Solution of the problem "Stearin engine"

presented by
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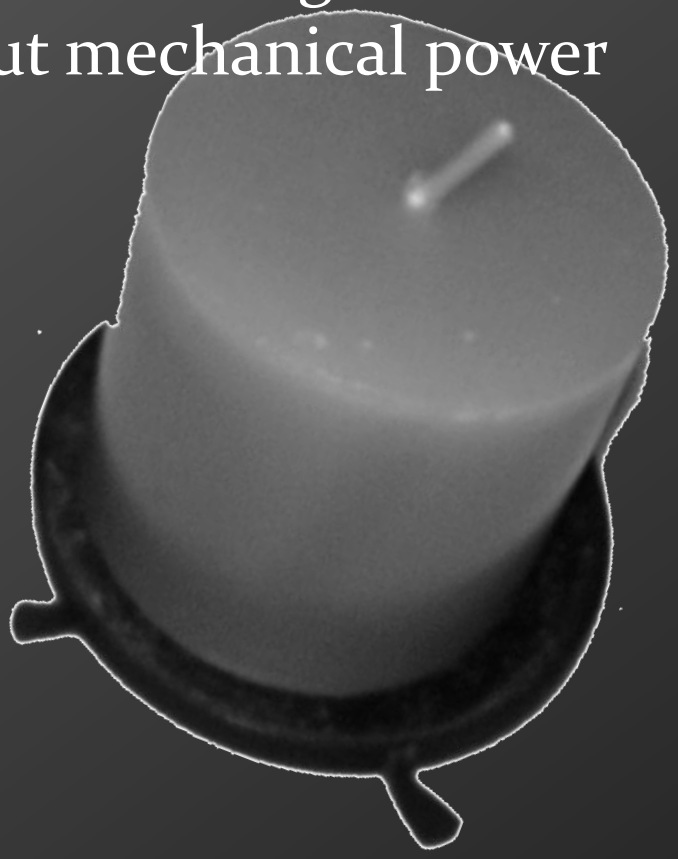
Nankai University, 2009

Formulation of the problem

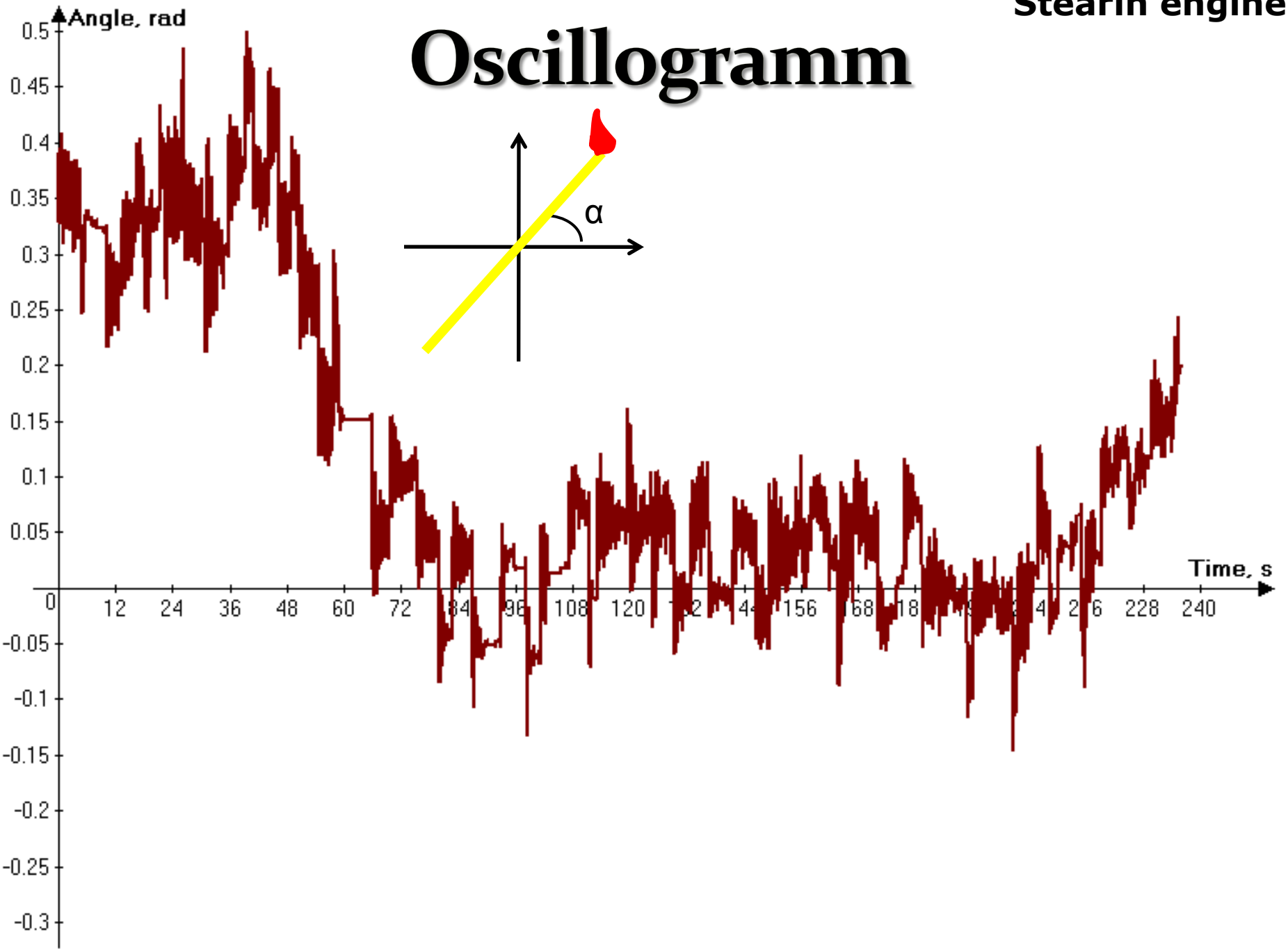
A candle is balanced on a horizontal needle placed through it near its centre of mass. When the candle is lit at both ends, it may start to oscillate. Investigate the phenomenon. Maximize the output mechanical power of the system.



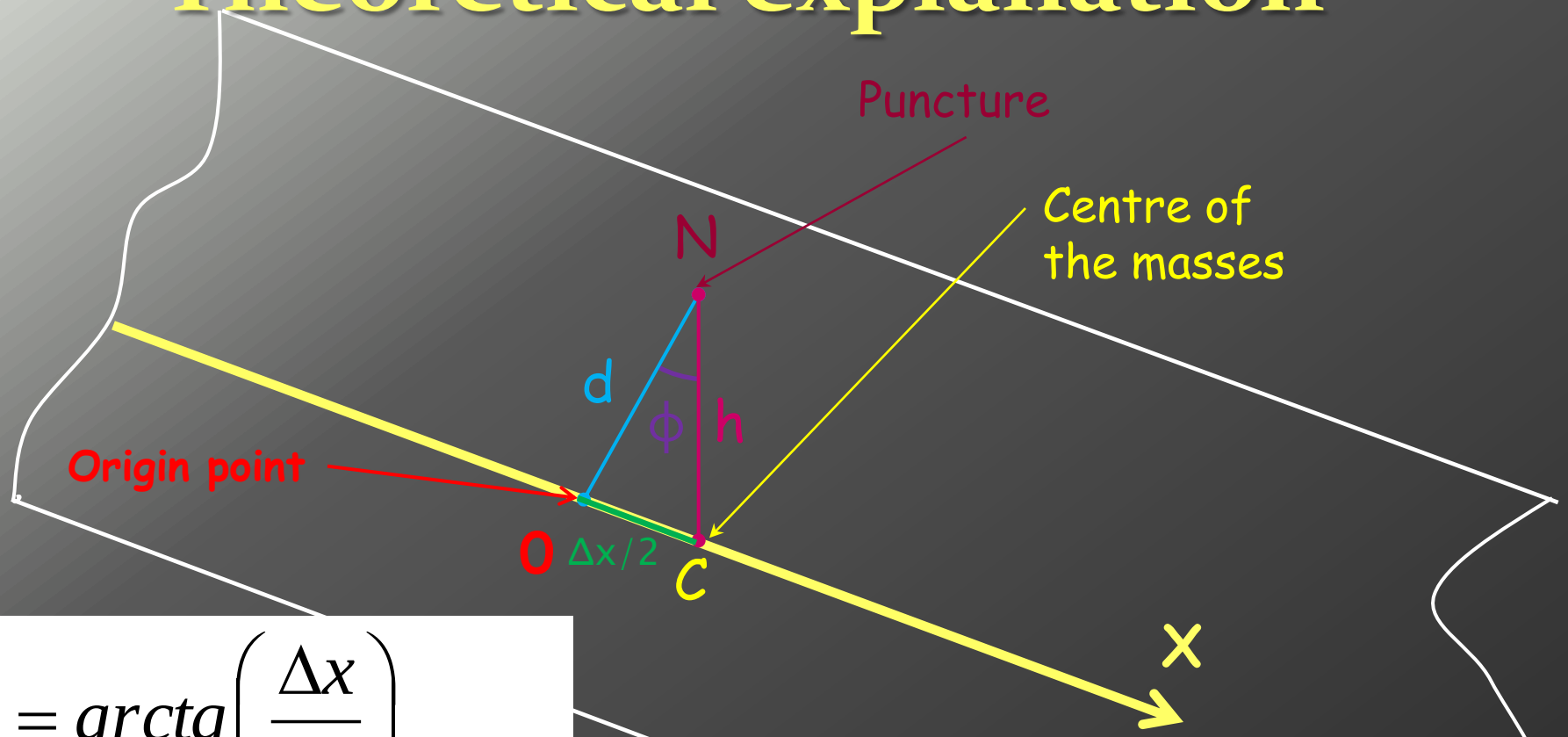
Demonstration of the effect

Stearin engine

Oscillogramm



Theoretical explanation



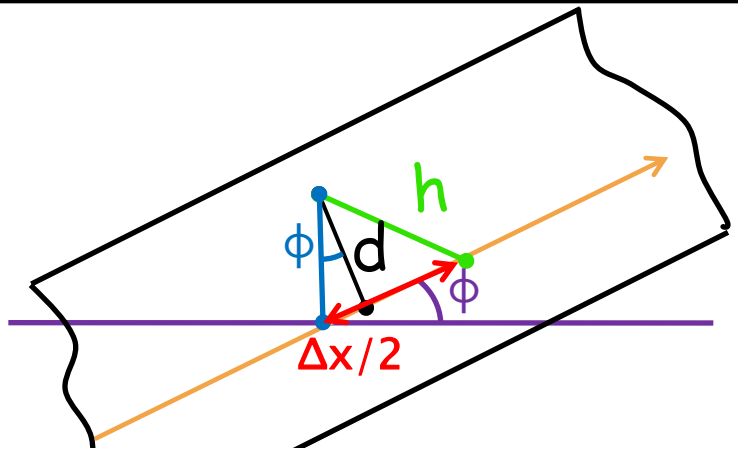
$$\varphi = arctg\left(\frac{\Delta x}{2d}\right)$$

$$A = |\varphi| = \left|arctg\left(\frac{\Delta x}{2d}\right)\right|$$

$$\Delta x = \frac{\Delta m}{m} l = \frac{\Delta m}{\rho_l}$$

Complicated Explanation

Δx > 0 when drops from right end

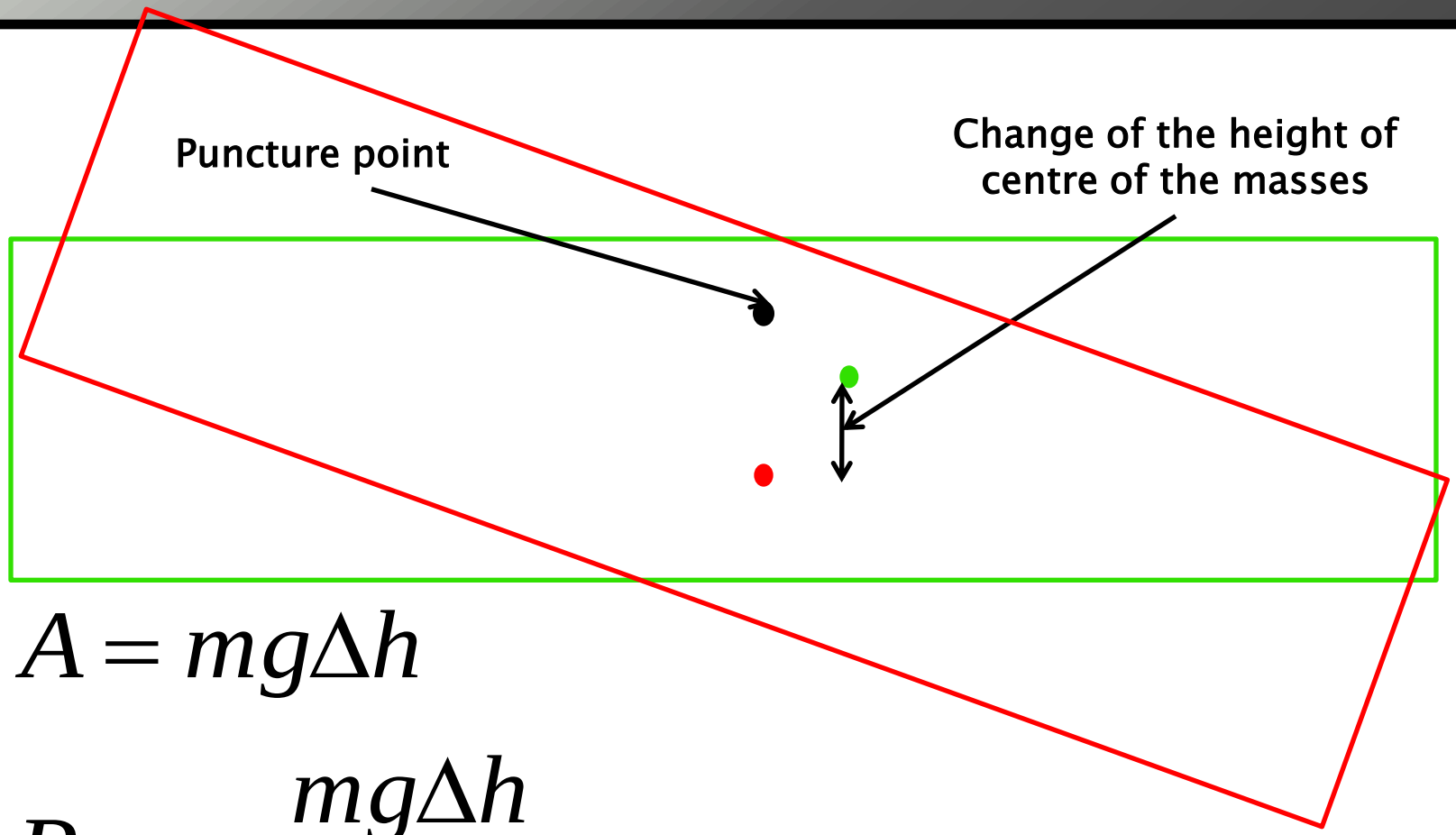


$$\varphi = arctg\left(\frac{\Delta x/2 - d * tg\,\varphi_0}{d}\right) - \varphi_0$$

$$A = |\varphi| = \left| arctg\left(\frac{\Delta x/2 - d * tg\,\varphi_0}{d}\right) - \varphi_0 \right|$$

Puncture near the centre of the masses

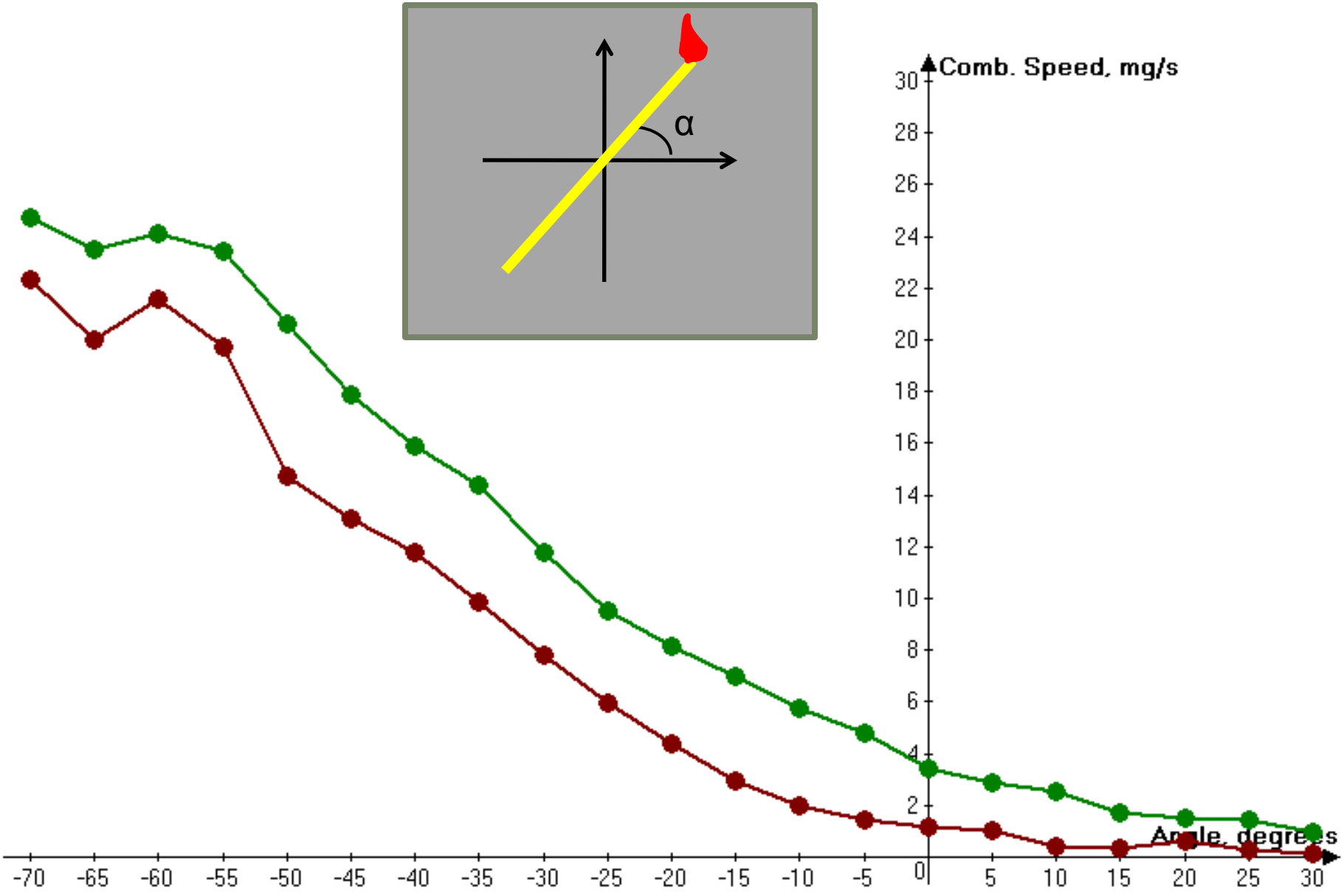
Maximal Power



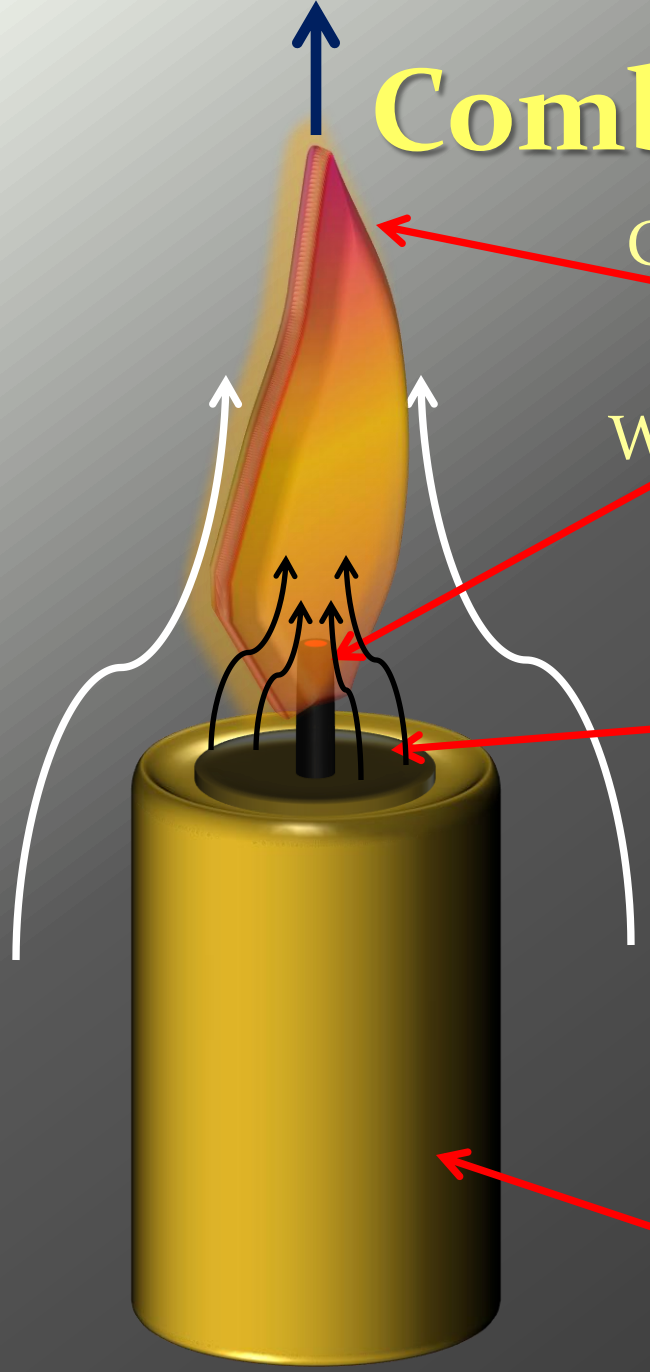
$$A = mg\Delta h$$

$$P_{\max} = \frac{mg\Delta h}{T_{drop}}$$

Plot combustion speed (ξ) against angle



Combustion speed



Gazes are mostly burning near the point, where temperature is maximal

Wick is in low temperature area, where it is unable to burn

Under the wick – a puddle of liquid paraffin. The gaze goes up.

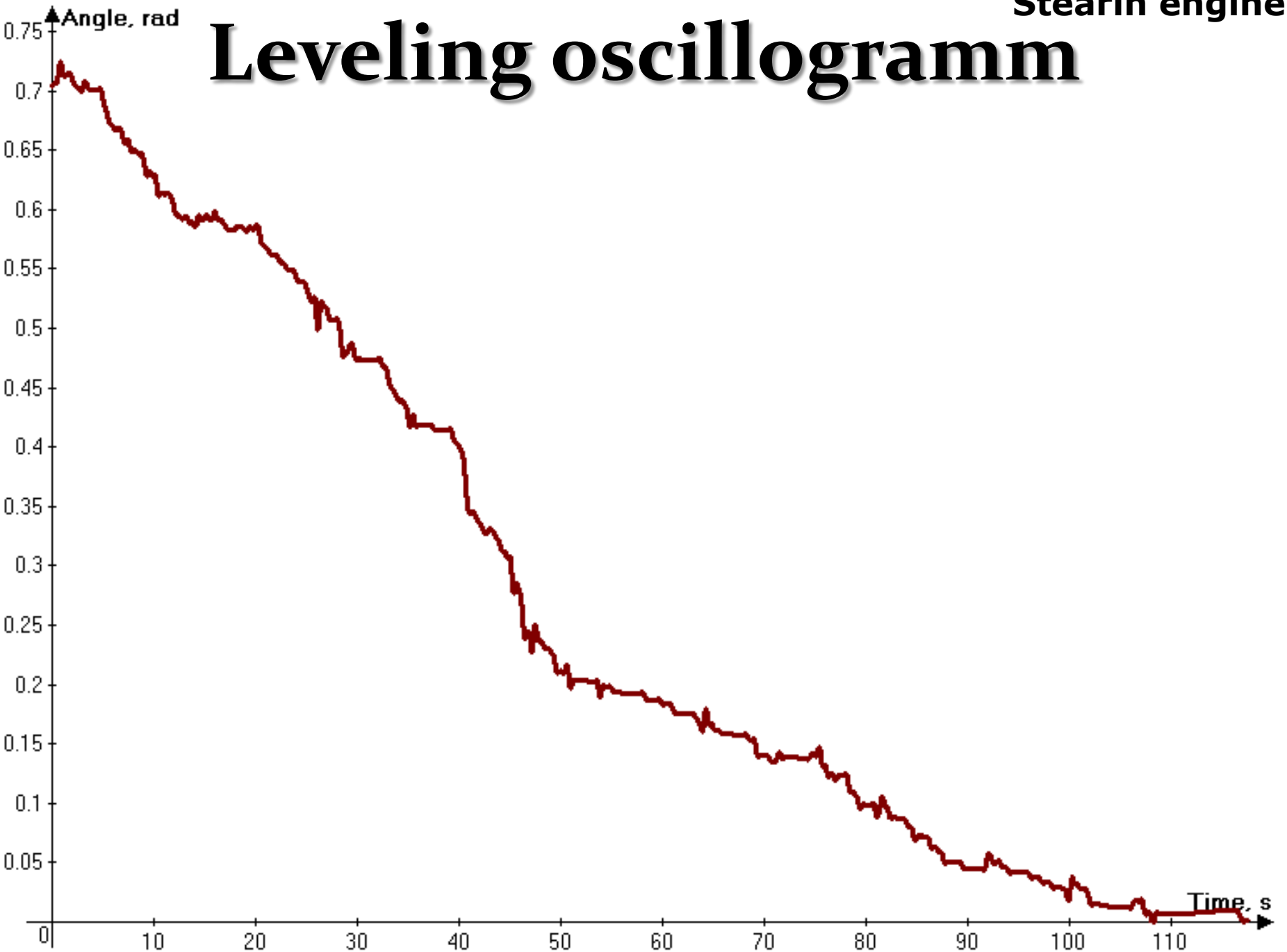
Oxygen for the combustion goes from bottom upwards

The candle material is the main source of energy

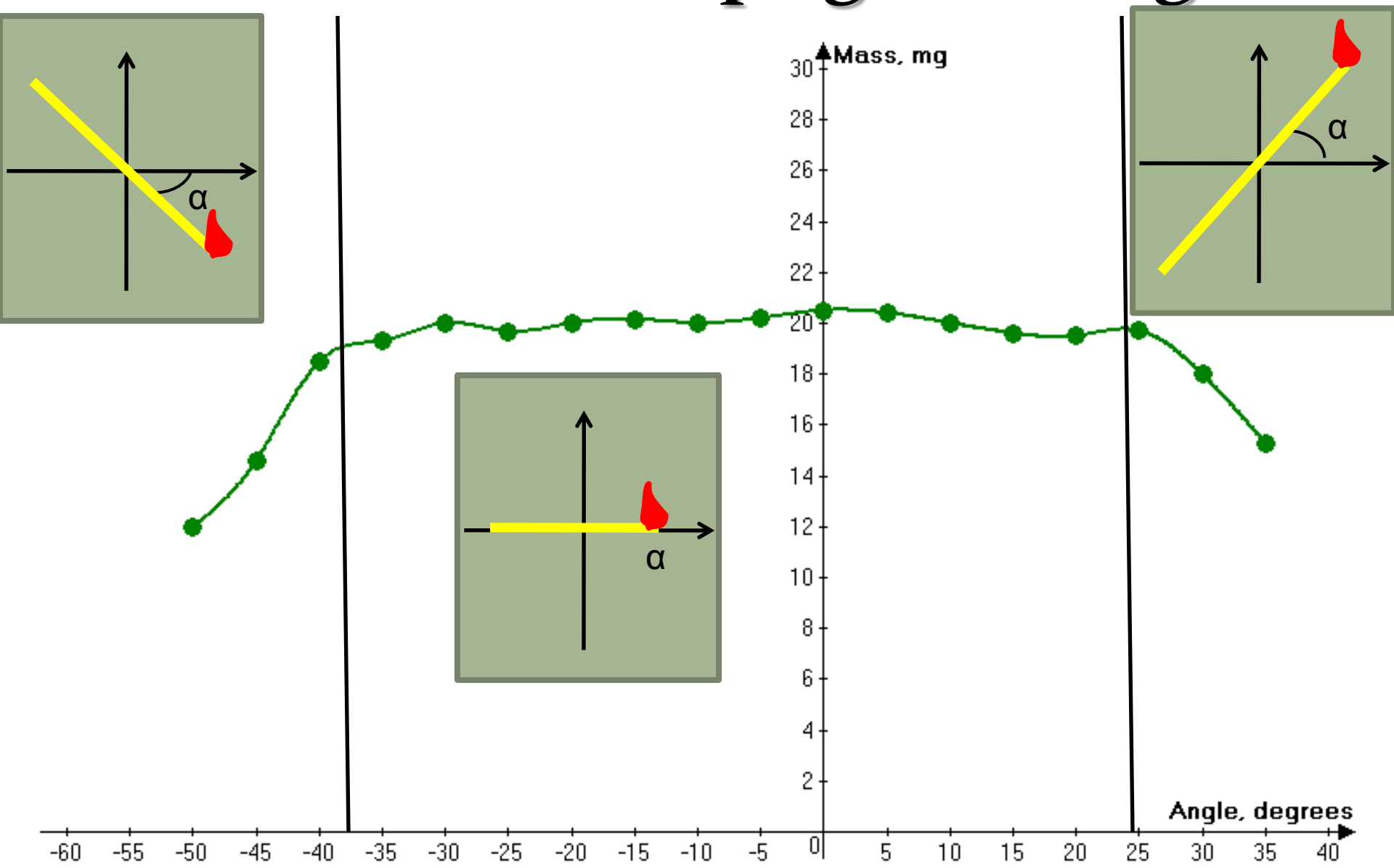
Leveling

Stearin engine

Leveling oscillogramm

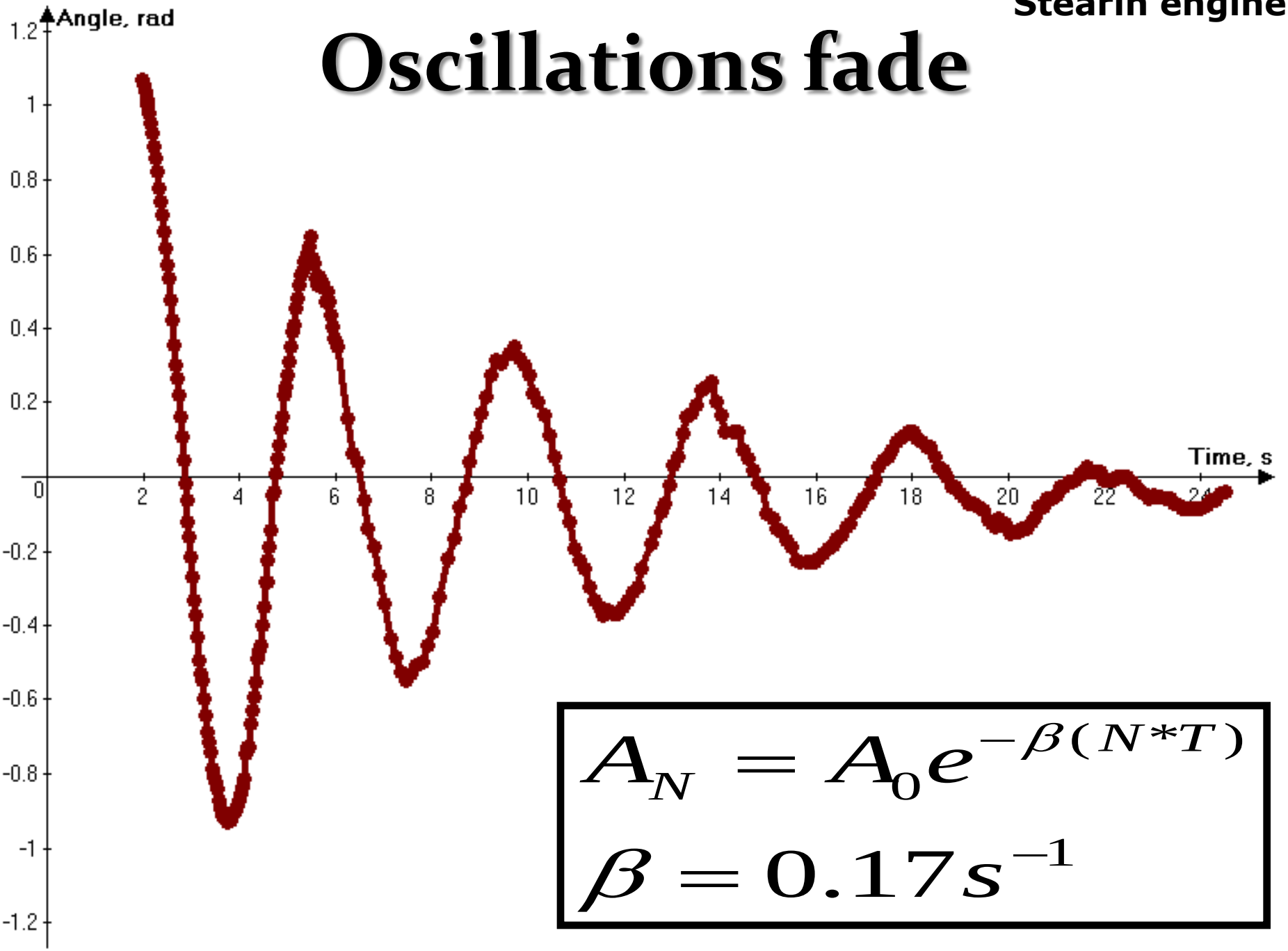


Plot mass of drop against angle



Oscillations fade

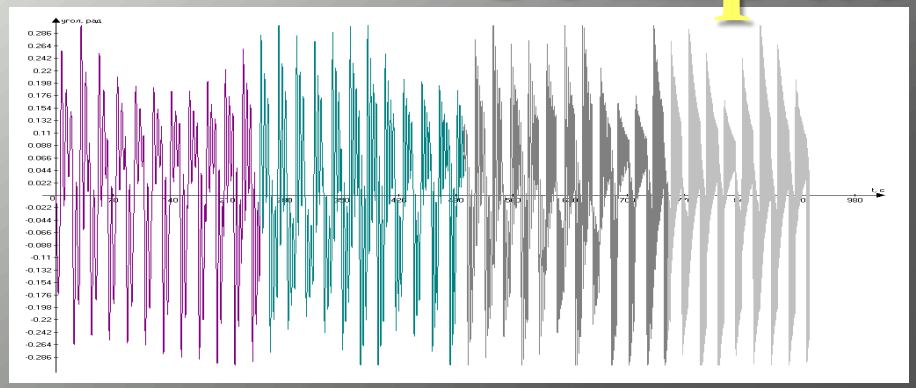
Oscillations fade



$$A_N = A_0 e^{-\beta(N*T)}$$

$$\beta = 0.17 s^{-1}$$

Computer Model



Input parameters (mass, length, angle, puncture)

Constants (viscosity, combustion speed)

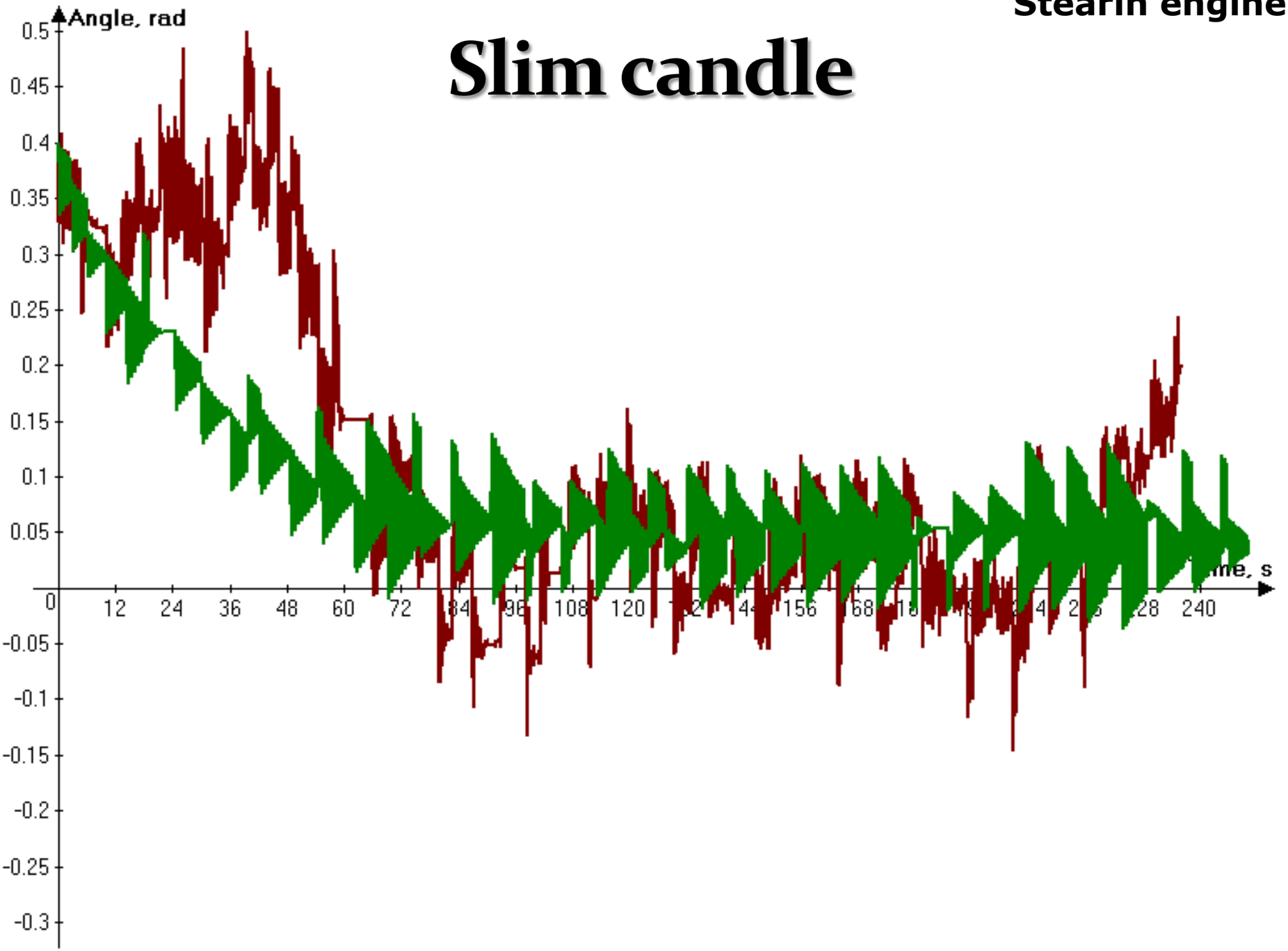
Combustion speed is thought to be random with average value equal to measured

Time is changed for small periods, for each we recount position of the candle

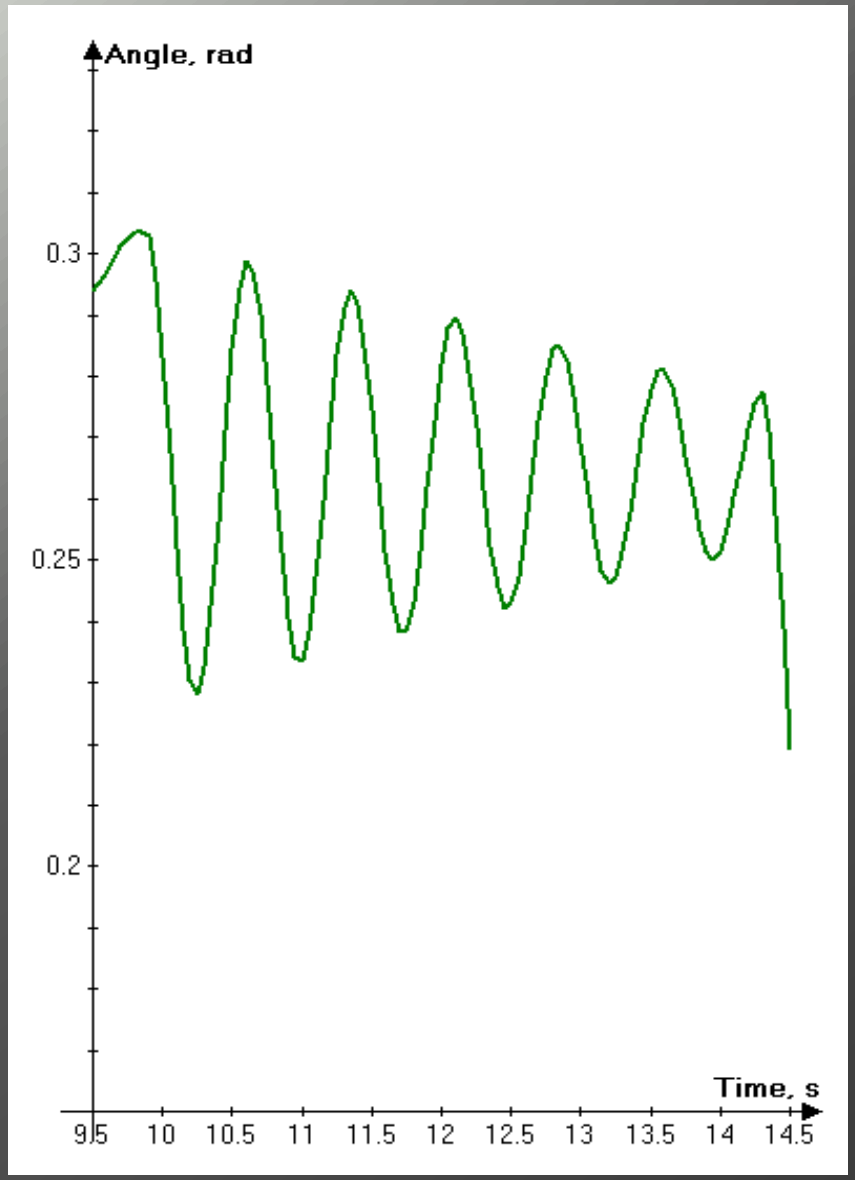
When the mass on one of the ends reaches critical, it drops

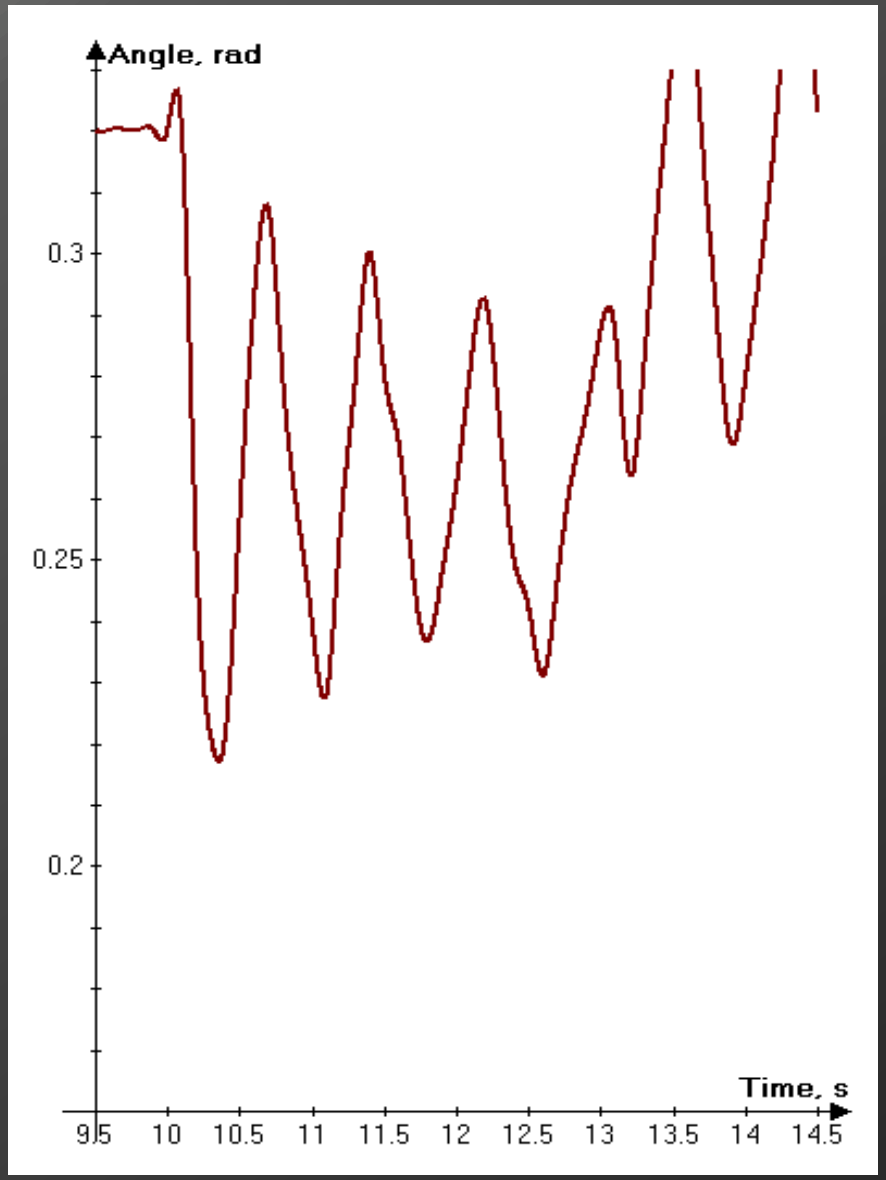
Stearin engine

Slim candle



Comparing


Angle, rad
0.3
0.25
0.2
Time, s
9.5
10
10.5
11
11.5
12
12.5
13
13.5
14
14.5


Angle, rad
0.3
0.25
0.2
Time, s
9.5
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10.5
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11.5
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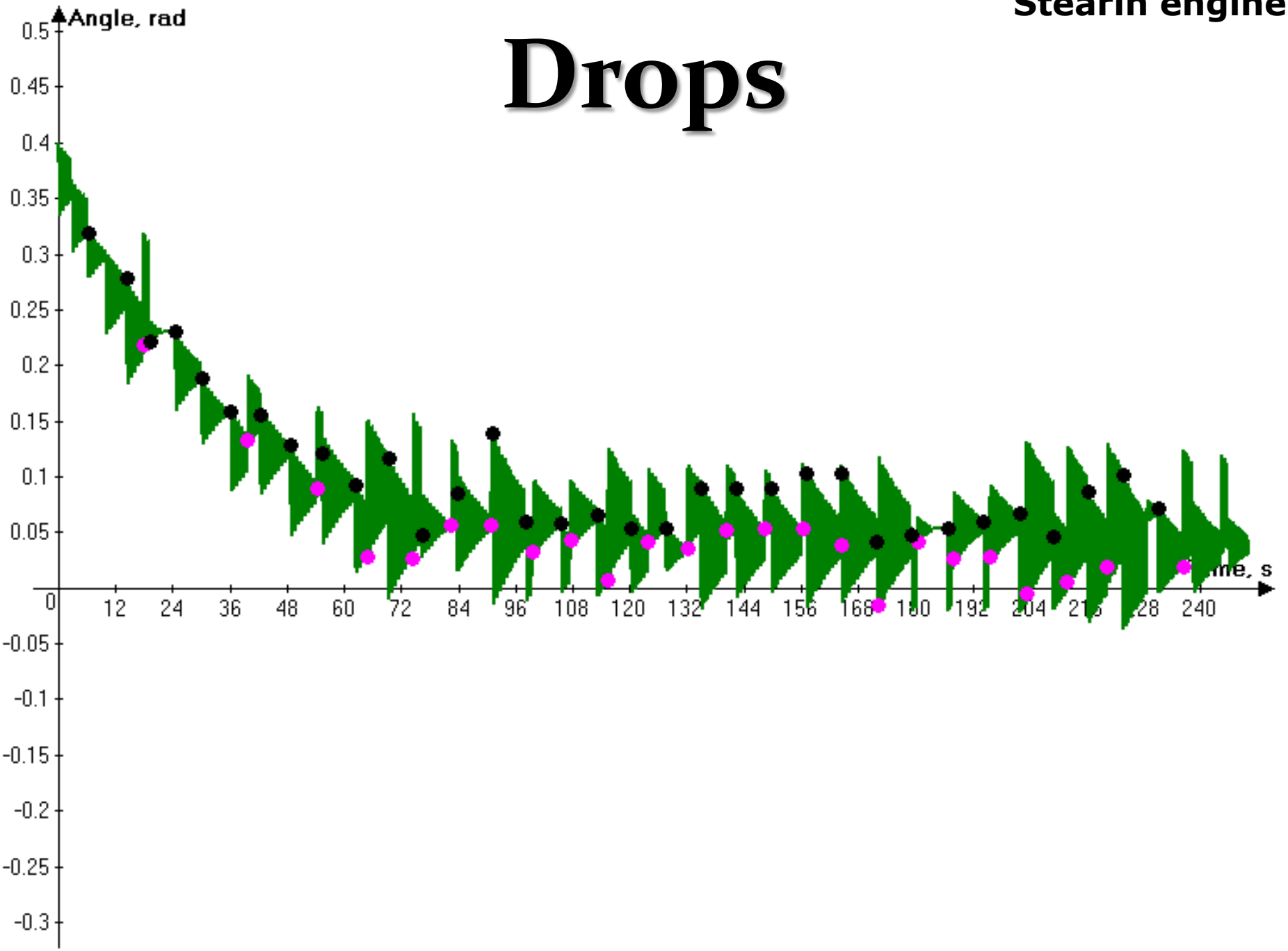
12.5
13
13.5
14
14.5

Stearin engine

Drops



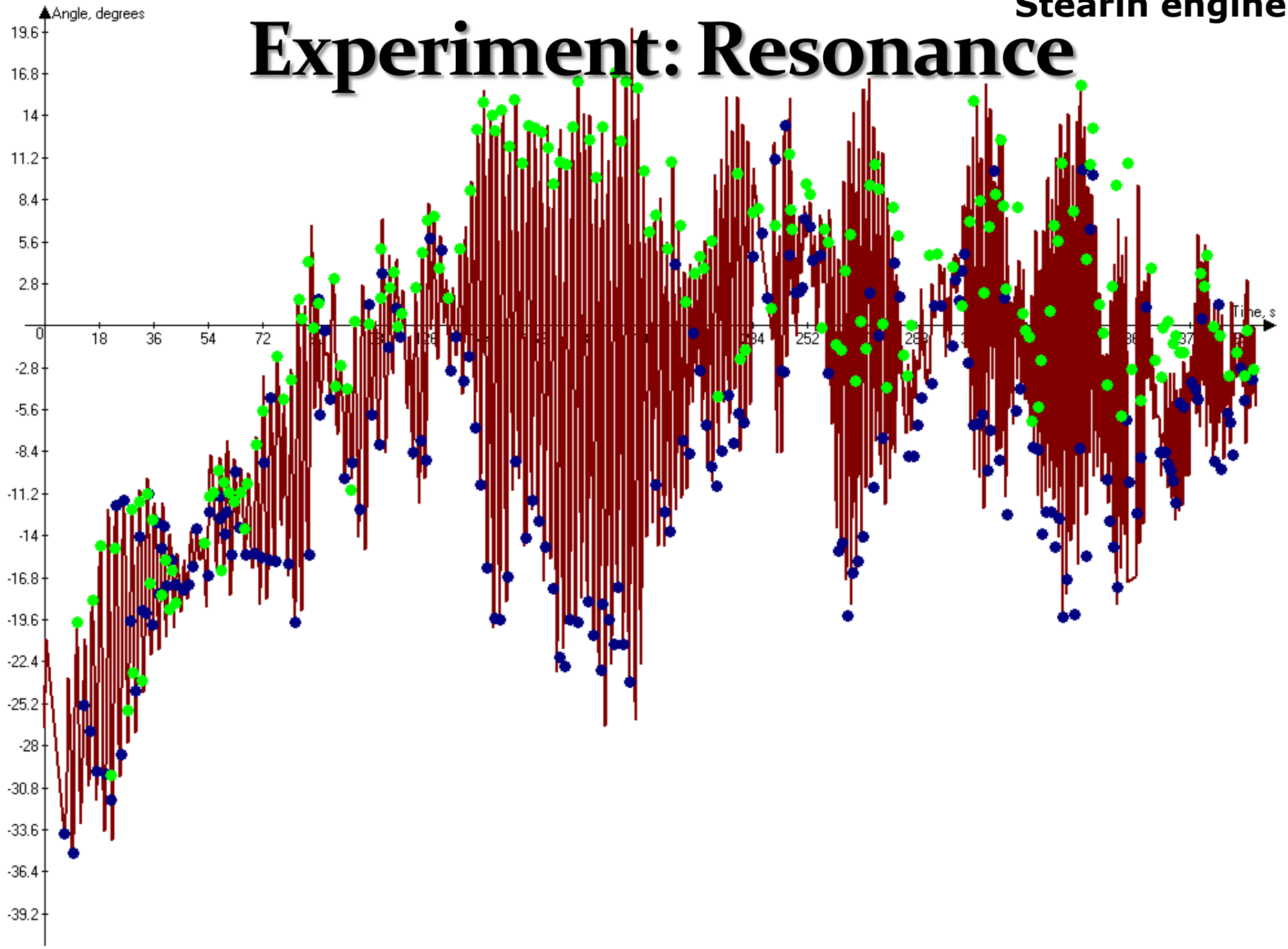
Period of oscillations

Free oscillation period of a candle:

$$T = 2\pi\sqrt{\frac{I}{mgd}} = \pi\frac{l}{\sqrt{3gd}}$$

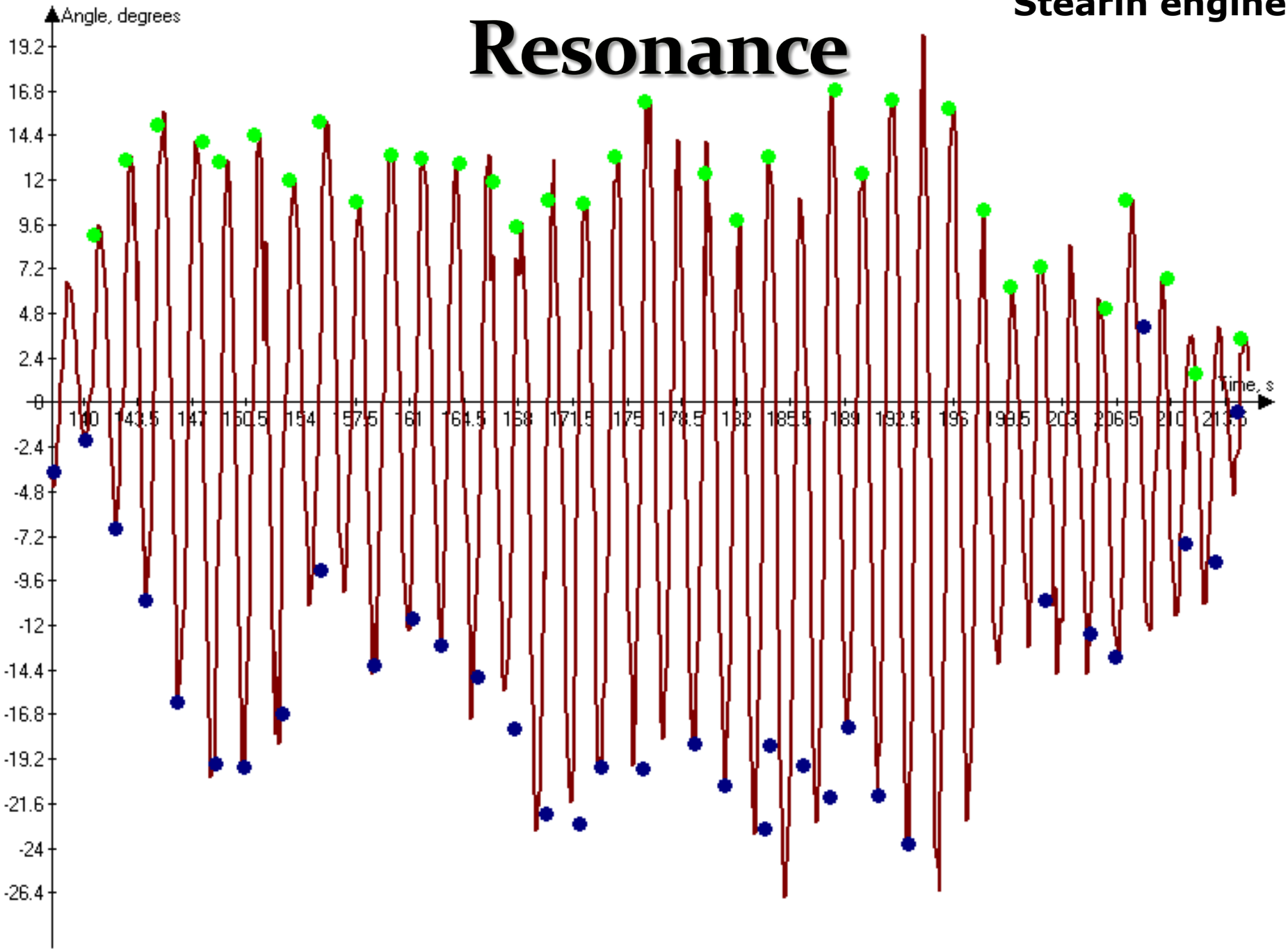
Resonance

Experiment: Resonance



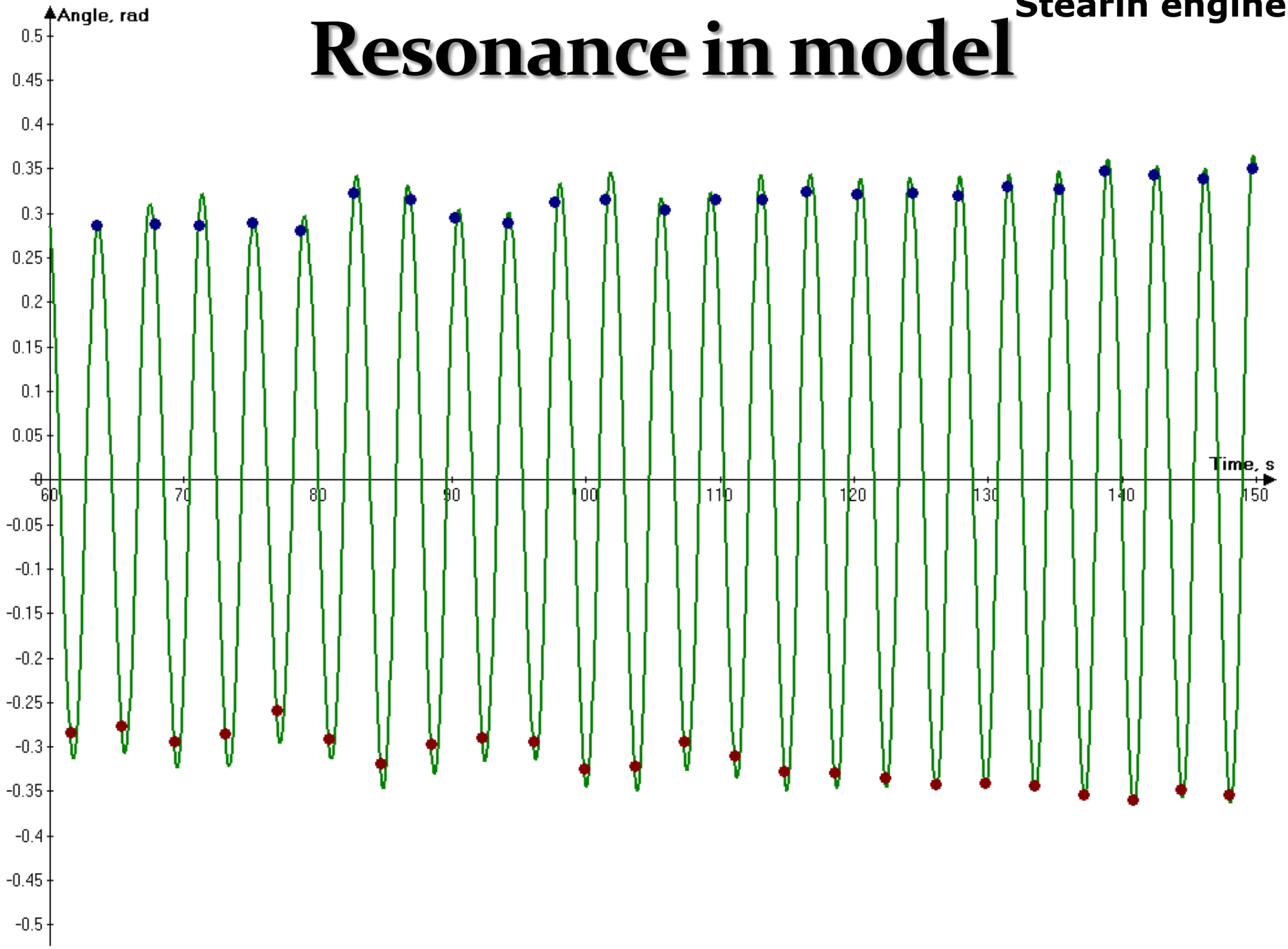
Resonance

Stearin engine

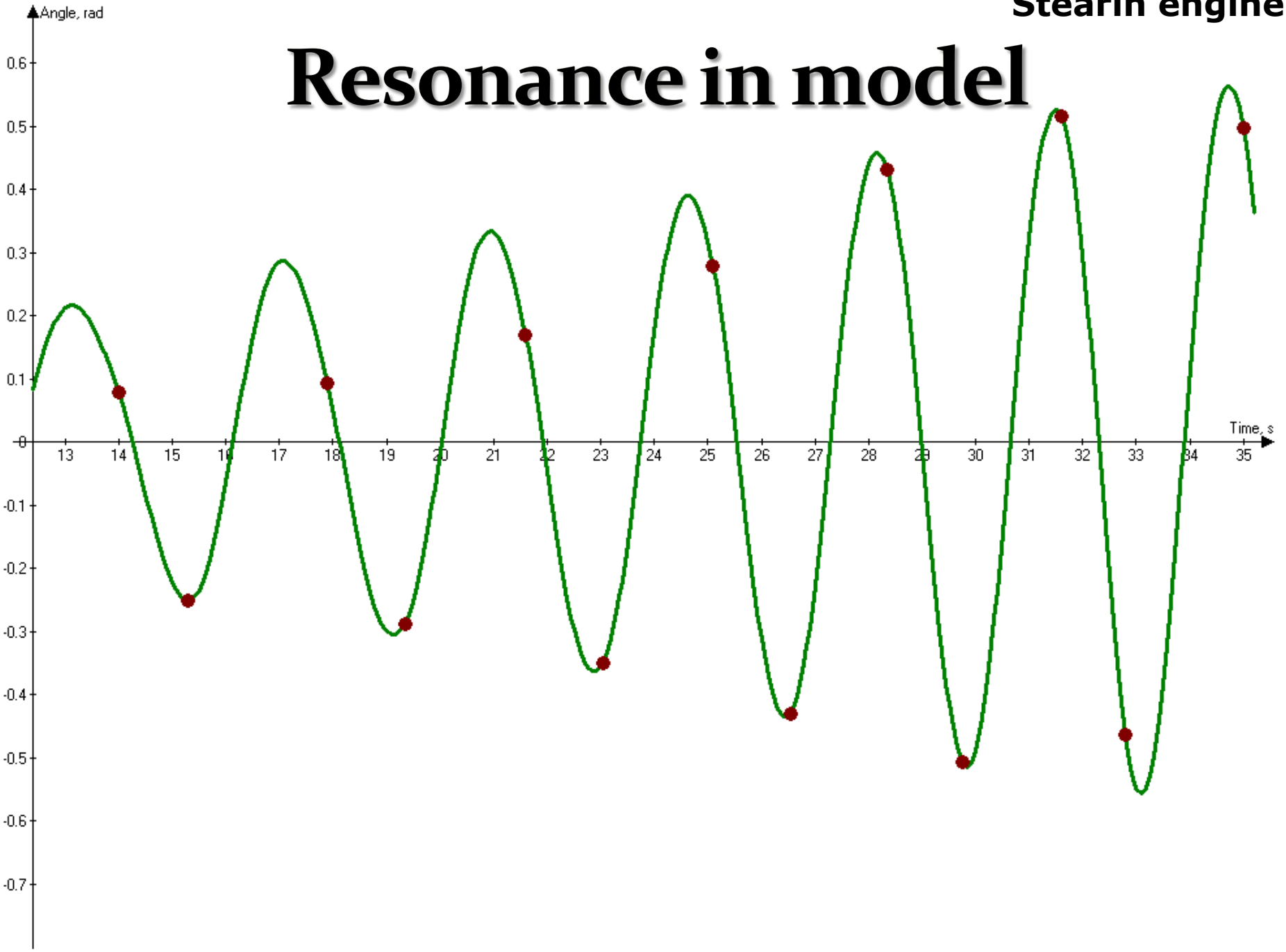


Resonance in model

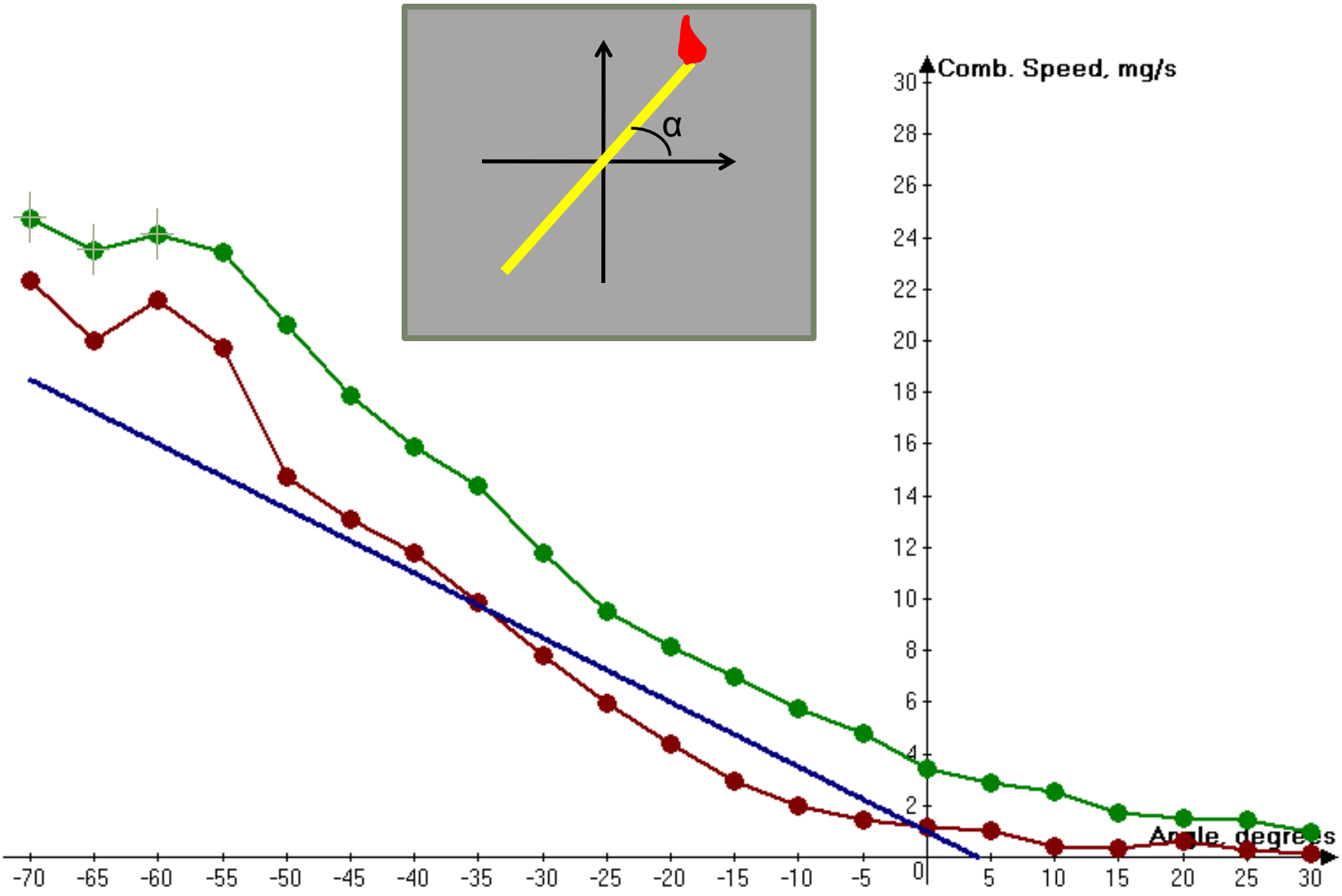
Stearin engine



Resonance in model



Plot ξ against angle



Resonance conditions

We'll suggest, that resonance occurs, when $T_{free} = T_{drop}$, and will find conditions, needed for the resonance. To do this, we'll find T_{drop}:

$$m_d = \int_0^{T_{drop}} \xi_i dt = \int_0^{T_{drop}} (2.85 - 0.25A\sin(\zeta_0 + \omega t))dt$$

Resonance conditions

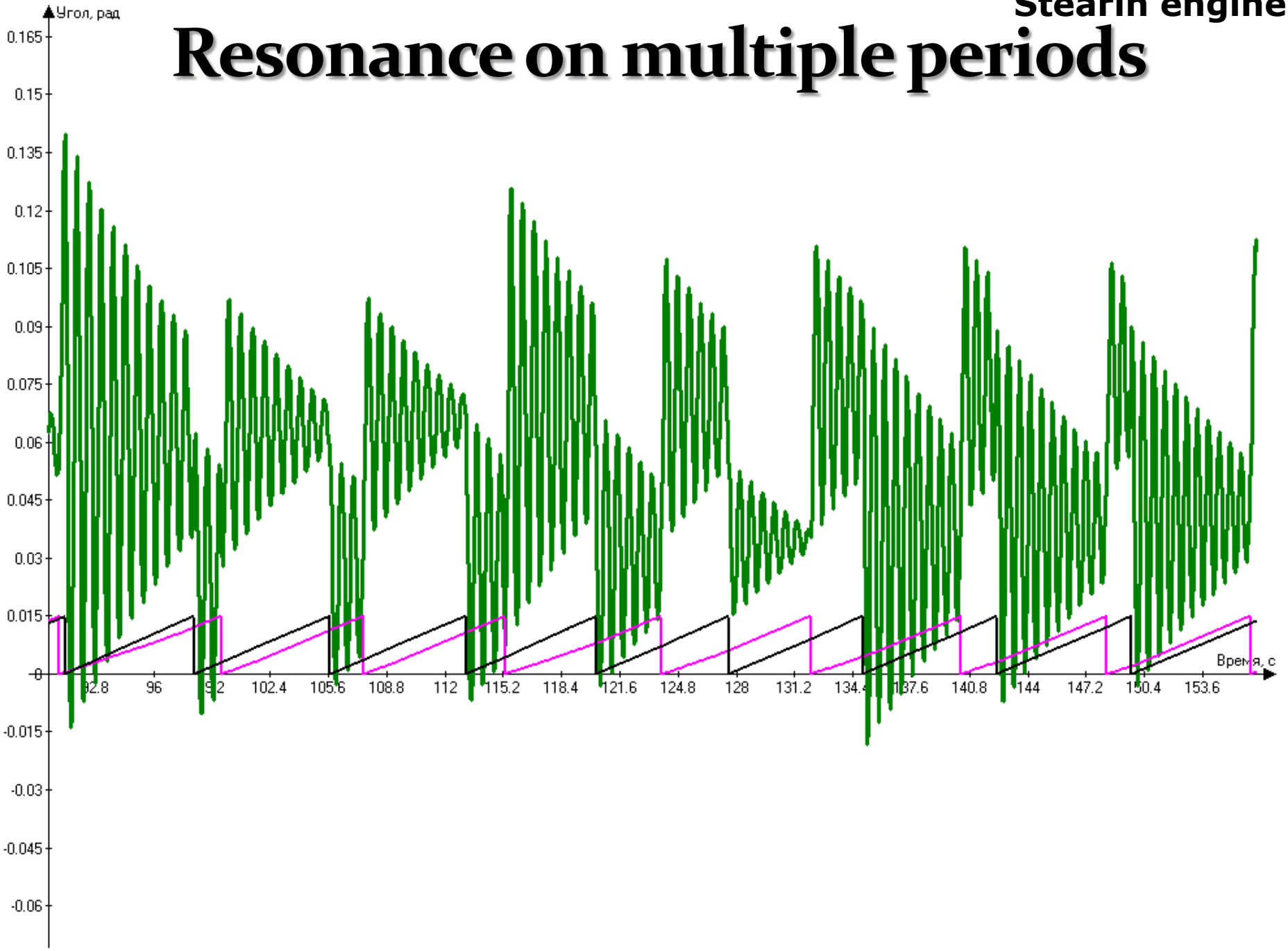
$$m_d = 2.85T + \frac{A\cos(\zeta_0 + 2\pi)}{8\pi}T;$$

$$m_d = \frac{l}{\sqrt{3gd}}\left(2.85\pi + \frac{\left|arctg\left(\frac{m_d}{2d\rho_l}\right)\right|\cos\zeta_0}{8}\right);$$

$l \approx 250sm$ ***d*=0.001 m *R*=0.003 m**

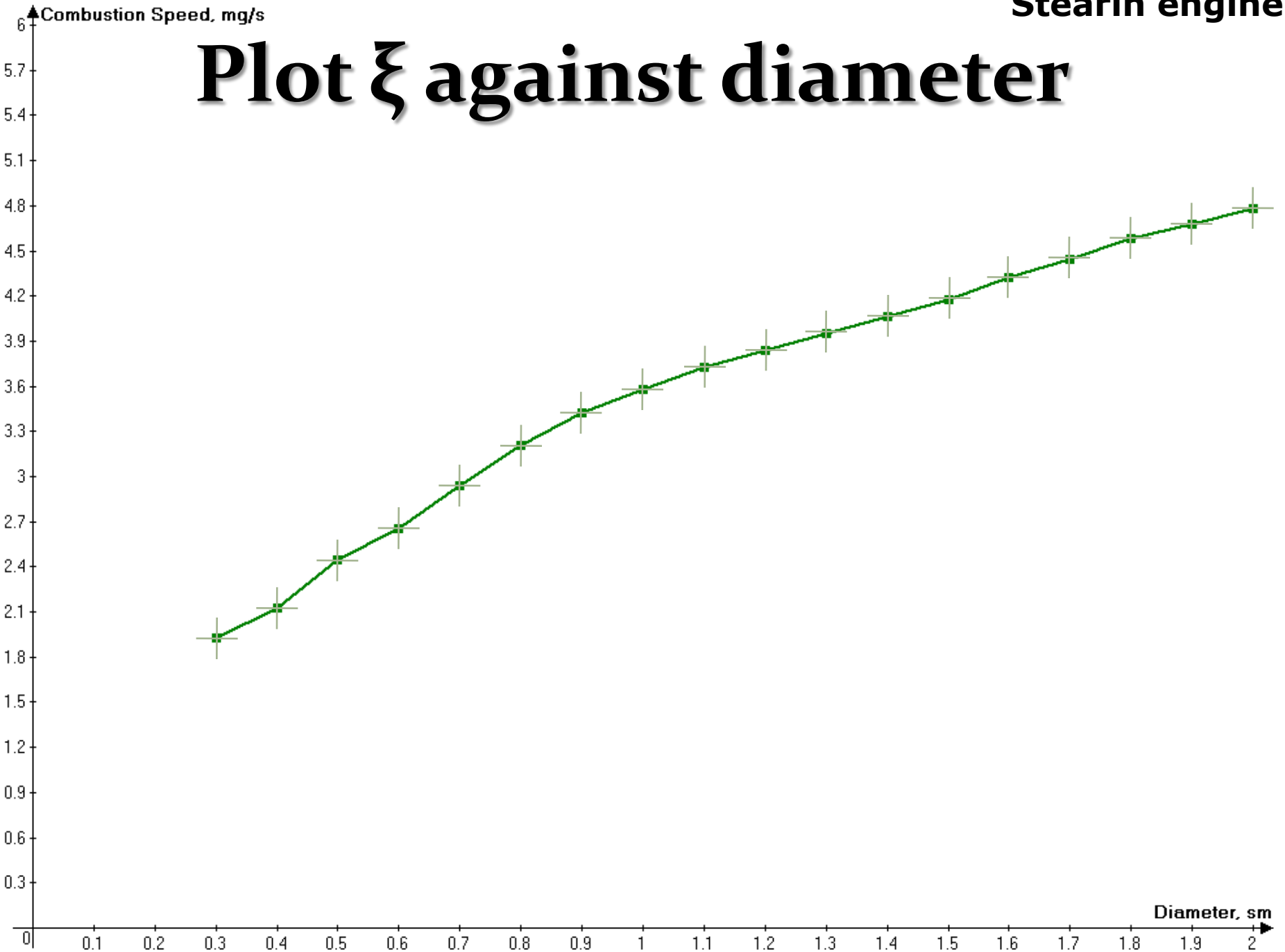
Stearin engine

Resonance on multiple periods

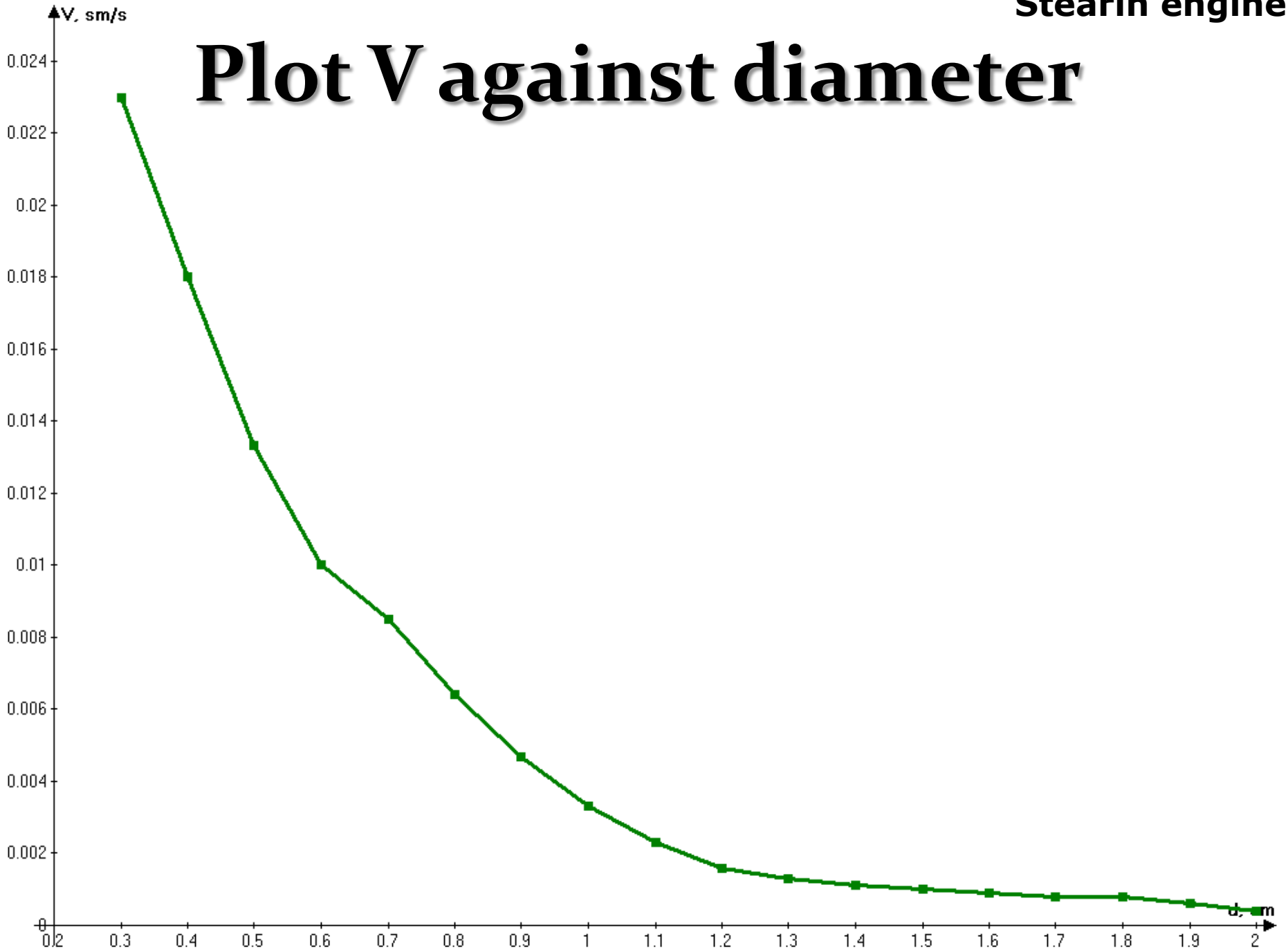


Stearin engine

Plot ξ against diameter

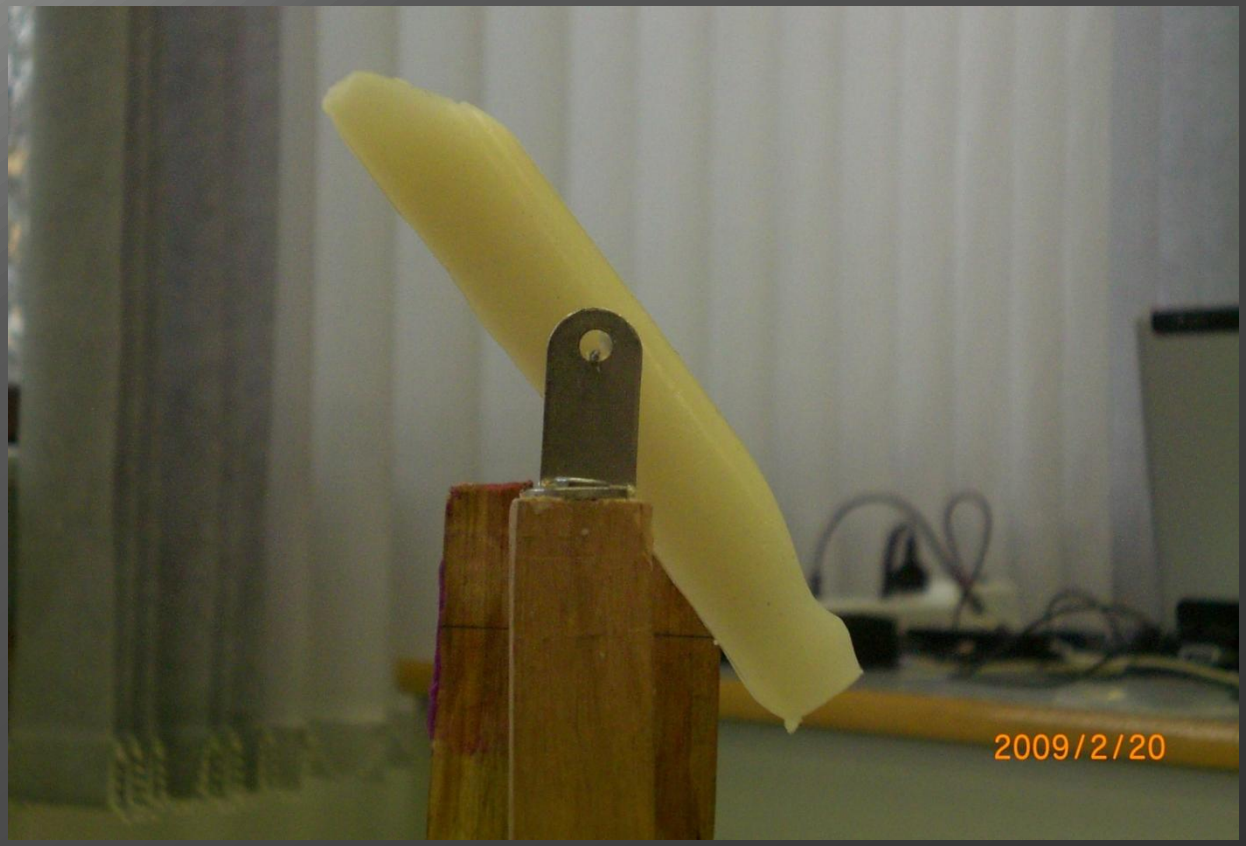


Plot V against diameter

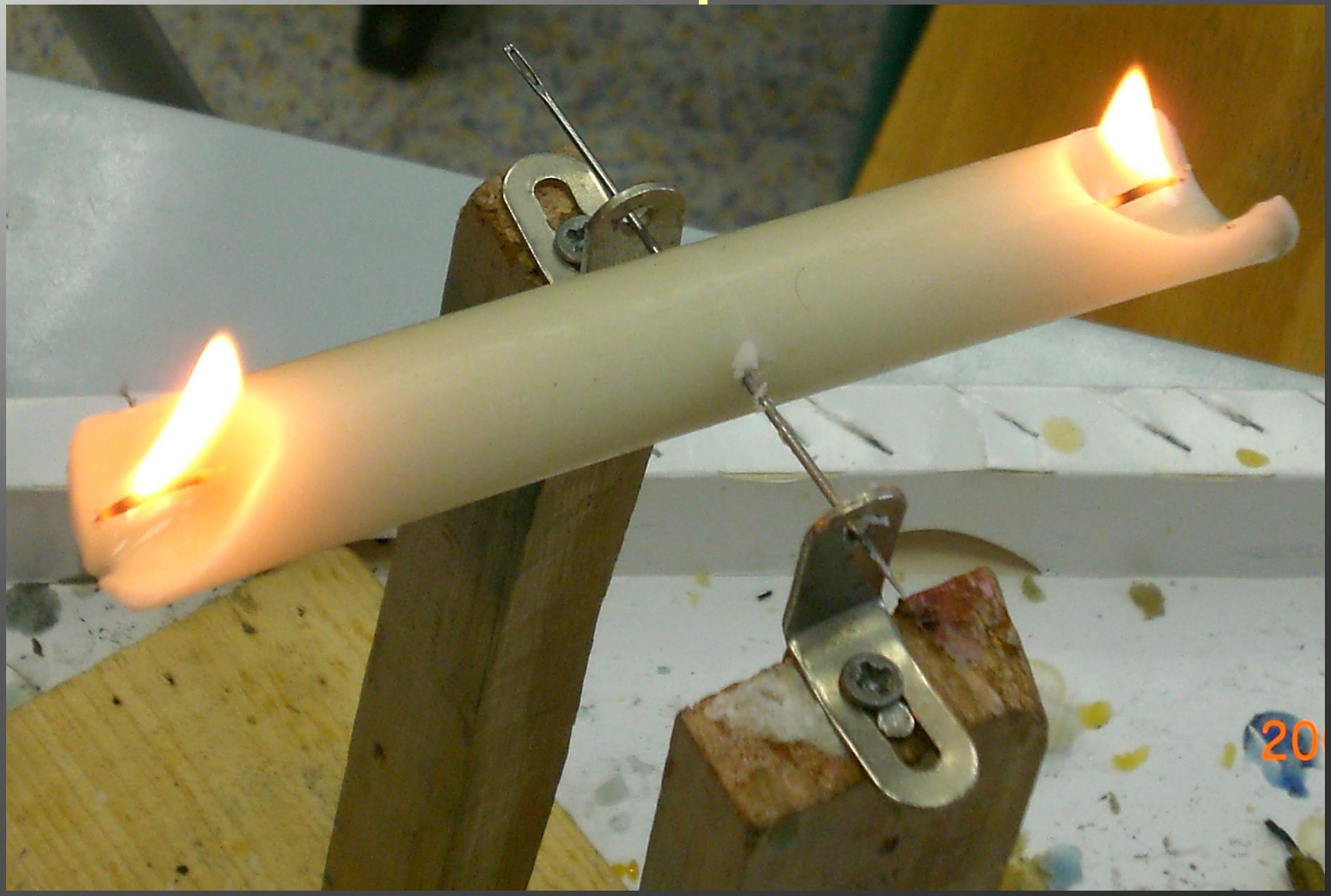


Massive candles

We will separate a class of massive candles. Massive candles are such ones, which can't move when one drop falls. It means that the moment of the force is less than moment of frictional forces. Such candle may not oscillate in basic mode, even if punctured at the center of the masses.



«Scoops»



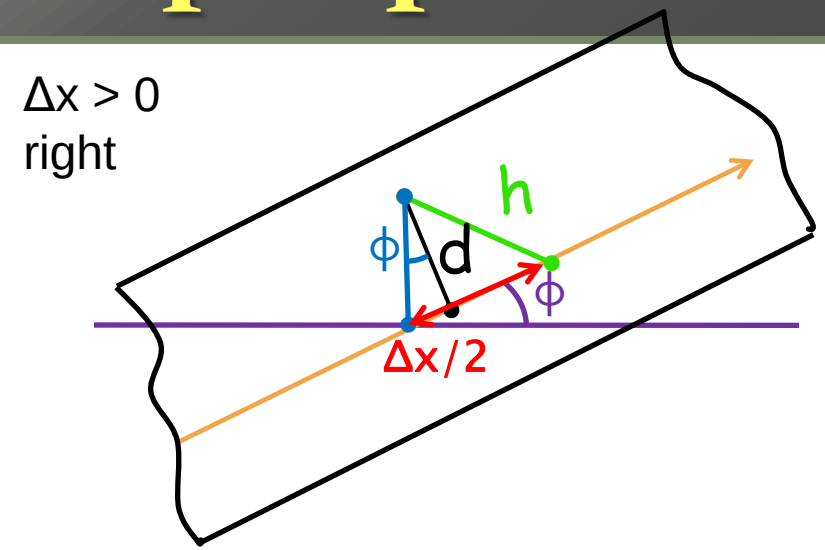
«Scoops»

Counting output power

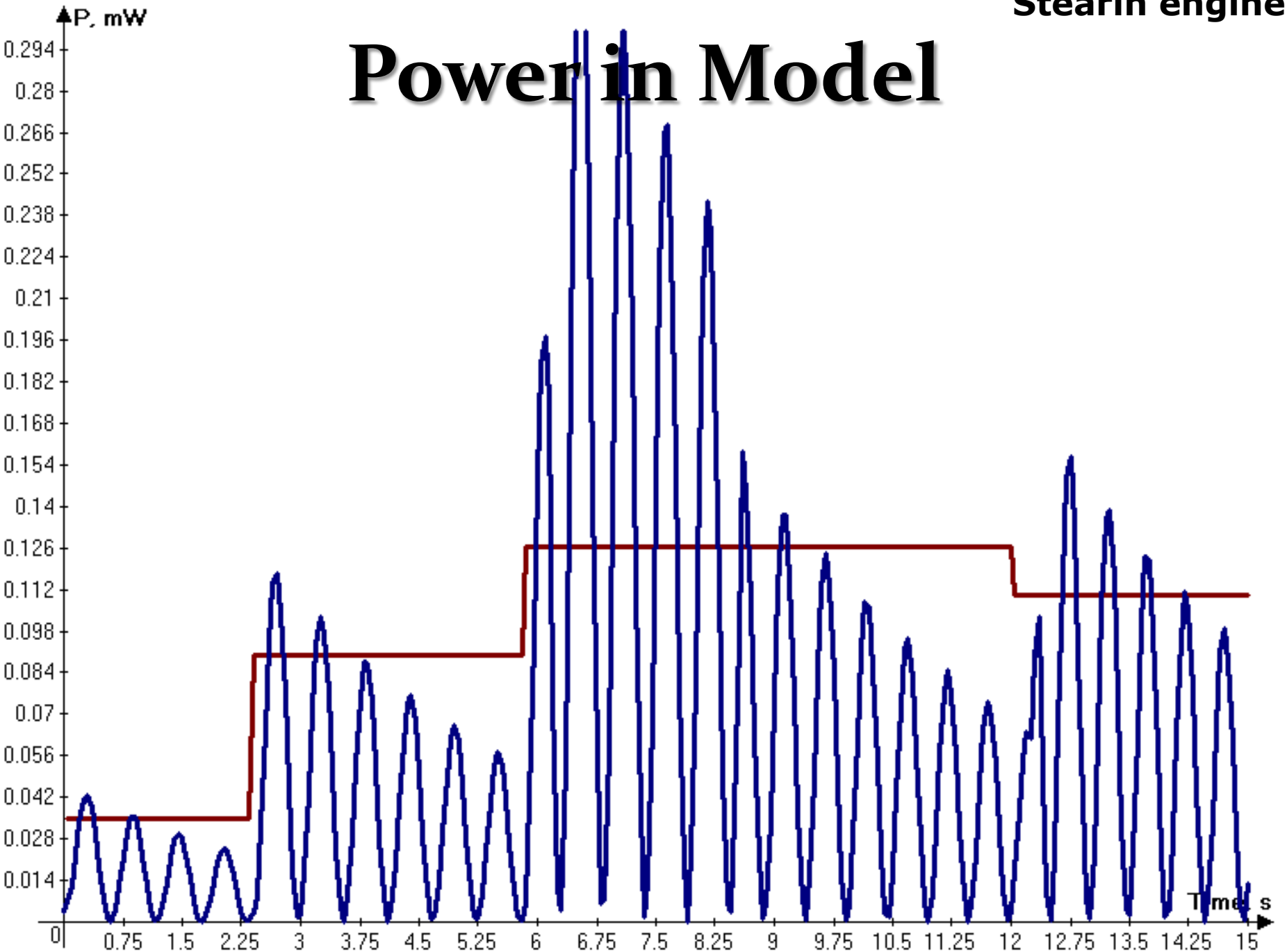
$$A_i = m_c g (h_2 - h_1)$$

$$P_i = \frac{A_i}{T_{drop}} = \frac{A_i}{N * T} =$$

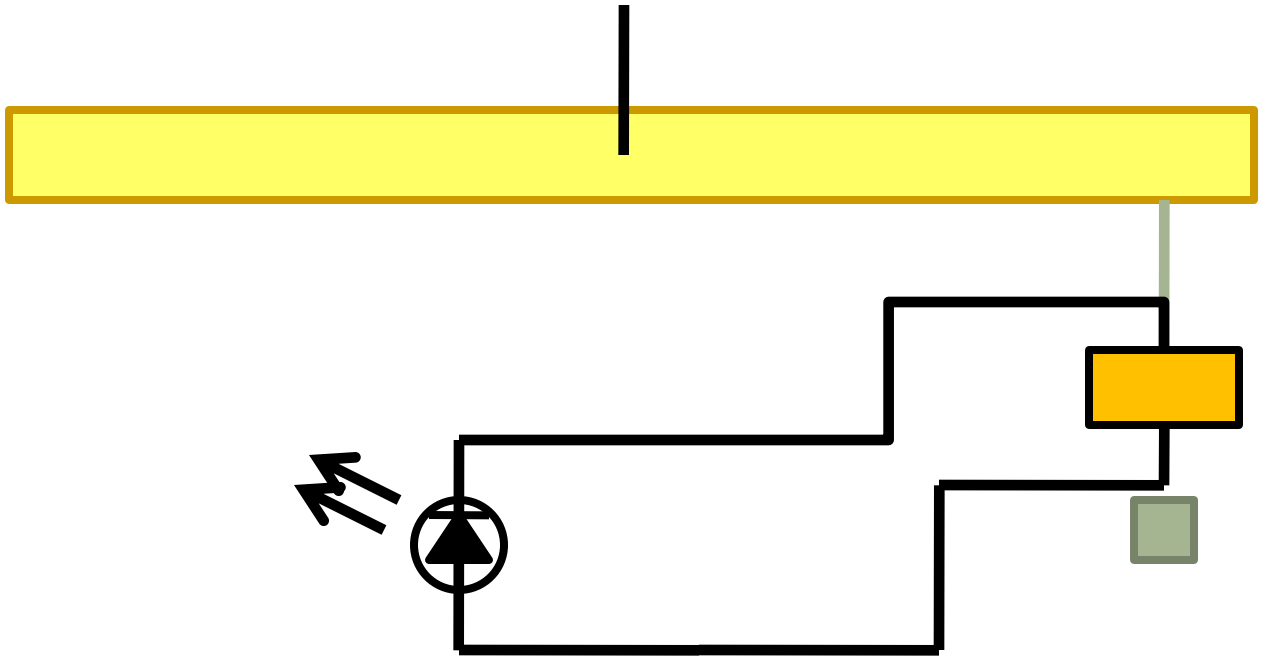
$$= \frac{mg\sqrt{d^2 + (\frac{\Delta x}{2} - d * tg\varphi)^2} - \frac{d}{\cos\varphi} + \frac{\Delta x}{2}\sin\varphi}{N * \pi l / \sqrt{3g\sqrt{d^2 + (\frac{\Delta x}{2} - d * tg\varphi)^2}}}$$



Power in Model

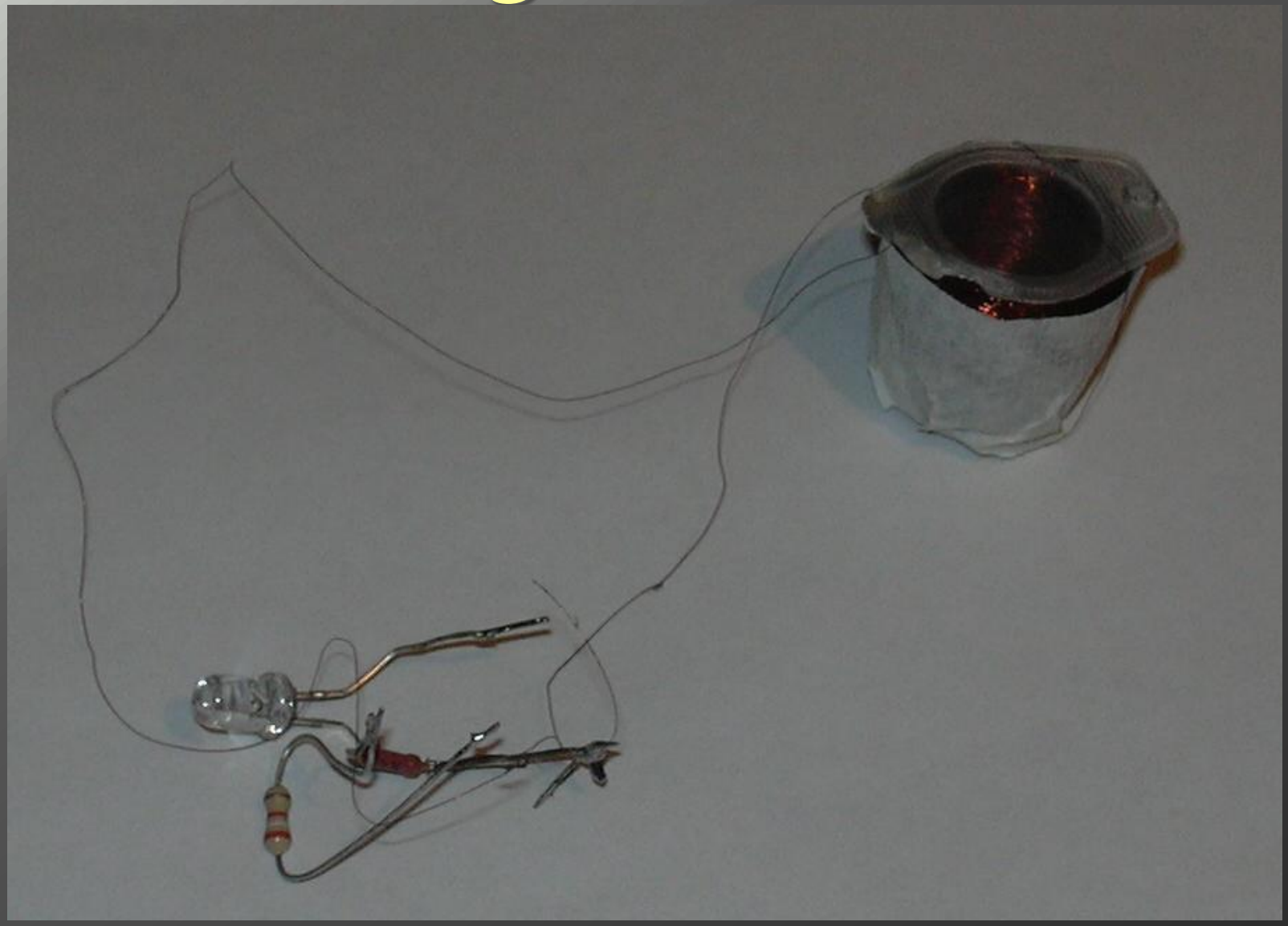


Usage of Power



$$P = \frac{<\varepsilon>^2}{R}; <\varepsilon> \sim \frac{\Phi_0}{T} \sim A\omega; P \sim A^2\omega^2$$

Usage of Power

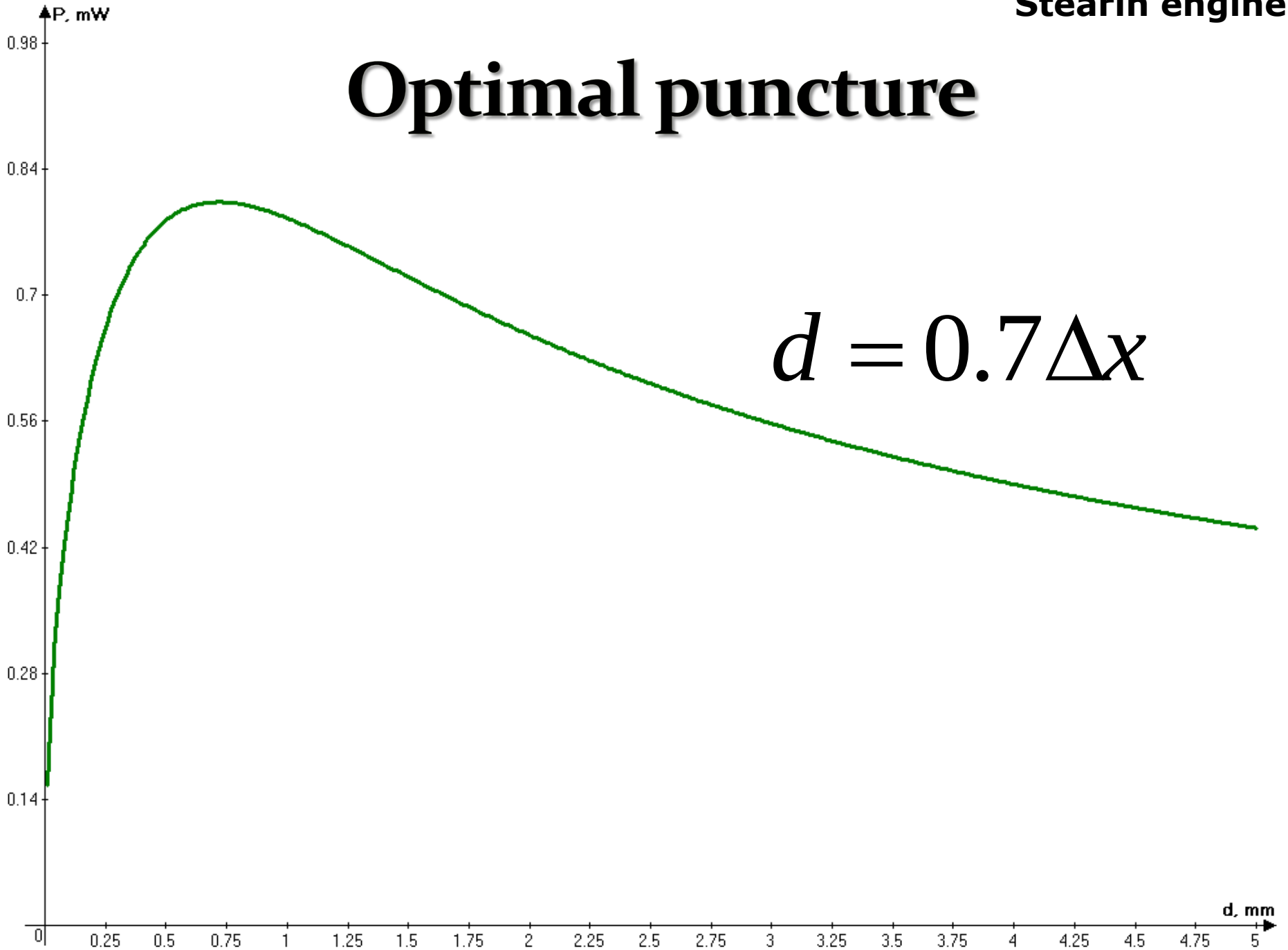


Output Power

$$P \sim A^2\omega^2 \sim \frac{arctg\left(\frac{\Delta x}{2d}\right)\sqrt{d}}{l}$$

$$\Delta x = \frac{\Delta m}{\rho \pi R^2}$$

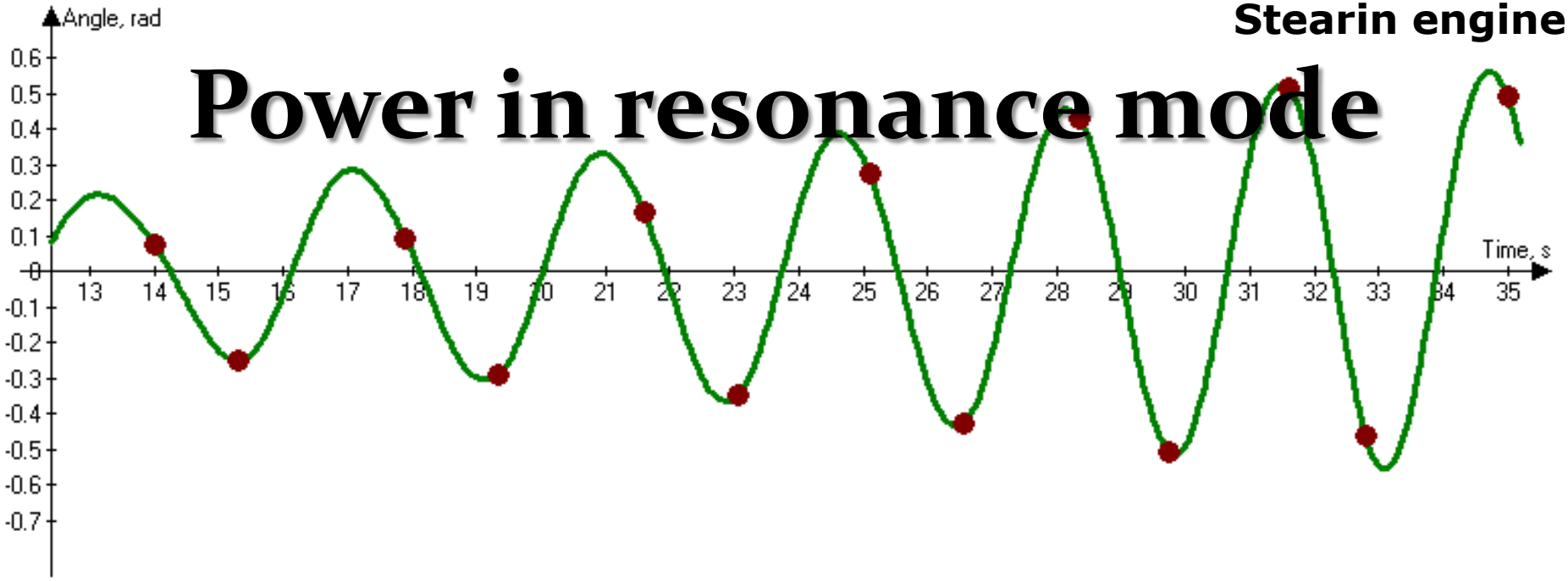
Optimal puncture

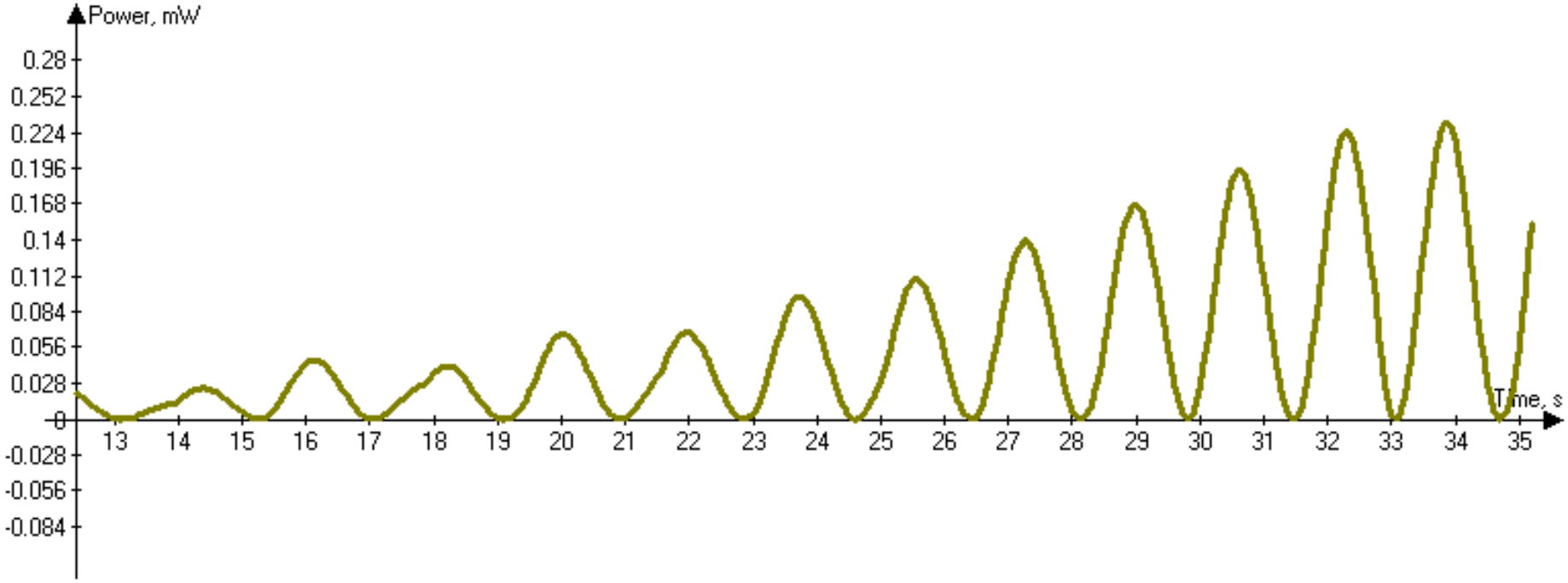


$$d = 0.7\Delta x$$

Stearin engine

Power in resonance mode





Conclusions:

- Stearin engine is not a heat machine
- We need to use candles from paraffin of about 1sm wide.
- The resonance effect is the basic of getting maximal power.
- The optimal puncture is needed for the power to be maximal.
- The best way of using Stearin engine is transforming its mechanical energy into electromagnetic.

Results:

- The effect of oscillations was explained
- We created a computer model, which lets us imitate effect of the problem
- We found the parameters, which the oscillations depend on.
- We defined optimal parameters for achieving maximal output mechanical power of the system.
- We created a device, transforming the power of oscillation into electromagnetic oscillations.

Literature:

- Slobodjanuk A. I. “Advance high school physic”
- Slobodjanuk A. I. “Computer model of physical processes for high school students”
- Y. A. Smorodinsky “Temprature”


Thanks for your
attention!

Some theoretical data

Температура плавления воска находится в пределах от 61 до 65 градусов.
При повышении температуры воска на один градус плотность воска уменьшается на 0,0008. При нормальных условиях плотность чистого воска колеблется от 0,96 до 0,98
Поверхностное натяжение - парафин 18-28 мН/м
Плотность парафина – 0.9
MFтр = 1*10-4 Н*м

How fast the candle burns

$$\xi = 50\sqrt{R} * e^{-0.04*\varphi}$$

Viscosity

$$M_{res} = 2 * \int_0^{l/2} kV_i l dl = 2\int_0^{l/2} k\omega l^2 dl = 2k\omega \frac{l^3}{3*8}$$

$$\Rightarrow \beta = \frac{kl^3}{12I} = \frac{kl^3}{ml^2} = \frac{k}{\rho_л} = \frac{k}{\rho * \pi R^2}$$

$$M_{fr} = \mu N * r = \mu mgr = \mu \rho l \pi R^2 r$$

The equation of candle movement

$$\varepsilon I = M_{mg} + M_{\Delta m} - M_{res} - M_{fr}$$

$$M_{mg} = mgd \sin\varphi$$

$$M_{\Delta m} = \Delta mgl / 2 \cos\varphi$$

$$\varepsilon I = M_o - M_{res} - M_{fr}$$

$$M_o = mg\sqrt{d^2 + \Delta x^2} \cos(\varphi_a - \varphi)$$