Problem:

- Look down a cylindrical metal tube which is shiny on the inside. You will notice dark and light bands. Investigate the phenomenon.
Plan of investigations:

- Define, if the problem can be explained using just geometrical optics, without involving wave effects;
- Explain the formation of the bands;
- Find a way of obtaining an image;
- Investigate the features of the image both for the point lamp and the “real” image (many lamps);
Part 1: One Radiator
To see the band we can use lens with a screen, use hole with a screen, or simply look with an eye.
Setup №1: Hole
No1: Obtained images
Setup №2: A lens
No2: Obtained images
The dependence on a wave length
Ray tracing

Visualizing the beams paths
Visualizing the beams paths and imaginary radiators
Angular position

The eye's position

\( \alpha \)
The angular position of the first ring according to the geometric optics:

\[ \alpha = \arctan\left( \frac{2 \cdot R}{L} \right) \]

\[ S = \frac{R}{L} \]

\[ \alpha = \arctan(2 \cdot S) \]

Angular position of the nth ring:

\[ \alpha = \arctan\left( 2n \cdot S \right) \]
Position of the first ring
Non-point light source

The angular “width” of the first ring:

\[
\alpha = \arctan \left( \frac{2 \cdot (R + r)}{L} \right) - \arctan \left( \frac{2 \cdot (R - r)}{L} \right)
\]

But the imaginary width is constant:

\[
h = \text{const} = 2 \cdot r
\]
Non-point light source
Changing the tube length
The lamp out of axis
Optical Tube

Computer model
Comparing Optical Tube
Changing lamp’s position
Boundaries
Part 2: Image
The real image

Direct image («zero zone»)
1st Boundary
1st Zone
2nd Boundary
2nd Zone
3rd Boundary
3rd Zone
\[ \alpha = \arctan((2 \cdot n + 1)S) \]

Zone beginning

Zone end

\[ \alpha = \arctan((2 \cdot n - 1)S) \]
For our case $\alpha_1 = 0.36 \text{ rad}$.

\[ \alpha = 2 \times \arctan \left( \left( n + 1 \right) S \right) \]

$n$ – Zone number

These rays are seen in the direct image.
Angle of view increase
Angle of view increase

Lamp

Metal tube

The axis of the tube
Angle of view increase
Observed number of zones
Because of the human’s vision properties we cannot see object outside the zone of 35 degrees. The angle on the n^{th} ring (for central point lamp):

$$\alpha = \arctan(2n \cdot S)$$

So, the number of rings:

$$n = 0.32 \frac{1}{2 \cdot S}$$
Intensity fading
Fading in a model
Fading in a model

Optical Tube
Experimental fading
Glass Tube
Interesting effects
The bands formation can be explained without involving wave effects. The bands are formed because of reflection on the inner side of the tube. We can divide an obtained image into zones. Each zone is formed by rays, which reflected same times. The boundary between zones is an image of the tube boarder. We cannot observe all the image zones due to the low intensity.
The formation of the bands is explained.
The computer model, which can count the image for different parameters of the tube is created.
The equations for counting the “zones” are obtained.
Literature

- Wikipedia: Whispering galleries
- Slobodjanuk A. I. “Advance high school physic”
- Slobodjanuk A. I. “Computer model of physical processes for high school students”
- Landsberg “Optics”
Thanks for your attention!
Some data

- Best vision zone 1.5 degrees.
- Clear vision zone 15 degrees.
- Maximum vision zone 35 degrees.

- The first ring = 0.19 rad
- S = 3mm
- First zone = 0.095 rad
- Second zone = 0.3 rad
The most common equation

\[ \alpha = \arctan \left( \frac{2 \cdot R \pm r}{L + l} \right) \]
The hole
3-D Graph $a(R,L)$
From where can we see it?

\[ \alpha = \arctan((n + 1)S) \]

\( n - \text{Zone number} \)

From here we can see the direct image & all zones
If we are away from tube

\[ N = \frac{l}{(l + L)} - 1 \]

\[ 35 > \arctan\left(\frac{R}{l}\right) \]