Solution of the problem "Stearin engine"

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Formulation of the problem

A candle is balanced on a horizontal needle placed through it near its centre of mass. When the candle is lit at both ends, it may start to oscillate. Investigate the phenomenon. Maximize the output mechanical power of the system.
Demonstration of the effect
Theoretical explanation

Puncture

Centre of the masses

Origin point

\[ \phi = arctg \left( \frac{\Delta x}{2d} \right) \]

\[ A = |\phi| = \left| arctg \left( \frac{\Delta x}{2d} \right) \right| \]

\[ \Delta x = \frac{\Delta m}{m} \cdot l = \frac{\Delta m}{\rho_l} \]
\[ \Delta x > 0 \text{ when drops from right end} \]

\[ \varphi = \arctg \left( \frac{\Delta x / 2 - d \cdot \tan \varphi_0}{d} \right) - \varphi_0 \]

\[ A = |\varphi| = \left| \arctg \left( \frac{\Delta x / 2 - d \cdot \tan \varphi_0}{d} \right) \right| - \varphi_0 \]
Puncture near the centre of the masses
Maximal Power

\[ A = mg \Delta h \]

\[ P_{\text{max}} = \frac{mg \Delta h}{T_{\text{drop}}} \]
Plot combustion speed ($\xi$) against angle $\alpha$. 

Stearin engine
Combustion speed

Gazes are mostly burning near the point, where temperature is maximal.

Wick is in low temperature area, where it is unable to burn.

Under the wick – a puddle of liquid paraffin. The gaze goes up.

Oxygen for the combustion goes from bottom upwards.

The candle material is the main source of energy.
Leveling
Plot mass of drop against angle

Stearin engine

\[ \alpha \]
Oscillations fade

Stearin engine
Oscillations fade

\[ A_N = A_0 e^{-\beta(N*T)} \]
\[ \beta = 0.17 \text{ s}^{-1} \]
When the mass on one of the ends reaches critical, it drops

Time is changed for small periods, for each we recount position of the candle

Combustion speed is thought to be random with average value equal to measured

Input parameters (mass, length, angle, puncture)

Constants (viscosity, combustion speed)
Stearin engine

Slim candle
Comparing Stearin engine
Drops

Stearin engine
Period of oscillations

Free oscillation period of a candle:

\[ T = 2\pi \sqrt{\frac{I}{mgd}} = \pi \frac{l}{\sqrt{3gd}} \]
Resonance

Stearin engine
Experiment: Resonance
Resonance in model

Stearin engine
Resonance in model
Plot $\xi$ against angle

Stearin engine
Resonance conditions

We’ll suggest, that resonance occurs, when \( T_{\text{free}} = T_{\text{drop}} \), and will find conditions, needed for the resonance. To do this, we’ll find \( T_{\text{drop}} \):

\[
m_d = \int_0^{T_{\text{drop}}} \xi_i \, dt = \int_0^{T_{\text{drop}}} (2.85 - 0.25A \sin(\zeta_0 + \omega t)) \, dt
\]
Resonance conditions

\[ m_d = 2.85T + \frac{A \cos(\zeta_0 + 2\pi)}{8\pi} T; \]

\[ m_d = \frac{l}{\sqrt{3gd}} \left( 2.85\pi + \arctg\left(\frac{m_d}{2d\rho l}\right) \right) \cos \zeta_0; \]

\[ l \approx 250 sm \quad d = 0.001 m \quad R = 0.003 m \]
Stearin engine

Resonance on multiple periods
Plot $\xi$ against diameter

Stearin engine

Combustion Speed, mg/s

Diameter, sm
Plot V against diameter
Massive candles

We will separate a class of massive candles. Massive candles are such ones, which can’t move when one drop falls. It means that the moment of the force is less than moment of frictional forces. Such candle may not oscillate in basic mode, even if punctured at the center of the masses.
Stearin engine

«Scoops»
Сочинский кулон
Counting output power

\[ A_i = m_c g (h_2 - h_1) \]

\[ P_i = \frac{A_i}{T_{drop}} = \frac{A_i}{N \cdot T} = \]

\[ mg \sqrt{d^2 + \left(\frac{\Delta x}{2} - d \cdot \tan \phi \right)^2} - \frac{d}{\cos \phi} + \frac{\Delta x}{2} \sin \phi \]

\[ = \frac{N \cdot \pi l}{\sqrt{3g \sqrt{d^2 + \left(\frac{\Delta x}{2} - d \cdot \tan \phi \right)^2}} + \frac{\Delta x}{2} \sin \phi} \]
Power in Model

Stearin engine

Graph showing the power output in model.
Usage of Power

\[ P = \frac{\langle \varepsilon \rangle^2}{R}; \langle \varepsilon \rangle \sim \frac{\Phi_0}{T} \sim A \omega; \quad P \sim A^2 \omega^2 \]
Usage of Power

Stearin engine
Output Power

\[ P \sim A^2 \omega^2 \sim \arctg\left(\frac{\Delta x}{2d}\right) \sqrt{d} \]

\[ \Delta x = \frac{\Delta m}{\rho \pi R^2} \]
Optimal puncture

\[ d = 0.7\Delta x \]
Power in resonance mode

Stearin engine
Conclusions:

- Stearin engine is not a heat machine.
- We need to use candles from paraffin of about 1sm wide.
- The resonance effect is the basic of getting maximal power.
- The optimal puncture is needed for the power to be maximal.
- The best way of using Stearin engine is transforming its mechanical energy into electromagnetic.
Results:

- The effect of oscillations was explained.
- We created a computer model, which lets us imitate the effect of the problem.
- We found the parameters, which the oscillations depend on.
- We defined optimal parameters for achieving maximal output mechanical power of the system.
- We created a device, transforming the power of oscillation into electromagnetic oscillations.
Literature:

- Slobodjanuk A. I. “Advance high school physic”
- Slobodjanuk A. I. “Computer model of physical processes for high school students”
- Y. A. Smorodinsky “Temprature”
Thanks for your attention!
Температура плавления воска находится в пределах от 61 до 65 градусов.
При повышении температуры воска на один градус плотность воска уменьшается на 0,0008. При нормальных условиях плотность чистого воска колеблется от 0,96 до 0,98
Поверхностное натяжение - парафин 18-28 мН/м
Плотность парафина – 0.9
МFтр = 1*10^-4 Н*м
How fast the candle burns

$$\xi = 50 \sqrt{R} \star e^{-0.04 \phi}$$
Viscosity

\[ M_{res} = 2 \int_{0}^{l/2} kV_i l dl = 2 \int_{0}^{l/2} k\omega l^2 dl = 2k\omega \frac{l^3}{3*8} \]

\[ \Rightarrow \beta = \frac{kl^3}{12I} = \frac{kl^3}{ml^2} = \frac{k}{\rho_l} = \frac{k}{\rho \pi R^2} \]

\[ M_{fr} = \mu N * r = \mu m g r = \mu \rho l \pi R^2 r \]
The equation of candle movement

\[ \varepsilon I = M_{mg} + M_{\Delta m} - M_{res} - M_{fr} \]

\[ M_{mg} = mgd \sin \varphi \]

\[ M_{\Delta m} = \Delta mgl / 2 \cos \varphi \]

\[ \varepsilon I = M_o - M_{res} - M_{fr} \]

\[ M_o = mg \sqrt{d^2 + \Delta x^2} \cos(\varphi_a - \varphi) \]