

IYNT 2021
Team Nitro

Problem 6.
Tall Towers



Problem Statement

A tower is built by stacking rectangular bricks on top of each other.

Some people argue that the maximum height of the tower is limited by the human skill to place the bricks gently; other may say that the limiting factor is nonperfect shape of the bricks.



Summary



Theoretical Part

slides 4 - 6

- 1.2 Center of mass
 - 1.1 Torque
- 1.3 Friction Force



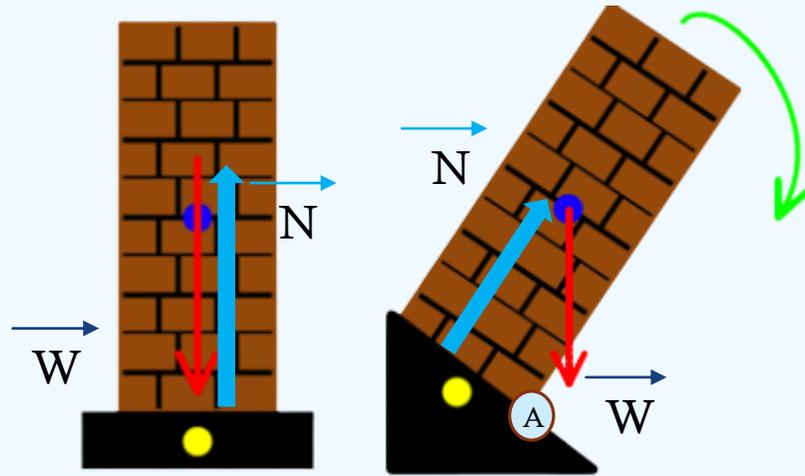
1.1 Torque

The diagram illustrates the equation for torque. It features three colored boxes on a purple background. The first box is green and contains the Greek letter τ , with the word "torque" written above it in green. This is followed by an equals sign. The second box is cyan and contains the symbol F_{perp} , with the words "perpendicular" and "force" written above it in cyan. The third box is magenta and contains the letter r , with the word "radius" written above it in magenta.

$$\tau = F_{\text{perp}} r$$

- is the rotational equivalent of linear force.
- can be thought as a twist to an object around a specific axis.

1.2 Center of mass

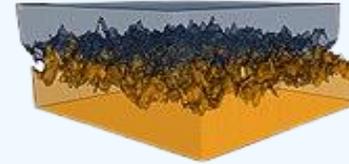


- is the unique point where the weighted relative position of the distributed mass sums to zero.
- if the vertical of the centre of mass falls outside the base area, the torque of the weight around point A would tend to rotate the block clockwise. But the torque of the normal reaction force already tends the block to rotate clockwise. So the resulted moment would rotate the block clockwise and will fall.
- On the other hand, if the vertical of the centre of mass will fall inside the base area, the block wouldn't fall.

1.3 Friction Force

$$f = \mu N$$

The formula of the Friction Force

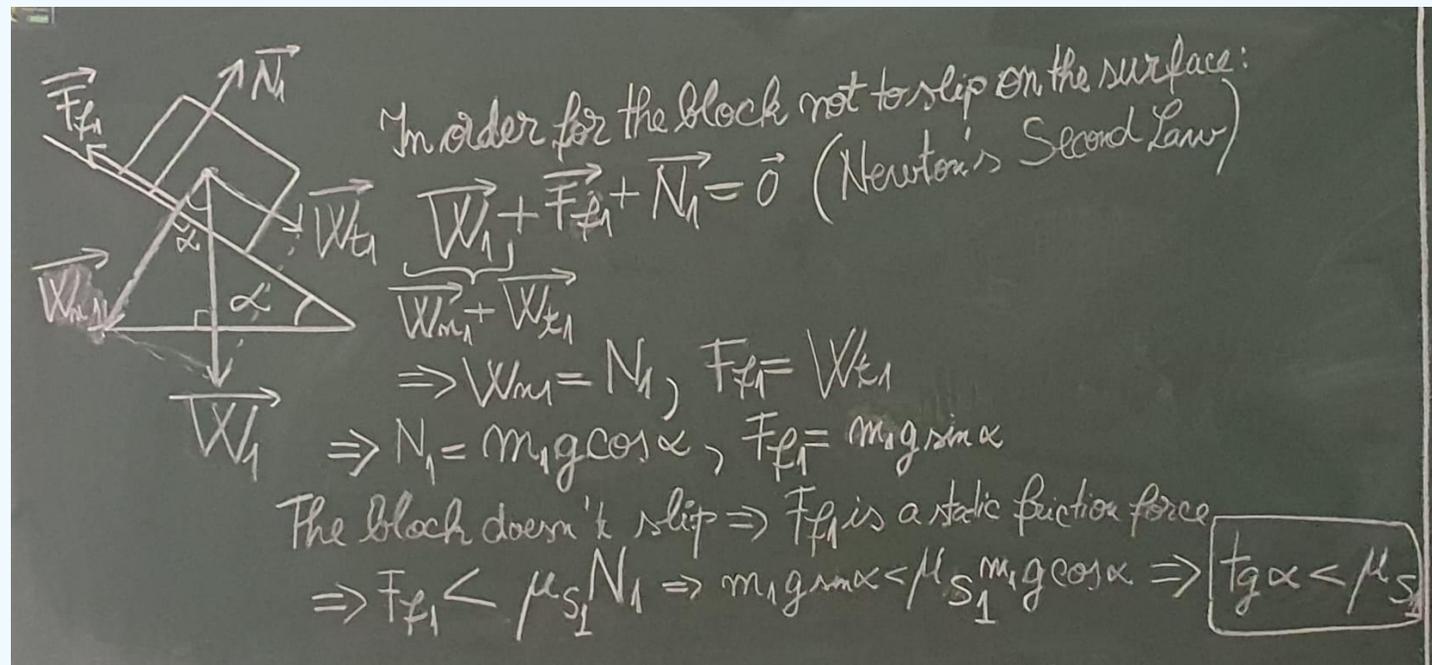


The friction between 2 solid on a microscopic level

- Friction is the force that prevents solid surfaces from moving against each other.
- Dry friction, the one that appears between our bricks, is a force that opposes the relative lateral motion of two solid surfaces in contact.

In order for the block not to slip on the surface:

Tan α must be smaller than the coefficient of friction of the surface



In order for the block not to slip on the surface:
Newton's Second Law:
$$\vec{W}_1 + \vec{F}_{f1} + \vec{N}_1 = \vec{0}$$

$$\vec{W}_{1x} + \vec{W}_{1y}$$

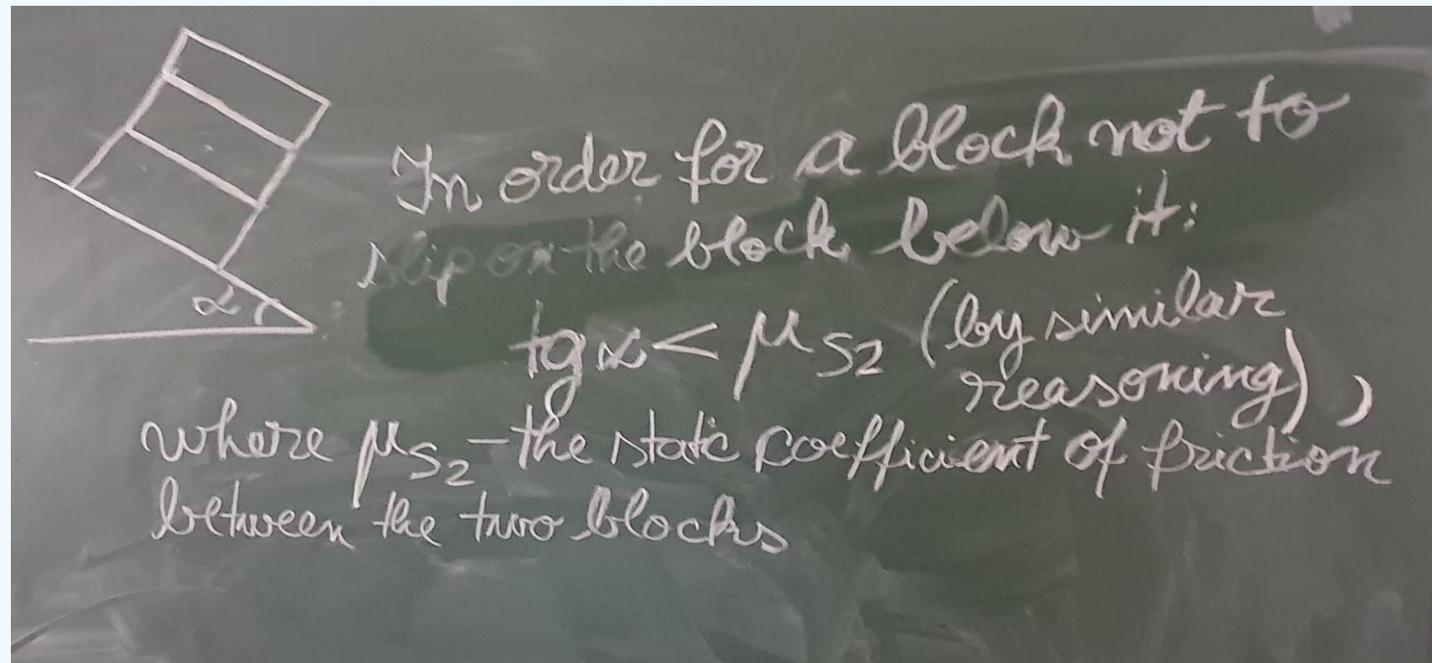
$$\Rightarrow W_{1y} = N_1, F_{f1} = W_{1x}$$

$$\Rightarrow N_1 = m_1 g \cos \alpha, F_{f1} = m_1 g \sin \alpha$$

The block doesn't slip $\Rightarrow F_{f1}$ is a static friction force
$$\Rightarrow F_{f1} < \mu_{S1} N_1 \Rightarrow m_1 g \sin \alpha < \mu_{S1} m_1 g \cos \alpha \Rightarrow \boxed{\tan \alpha < \mu_{S1}}$$

In order for a block not to slip on the block below it:

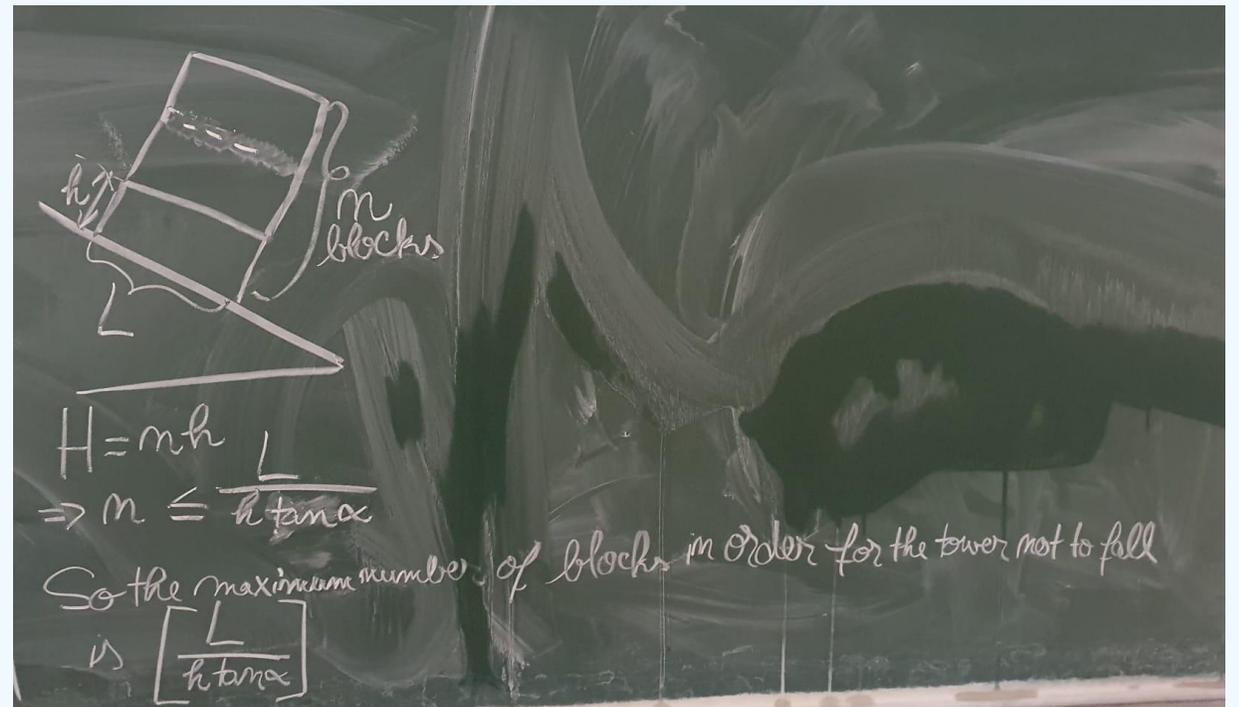
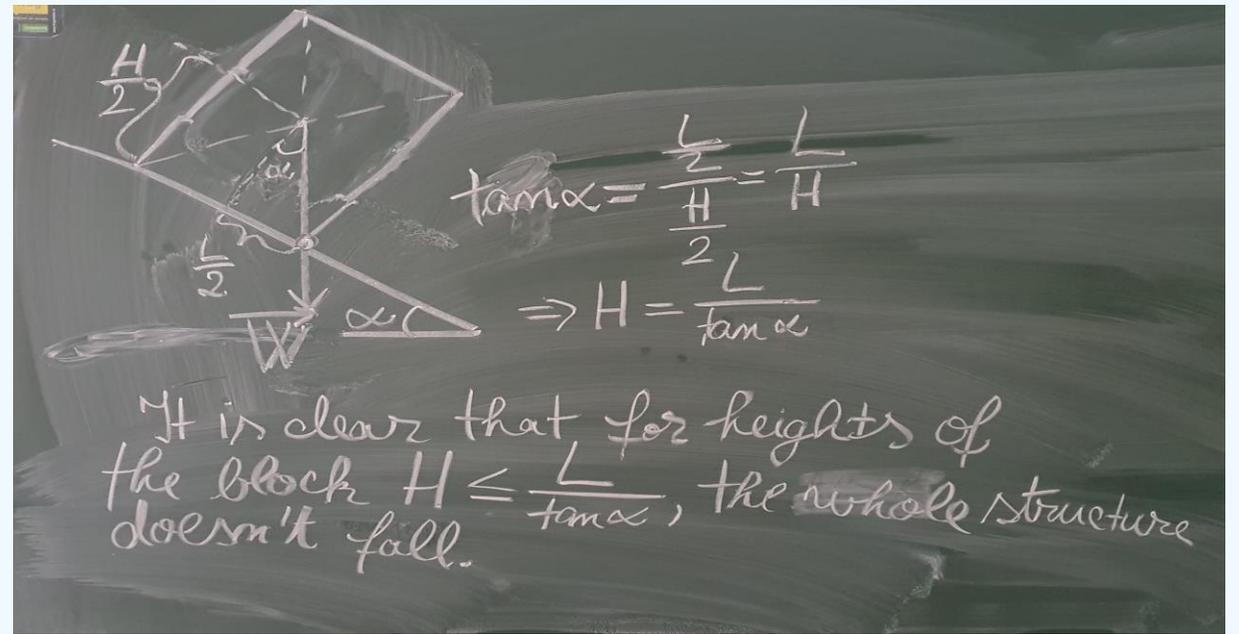
Tan α must be smaller than the coefficient of friction between the two blocks



In order for a block not to slip on the block below it:
$$\tan \alpha < \mu_{S2}$$
 (by similar reasoning),
where μ_{S2} - the static coefficient of friction between the two blocks

The maximum number of blocks in order for the tower to fall is:

$$L/h \cdot \tan \alpha$$



Experimental Part

slides 6-24

- 2.1 Hypothesis
- 2.2 Materials and Preparation
- 2.3 Results



2.1 Hypotheses

- Experiment 1 – trying to build a tower using a wooden frame for stability

Hypothesis: if the frame is used correctly the tower will not collapse

- Experiment 2 – trying to build towers using the most common building structures

Hypothesis: different buildings come with different stabilities so, I assume that the ones with the bigger base will have a better stability

- Experiment 3 – trying to build tower using different sized bricks to simulate and study each deformation that can occur in the building of a such tower.

Hypothesis: For a brick deformation to collapse a tower, that deformation should be visible with the free eye

2.2 Materials and Methods

Materials:

- Perfect table parallel to the ground 100cm*120cm
- Prebuild frame to hold the bricks
- Different dimensions and shapes bricks for simulating the bricks deformations
- Notebook to record the result and camera to take pictures

For each experiment:

1. Gather all needed materials on the working table that is perfectly flat.
2. Choose the appropriate types of wood brick, according to each experiment' purpose.
3. Be sure that the solid surface is empty.
4. Built the bricks tower, as planned.
5. Once with a frame and then without. Without the frame we will see which discussed tower will be the most tolerant o human errors.
6. Take pictures and videos.
7. Record the experiments' findings in the notebook.
8. Introduce and analyse data in the computer.
9. Empty the solid surface for the next experiment.

*Note: before these experiments, I spend several hours practicing how to better manipulate the bricks

Experiment materials – wood brick types

No	Type	Base Shape	Length(cm)	Width(cm)	Height(cm)
1		rectangle	9	1.4	2.8
2		rectangle	7.4	1	2.5
3		triangle	4	4	2.9
4		rectangle	7.2	1.2	2.4
5		rectangle	5	1	2.6
6		square	2.3	2.3	5
7		triangle	3.2	3.2	3.4
8		square	2.5	2.5	2.5

2.3 Results

Experiment 1

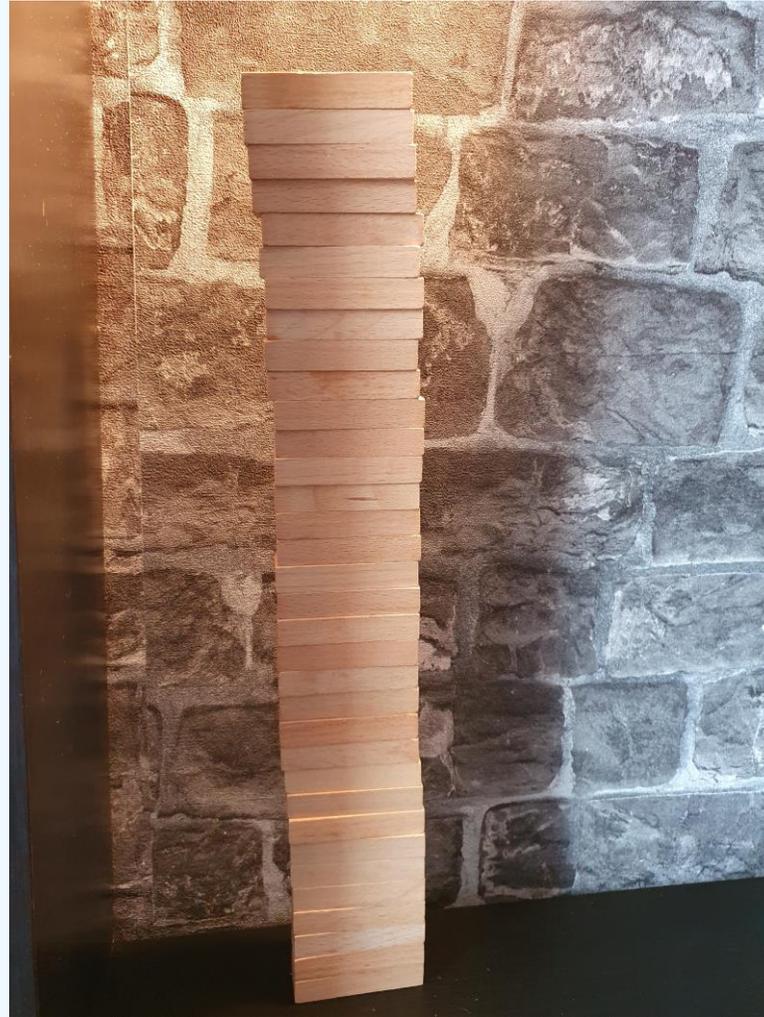
- The tower built using a frame didn't collapse.
- The frame was build using wood, had a 90 degree angle between the sides and the surface was flat.



Level for checking if the frame is perpendicular to the ground



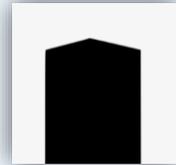
- I've tried this experiments without a frame.
- After the 20th brick the tower had a tendance of leaning forward.
- If we continued building, the vertical centre of mass would've fallen outside the base area and the whole structure with it.



Experiment 2

- Most common ways of building a brick tower:

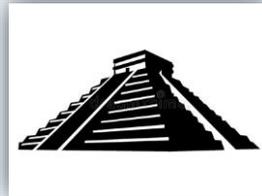
1. As a **column**, with a large base



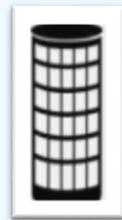
2. As a **column**, with a small base



3. In a shape of a **pyramid**



4. As a **cylinder**



5. **Size decreasing**(smaller at the top bigger at the bottom)





Way of building the bricks tower	Bricks used	Height	Stability (1->10)	The tower collapsed?	Comments	Conclusion
As a column, with a large base	34, number 4	40.8	7	No	The tower had a tendency of leaning forward	If we would have been building further on, the whole tower will fall.



Way of building the bricks tower	Bricks used	Height	Stability (1->10)	The tower collapsed?	Comments	Conclusion
As a column, with a small base	5, number 4	36	2	Yes	The tower has a low stability.	The tower felt after a overlapping maximum 5 bricks. I had 8 attempts to capture this picture.



Way of building the bricks tower	Bricks used	Height	Stability (1->10)	The tower collapsed?	Comments	Conclusion
Pyramid	48, number 4	8.4	9	No	The tower has a high stability. I do not think it will fall, no matter the scale of the experiment.	The structure has a great tolerance to human error abilities.



Way of building the bricks tower	Bricks used	Height	Stability (1->10)	The tower collapsed?	Comments	Conclusion
Cylinder	34, number 4 (plus 19, no 5)	14.4	7	No	The tower has a pretty good stability. As in exepriement 1, the tower has a tendency of leaning one side, especially when we use uneven bricks.	If we would have been building further on, the whole tower will fall.



Way of building the bricks tower	Bricks used	Height	Stability (1->10)	The tower collapsed?	Comments	Conclusion
Triangle shape	34, number 4, 9 number 6, 22 number 5 2 number 4	22.5	7	No	The tower has a pretty good stability, as the bricks were having almost the same width.	As the layers are smaller and smaller, concentrated to the center of the base, the center of mass is always remaining inside the base area. The human abilities in this care are very important.



Way of building the bricks tower	Bricks used	Height	Stability (1->10)	The tower collapsed?	Comments	Conclusion
Square base tower	33, number 4	13,2	10	No	The most tolerable structure regarding the human errors. In case of a misplacement, the next layer will correct it.	The most stable structure.



Way of building the bricks tower	Bricks used	Height	Stability (1->10)	The tower collapsed?	Comments	Conclusion
Mixed stucture	15, number 4	40.8	4	Yes	An upgrade of stability comparing to the experiment 1. The horizontal layer comes to stabilize the vertical brick.	Humar errors must be reduced in order to built successfully such a structure.

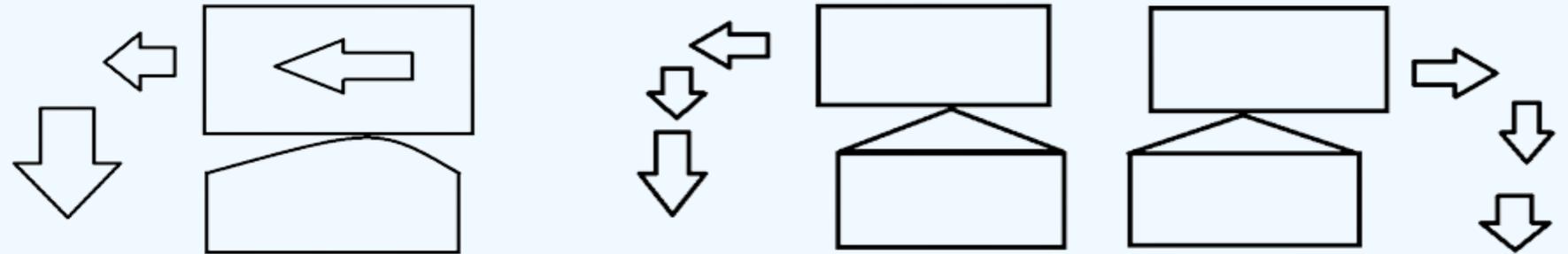
2.3 Results

Experiment 2

- The highest structure built was of 40.8 cm
- The human ability of overlapping the bricks is important, but it's not the only crucial factor when it comes to the falling of the tower
- Different tower structures come with advantages and disadvantages regarding stability, tower height, human error tolerance and construction costs
- When the base structure area is the largest used, we do have the chance to build the highest tower
- In an ideal world, when there will be no time and cost constraints, the human factor error can be reduced to 0

2.3 Results

Experiment 3



Studied Case	Comments	Conclusions
<ul style="list-style-type: none"> The effect of a middle and side deformation. 	<ul style="list-style-type: none"> If the deformation is in the middle of the brick, the next put bricks should be perfectly put on top of each other. This is nearly impossible. As what concerns a side deformation, the next put brick after the deformed one will slide outside the structure. 	<ul style="list-style-type: none"> If we would have been building further on, the whole tower will fall. As a conclusion the tower will definitely fall.

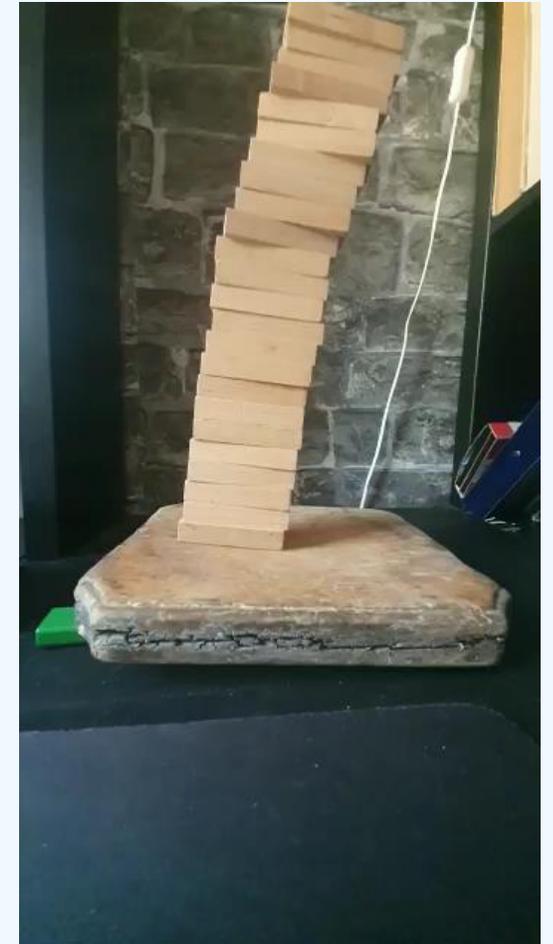




10 degree inclination

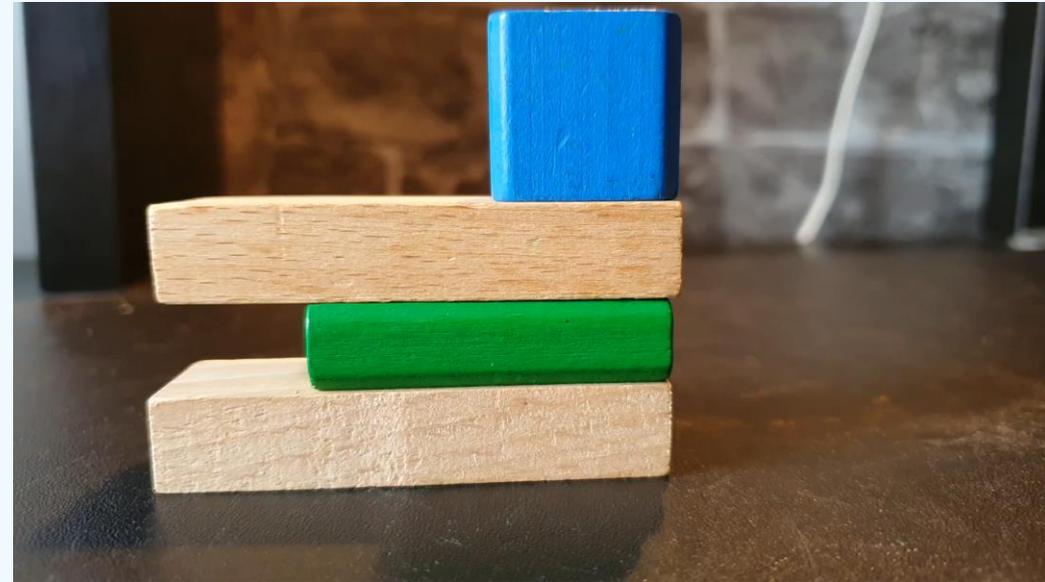
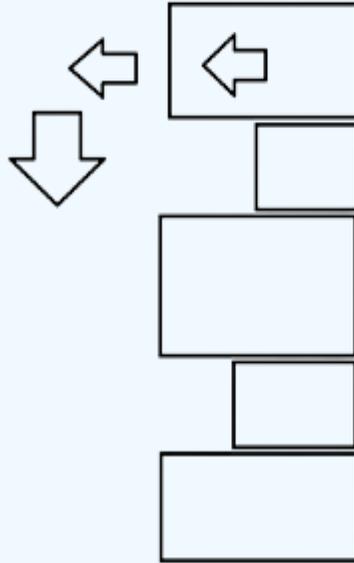


40 degrees inclination



5 degree inclination

Studied Case	Comments	Conclusions
<ul style="list-style-type: none"> The effect of base inclination 	<ul style="list-style-type: none"> In this experiment we will have a 5,10,40 degree angle inclination. Overlapping the bricks, the centre of mass is moving with each brick closer and closer to the edge. In this case the tower will surely fall. 	<ul style="list-style-type: none"> The bigger the base inclination is, the shorter the tower will be.



Studied Case	Comments	Conclusions
<ul style="list-style-type: none"> Different length of the bricks 	<ul style="list-style-type: none"> In this experiment we have overlapped several bricks with different lengths. When a shorter brick is added, the centre of mass comes closer to the right vertical alignment. When the next brick, the longer one, is overlapped, it will eventually fall down. For an edge deformation the rules remain the same: as long as the centre of mass remains in the base area, the tower will not fall. 	<ul style="list-style-type: none"> The more uniform the used bricks are, the higher the tower will be.

Final Part

slides 25-29

- 3.1 Conclusions
- 3.2 Errors and limitations
- 3.4 References



3.1 Conclusions

Experiments 1 and 2

- For sure, the **human ability of perfectly overlapping the bricks is crucial** in obtaining a stable tower and the human factor can be **minimized with a frame like we used.**
- I discovered that the **following tower structures are more tolerant to human errors: pyramid and square base tower.**

A tower build **using a frame** will go to the **infinite**. I didn't have **the necessary materials** for this but analysing the other experiments and the theory I am sure **that this will work.**

Our experiments proved that **the bigger the base is, more stabile the tower will be.**

Experiment 3

- **The bricks' shape uniformity is essential for the tower stability.**
- The smallest defect (for example, **a different angle** inclination of a brick, caused by the imperfect shape of the brick placed below it) can determine the **tower to fall.**

3.2 Errors and Limitations

- All experiments **were done indoor**. Therefore, we can assume that the towers were built in perfect conditions. **Wind and imperfections at the ground level were all absent**. On the other side, the **friction force is missing**, that could've helped us stabilizing the bricks better.
- The **wood bricks** were **not having perfect shapes**.
- The weight of the **bricks** was small enough to be **easily maneuvered**.

3.3 References

- <https://en.wikipedia.org/wiki/Archimedes>
- https://en.wikipedia.org/wiki/Center_of_mass
- <https://courses.lumenlearning.com/boundless-physics/chapter/center-of-mass/>
- https://en.wikipedia.org/wiki/Burj_Khalifa
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- <https://flothefof.github.io/fun-with-stacking-bricks.html>
- <https://en.wikipedia.org/wiki/Block-st>
- <https://en.wikipedia.org/wiki/Torque>
- <https://www.youtube.com/watch?v=b-HZ1SZPaQw>

