6. Tall Towers

A tower is built by stacking rectangular bricks on top of each other. Some people argue that the maximum height of the tower is limited by the human skill to place the bricks gently; others may say that the limiting factor is non-perfect shape of the bricks. Perform experiments to outline the factors that limit the maximum height of such a tower.

Interpreting the Problem

A tower is built by stacking rectangular bricks on top of each other. Some people argue that the maximum height of the tower is limited by the human skill to place the bricks gently; others may say that the limiting factor is non-perfect shape of the bricks. **Perform experiments to outline the factors that limit the maximum height of such a tower.**

A tower is a tall structure with a **higher proportional height in relation to its lateral dimensions.**

For this problem, I will perform experiments to propose a limit for the height of this tower and investigate the factors that affect its maximum height.

Variables

Human skill - this can greatly impact the degree of perfection that humans are able to place the bricks gently so that the tallest tower can be built

Friction of the brick surface - increased friction can make it more difficult for bricks to fall

Material strength of bricks - this can increase or decrease the maximum height of the tower that can be built

Centre of mass - this can increase or decrease the angle at which the tower needs to tilt for it to fall

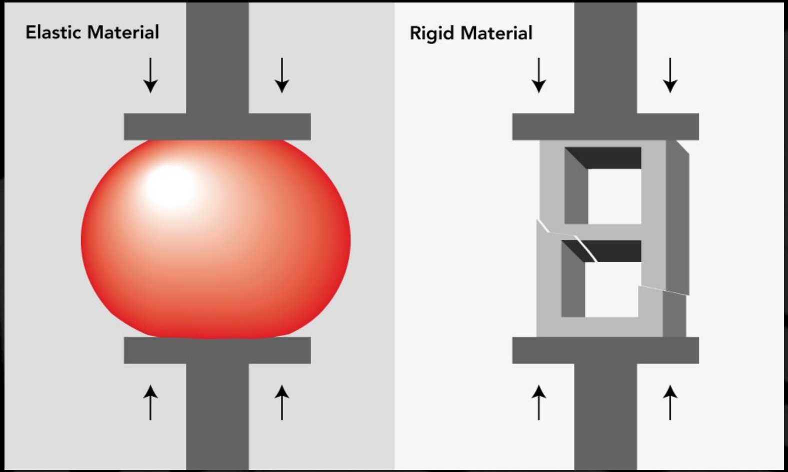
Theory

The maximum height of a tower is determined by several key factors:

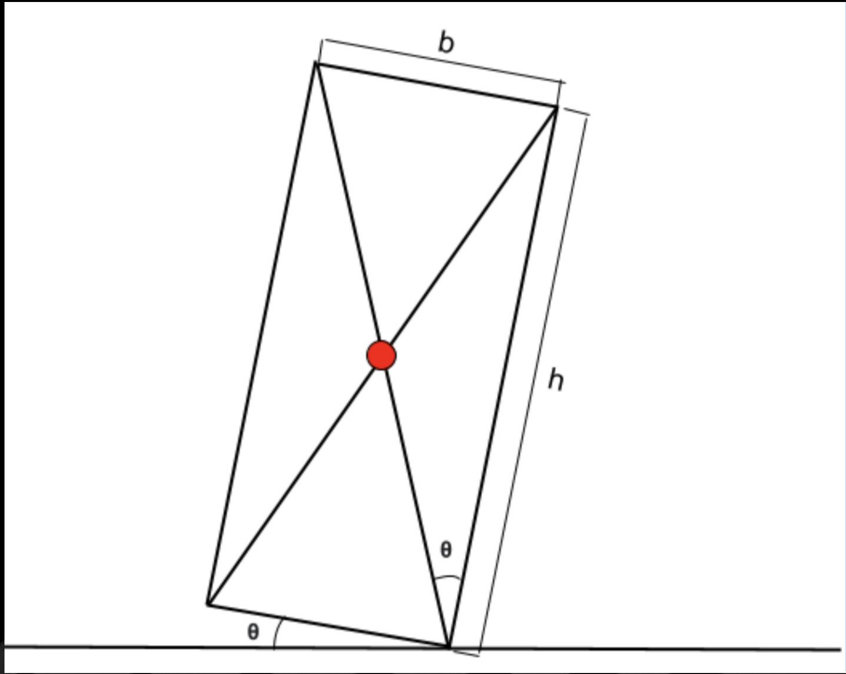
Friction, material strength, human skill, centre of mass

Material strength

- if the base material of the tower is not strong enough, the whole tower will collapse on itself.



Theory 2



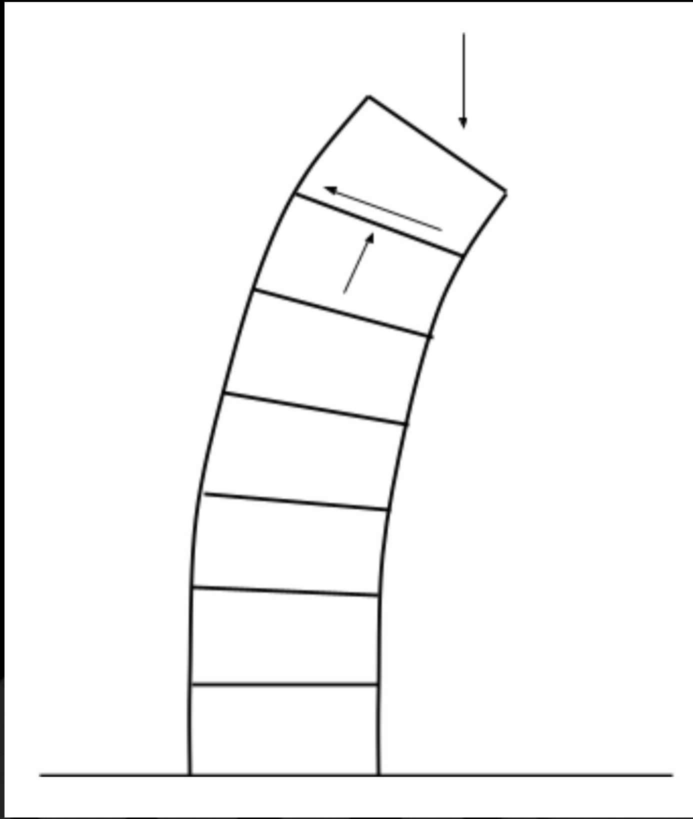
If all the blocks acted as a whole within the tower:

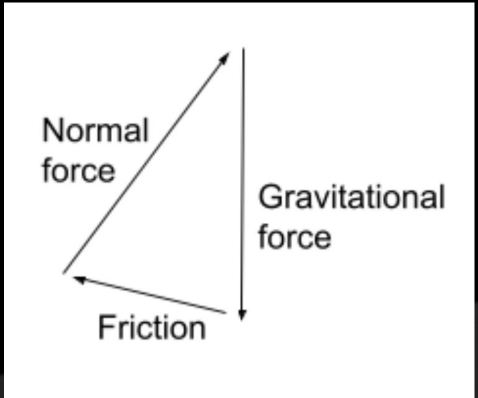
$$tan(\theta) = \frac{b}{h}$$

$$\theta = tan^{-1}\left(\frac{b}{h}\right)$$

$$\therefore \text{ if } h \text{ increases or } b \text{ decreases, } \theta \text{ decreases}$$
$$(\text{where } h \text{ is height and } b \text{ is the minimum base dimension})$$

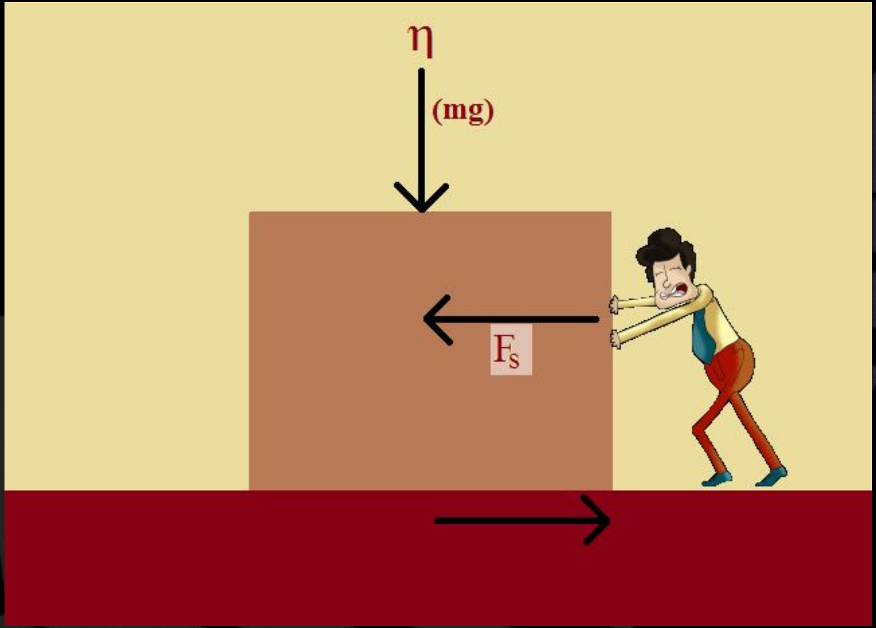
Theory 3





Theory 4

Static friction is a force that keeps an object at rest, which can be defined as the friction between two or more surfaces that are not moving relative to each other.



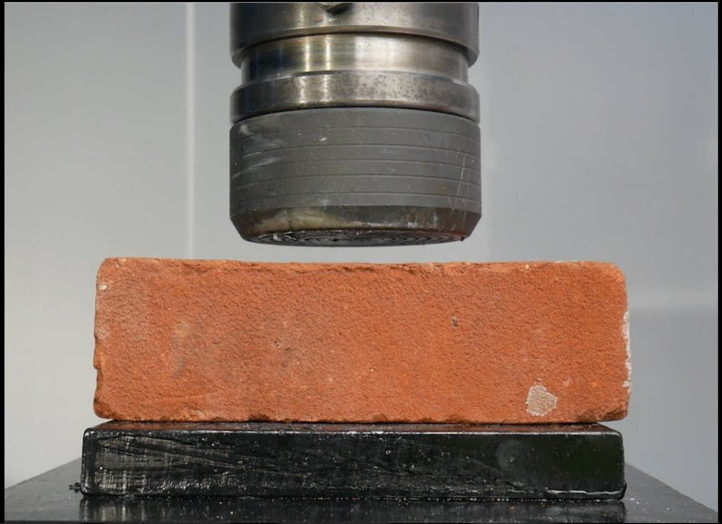
Predictions of Theory

Alder wood has a compressive strength of 5820psi

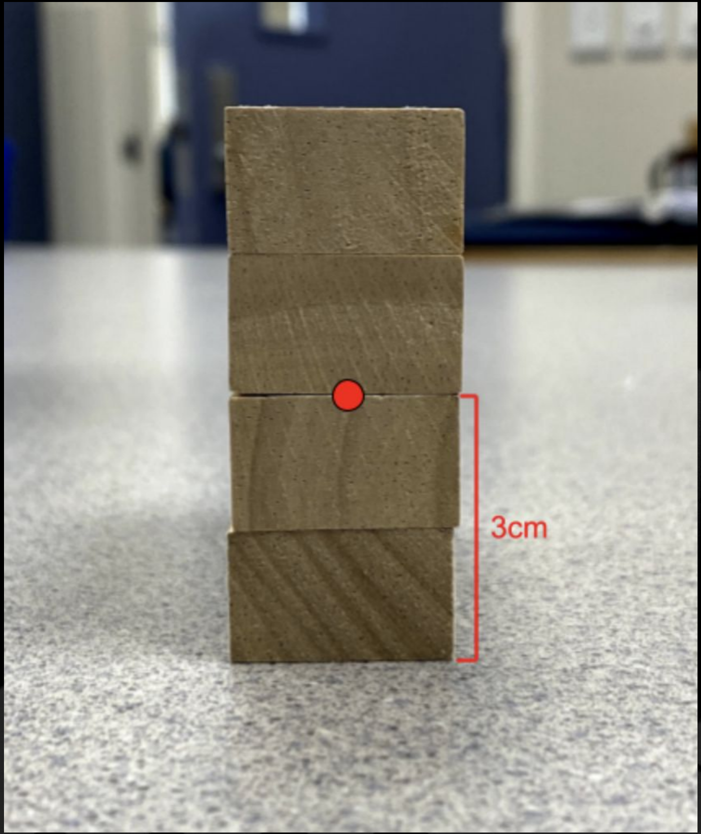
- one alder wood Jenga block can hold 7672kg
- theoretically meaning that approximately 42000 can be stacked upon each other without error.

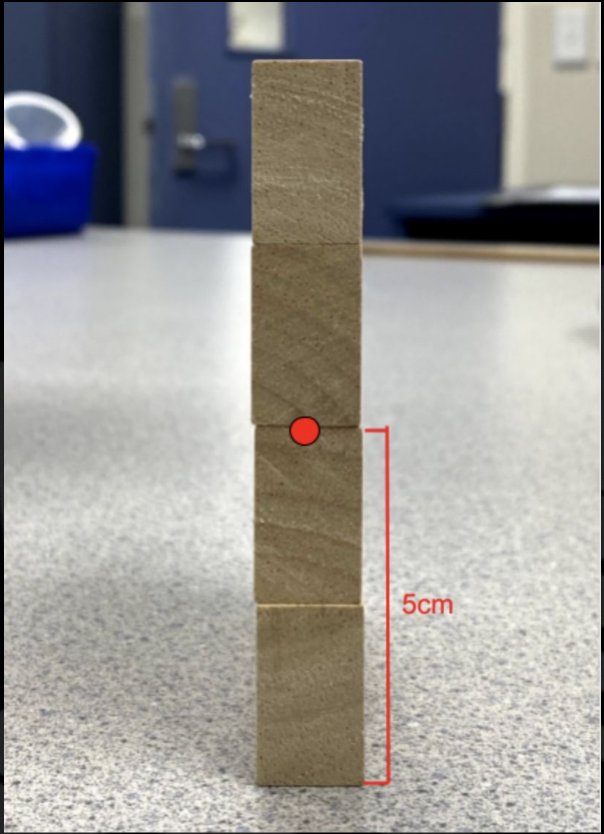
Brick has a compressive strength of 12000psi,

- A brick base of 253cm^2 could hold a weight of 213452kg
- approximately 5336 bricks

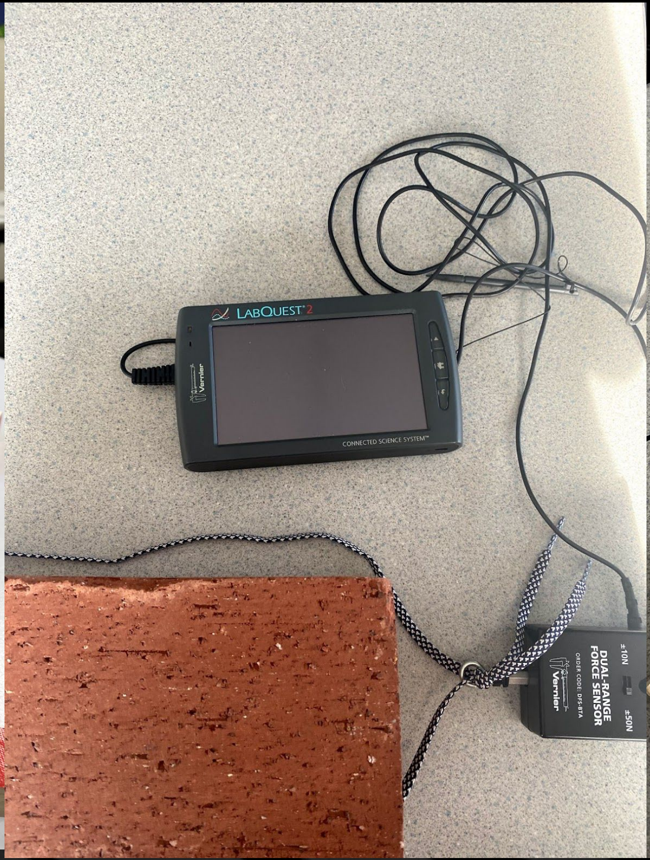


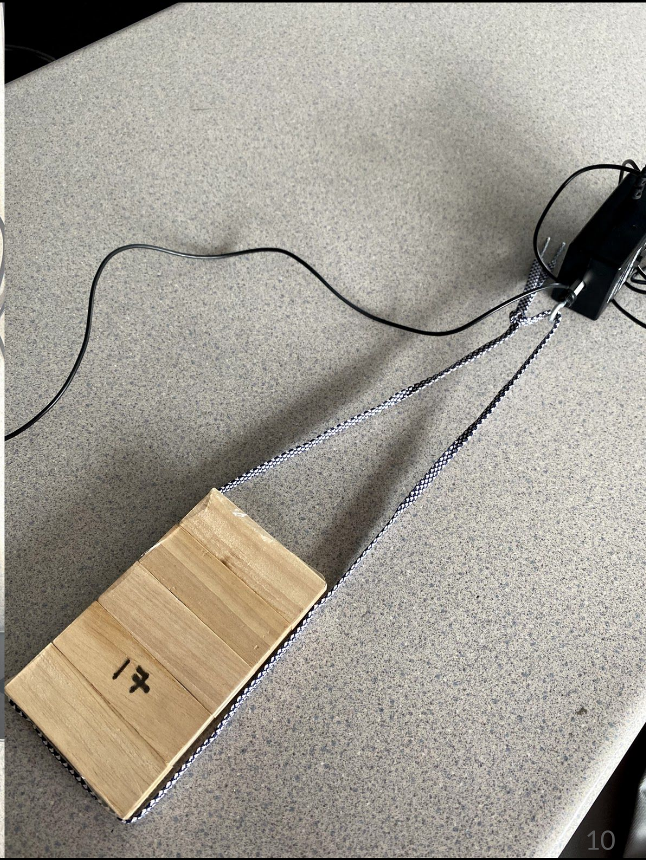
Predictions of Theory





Experimental evidence





Factors that limit the maximum height

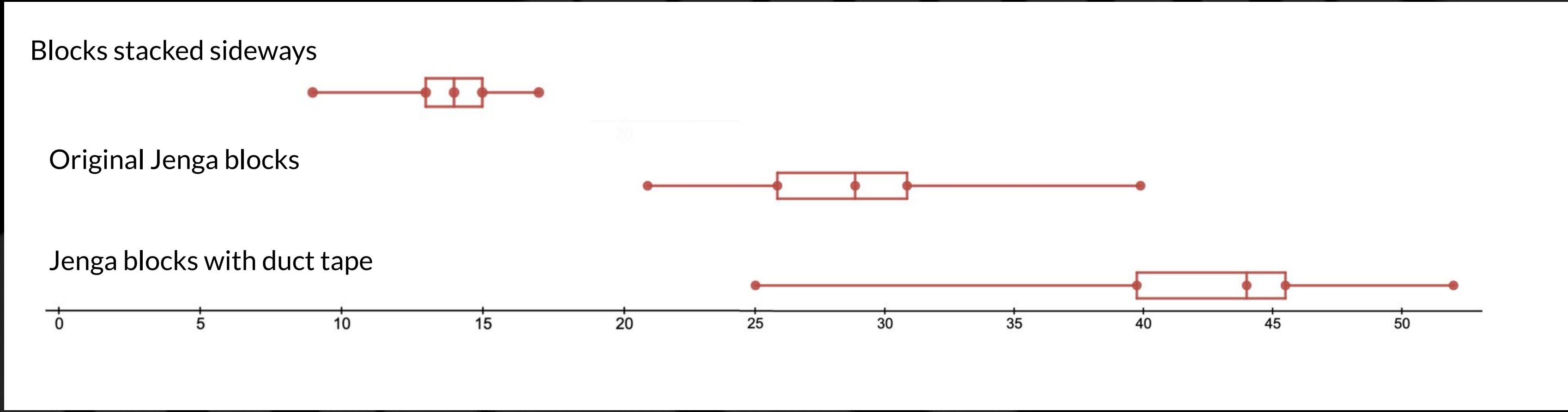
Coefficient of kinetic friction for a brick: **0.429**

Coefficient of kinetic friction for a block: **0.296**

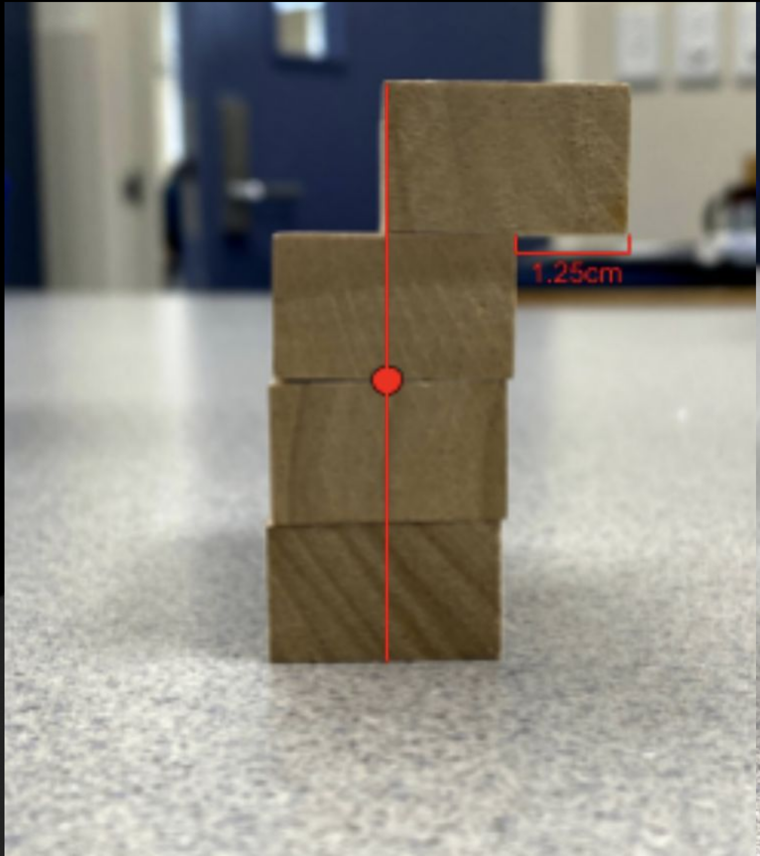
F(w)=ma and **F(n)=mg**, giving that the normal force of an object on a flat surface is equal to its weight.

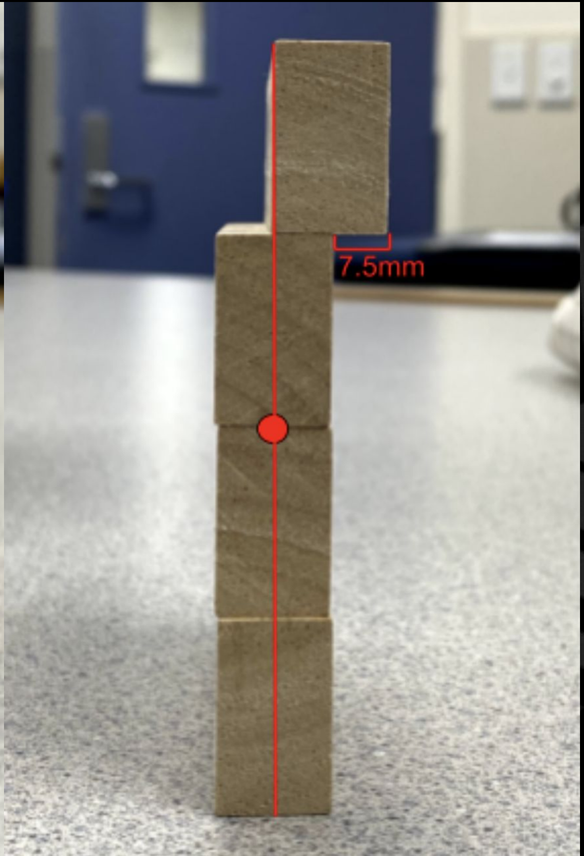
Coefficient of kinetic friction formula:

μ=f/N, where N is normal force, f is the force required to keep the block in motion, u is the coefficient of friction.



Factors that limit the maximum height





Controls

Controlled environment

Random sample of blocks

Maximum from 40 data samples

Testing the friction and weight multiple times, finding the average

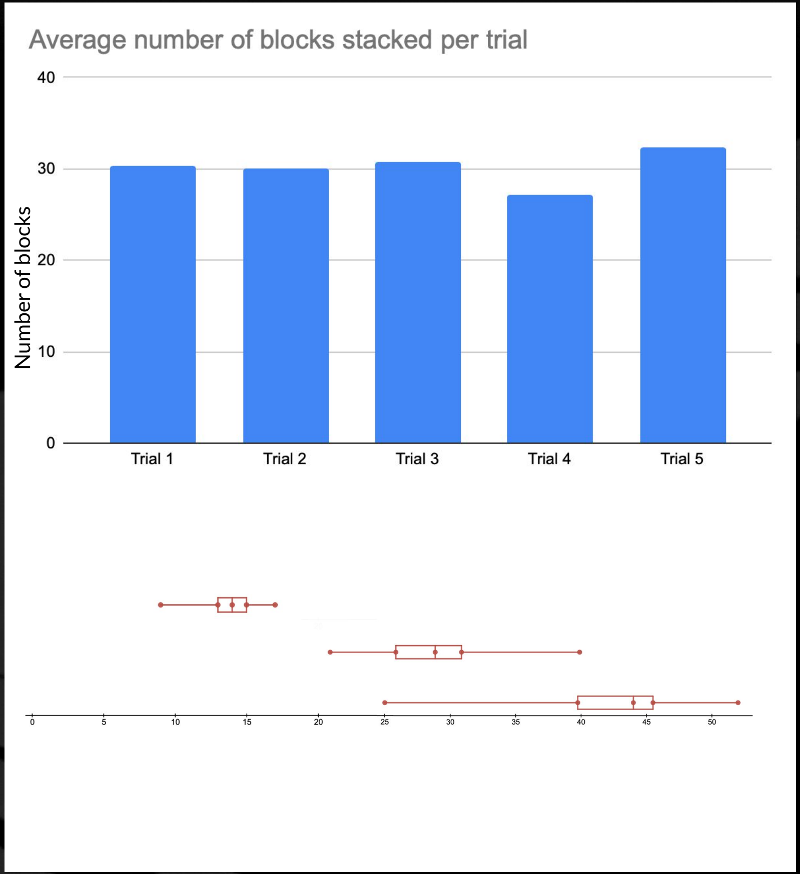
Data taken from and checked across various studies

Conclusion

Factors that affect the maximum height of a tower include material shape, strength, and the major aspect of human skill in placing the blocks.

Other factors:

- the improvement of human skill over time
- Mindset of the participant



Appendixes

https://datagenetics.com/blog/may32013/index.html
https://flothesof.github.io/fun-with-stacking-bricks.html
https://physics.stackexchange.com/questions/107484/hanging-a-brick-free-over-an-edge-by-stacking-them
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Appendixes

Material	Compressive strength (psi)	Coefficient of kinetic friction
Brick (commercial standards)	12000	0.42925
Jenga block (alder wood)	5820	0.296

Appendixes

Material	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
Tape	0.06	0.08	0.07	0.05	0.05	0.062
Block	0.05	0.06	0.05	0.05	0.06	0.054

Force required to move (N)

Weight (g)	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
Brick	4000	4000	4000	4000	4000	4000
Jenga block	17.07	17.02	18.71	19.73	18.71	18.25

Appendixes

Test	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
1	9	14	14	14	16	13.4
2	13	14	15	15	15	14.4
3	14	13	15	13	17	14.4
4	12	14	13	15	16	14
5	15	13	16	16	17	15.4
5	14	15	15	16	17	15.4
6	13	10	15	14	15	13.4
7	12	13	14	16	13	13.6
8	15	13	11	13	14	13.2

Appendixes

Jenga blocks stacked

Test	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
1	25	29	27	26	29	27.2
2	30	31	29	28	39	31.4
3	32	29	40	30	31	32.4
4	31	32	29	27	29	29.6
5	34	23	30	21	35	28.6
6	26	27	29	31	26	27.8
7	34	39	31	27	37	33.6
8	25	23	26	22	26	24.4

Appendixes

Test	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
1	28	26	35	37	37	32.6
2	40	45	38	45	49	43.4
3	47	45	45	49	49	47
4	25	50	49	44	52	44
5	42	49	42	50	50	46.6
6	39	44	44	44	42	42.6
7	40	40	40	39	40	39.8
8	38	45	42	45	45	43

Appendixes

12000=843.68kg/cm^2
5820= 409.19kg/cm^2
Surface area of brick:253cm^2
Surface area of block:18.75cm^2
Brick can hold 213452kg
Block can hold 7672kg
So theoretically, (213452/40) approximately 5336 bricks could be stacked upon each other without error.
(7672/0.1825) approximately 42000 blocks could be stacked upon each other without error.
Midland Brick Standard 110 Series Heritage Red
23cm*11cm*7.6cm
Volume: 1922.8cm^3
Weight per cm^3: 2.08g
Jenga block
7.5cm*2.5cm*1.5cm
Volume: 28.125cm^3
Weight per cm^3: 0.649g
Newton's 2nd Law
F(w)=ma
Acceleration is Earth's gravity: 9.807m/s^2
4000=9.807m
m=407.872

Newton's 3rd Law
F(n)=mg

Therefore the laws give that the normal force of an object on a flat surface is equal to its gravitational force, or weight

40N of normal force for mass of 407.872
f=μN
μ=f/N
μ=17.17/40
μ=0.42925
Coefficient of kinetic friction^
μ=tan(theta) where u is the coefficient of friction and theta is the angle
For static friction^
Newton's 2nd Law
F(w)=ma
Acceleration is Earth's gravity: 9.807m/s^2
18.25=9.807m
m=1.861
0.1825N for weight of 18.25
f=μN
μ=f/N
μ=0.054/0.1825
μ=0.296
Incorporating the physics of friction with the geometry of the inclined plane gives a simple formula for the coefficient of static friction: μ = tan(θ), where μ is the coefficient of friction and θ is the angle.