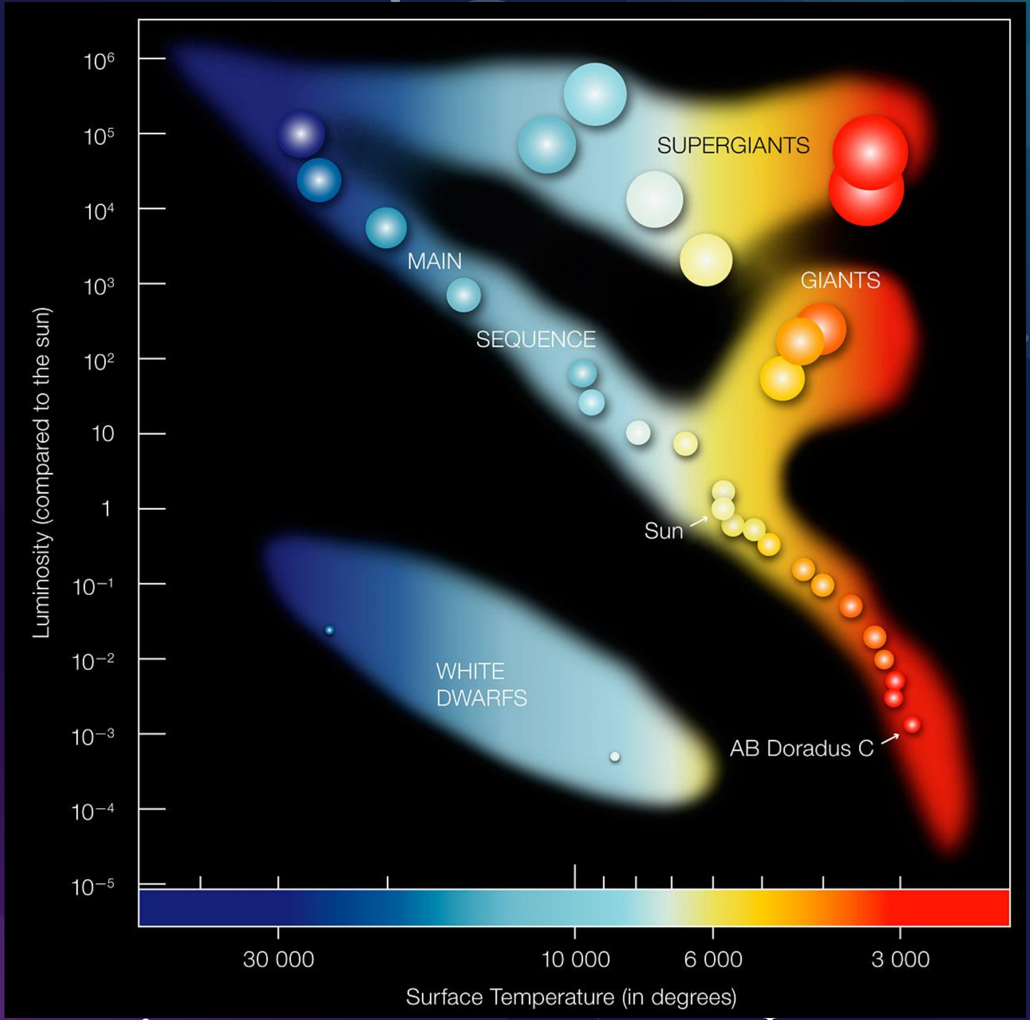
$$\lambda_{\mathrm{max}} T = b$$

$$b = 2.9 \cdot 10^{-3} \ \mathrm{mK}$$



https://en.wikipedia.org/wiki/Rayleigh%E2%80%93Jeans_law


a
Ω
R
R



Luminosity (compared to the sun)
10^6
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
SUPERGIANTS
MAIN
SEQUENCE
GIANTS
Sun
WHITE
DWARFS
AB Doradus C
30 000
10 000
6 000
3 000
Surface Temperature (in degrees)