BELARUSIAN TEAM PRESENTS

Roundabout

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Formulation of the problem

6. Roundabout

Put a plastic cup on a thin layer of liquid on a flat solid surface.

Make the cup rotate.

On what parameters does the rotational deceleration of the cup depend?

Contents:

I. The forces of deceleration

$$F_v \sim A \cdot \mu$$

Viscosity of water and air:

$$\mu_{water} = 1.00 \times 10^{-3}\, Pa \cdot s$$

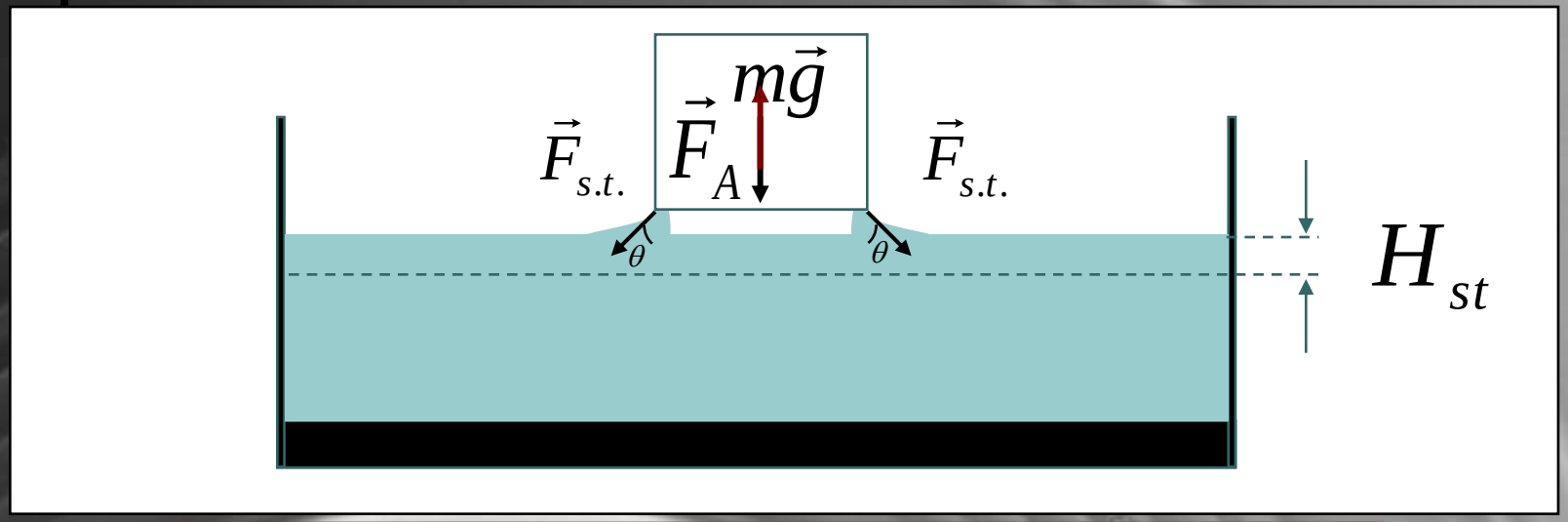
$$\mu_{air} = 17.4 \times 10^{-6}\, Pa \cdot s$$

$$\cancel{F_{air} \approx 8.7 \times 10^{-8}}$$

$$F_{water} \approx 4.5 \times 10^{-6}$$

The main forces of deceleration

Subsidence of static cup



$$H_{st} = \frac{m}{\pi \cdot \rho \cdot R^2} + \frac{2 \cdot \sigma}{\rho \cdot g \cdot R} \cdot \cos\theta$$

m – mass of the cup

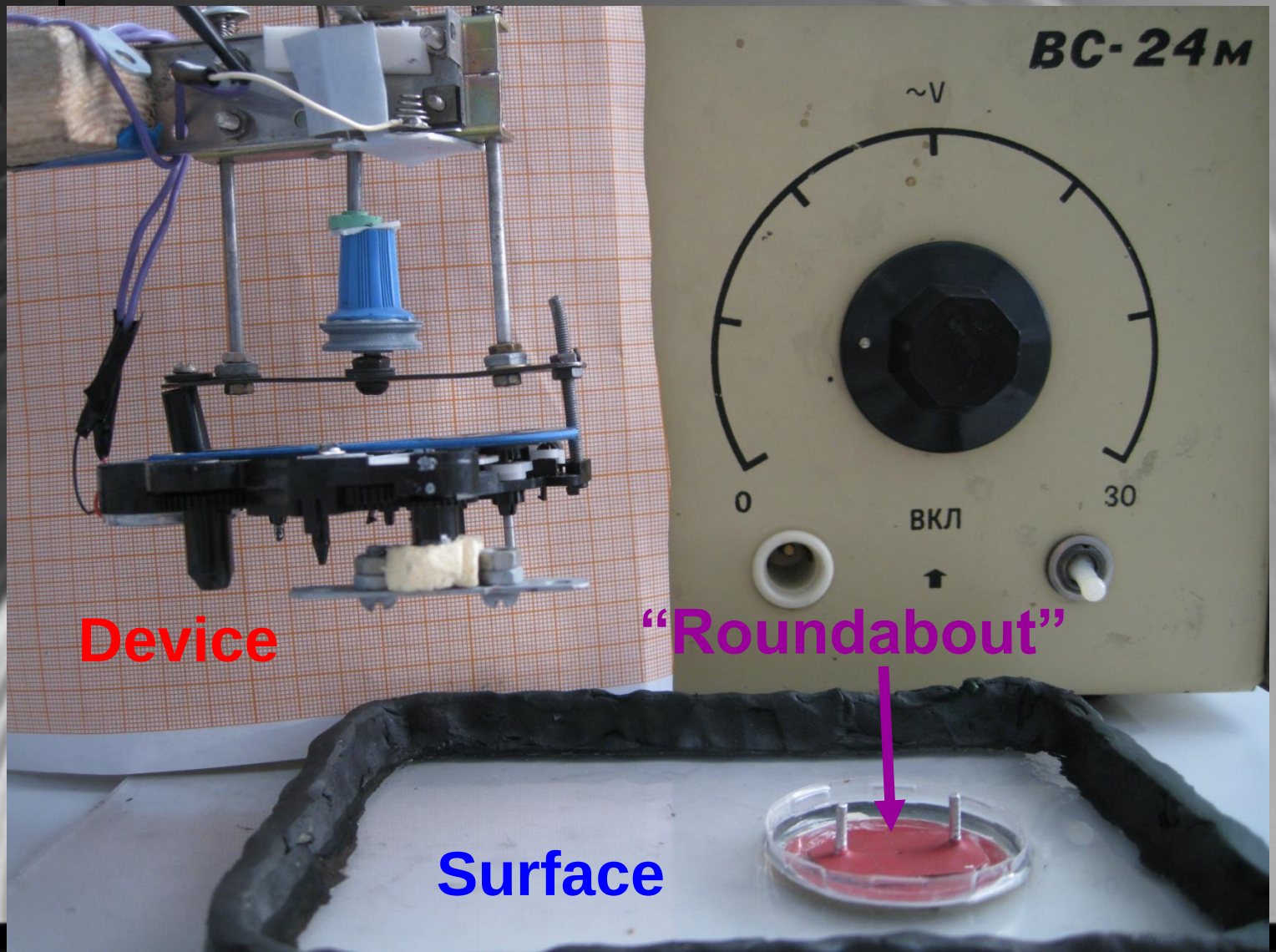
R – radius of cup

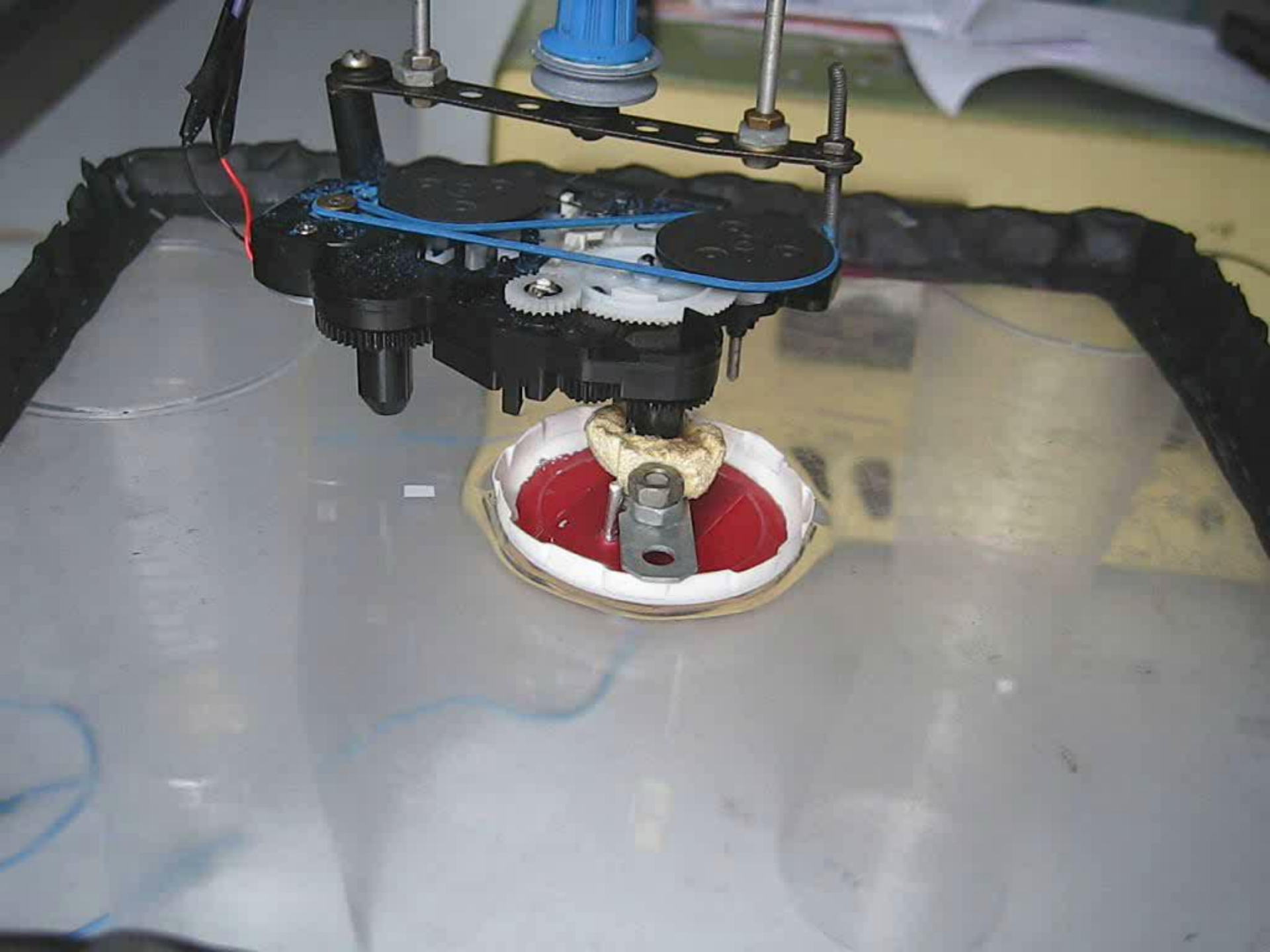
ρ – density of liquid

θ – angle of contact

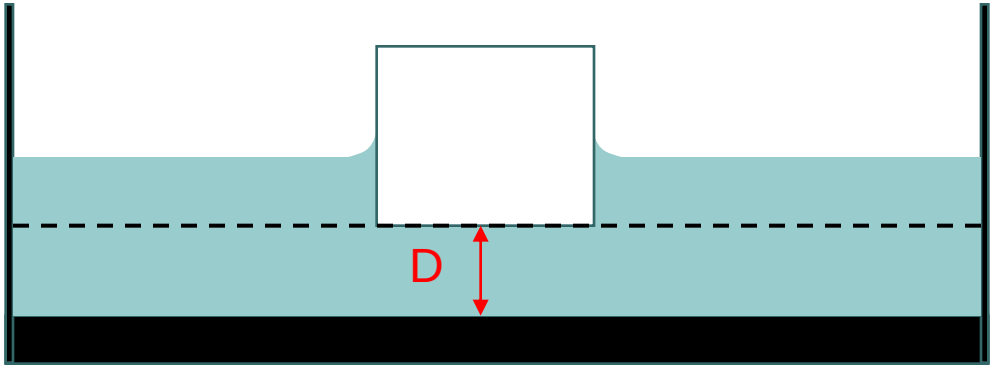
σ – surface tension of liquid

Experimental setup

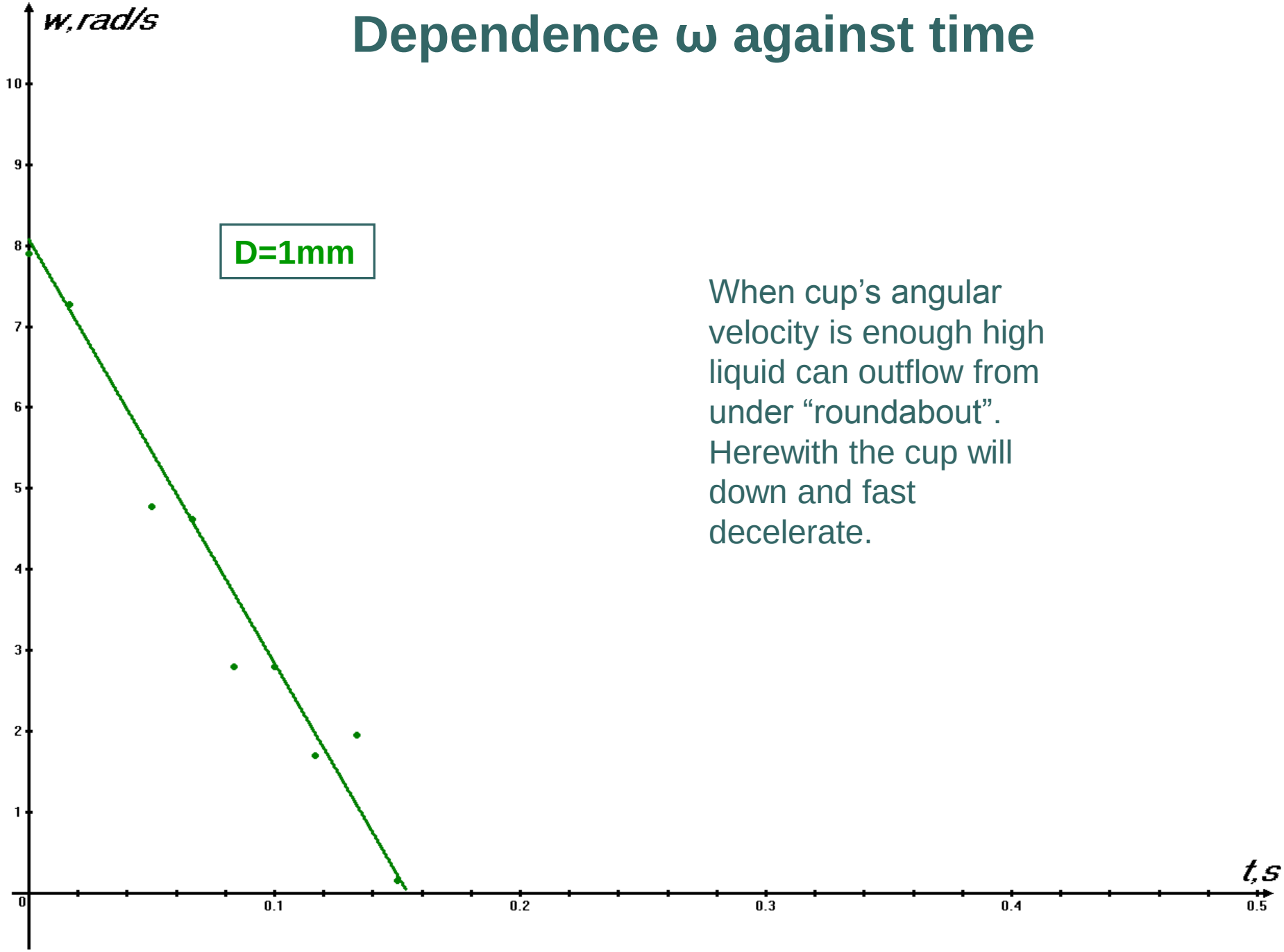




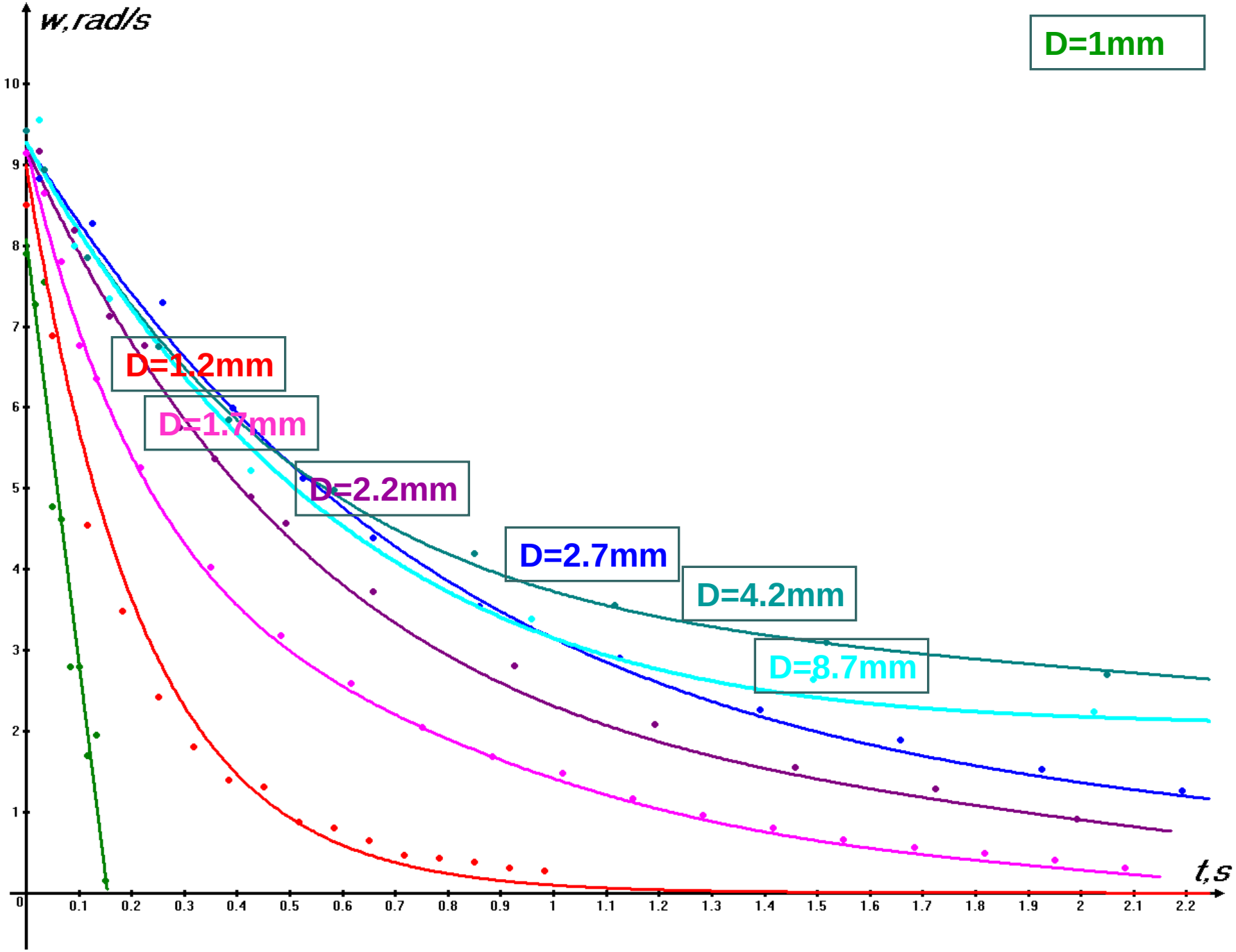
II. Dependence of deceleration against depth of layer of liquid



Dependence ω against time



When cup's angular velocity is enough high liquid can outflow from under "roundabout". Herewith the cup will down and fast decelerate.


w,rad/s
t,s
D=1mm
D=1.2mm
D=1.7mm
D=2.2mm
D=2.7mm
D=4.2mm
D=8.7mm
0
1
2
3
4
5
6
7
8
9
10
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
2.1
2.2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The linearization(prediction)

If flow under cup is laminar moment of viscous forces is proportional to the angular velocity.

The basic equation of rotational motion's dynamics:

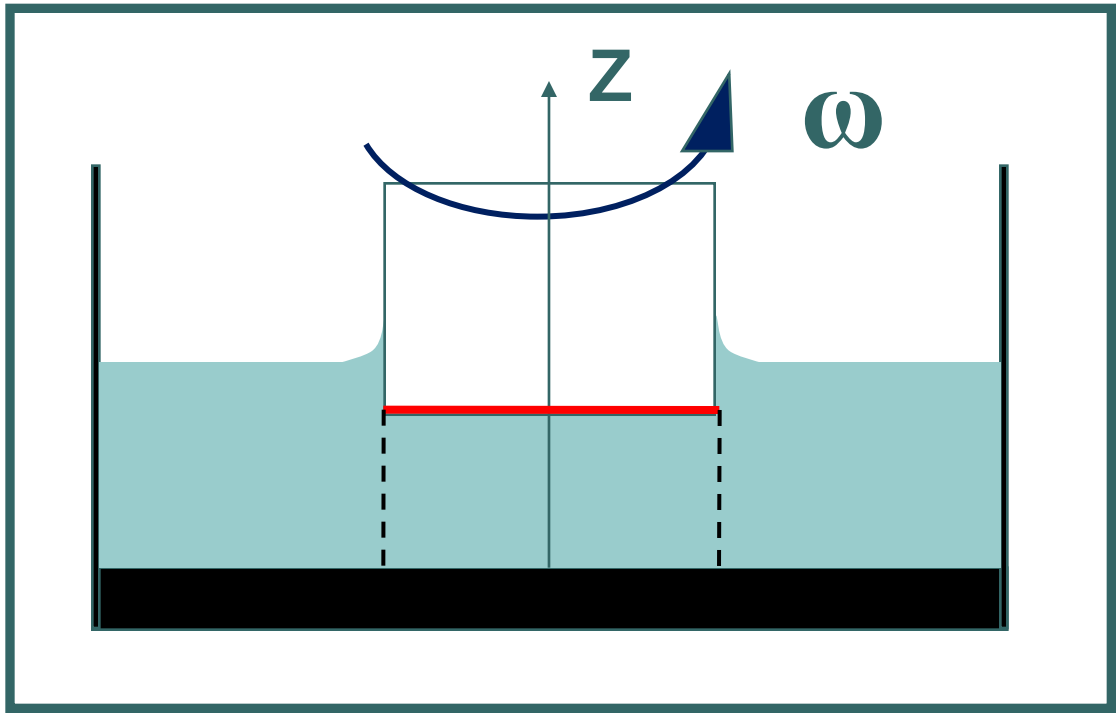
$$J\frac{d\omega}{dt} = -\alpha \cdot \omega$$

$$\omega = \omega_0 \exp\left(-\frac{t}{\tau}\right)$$

ω – angular velocity of cup t – time of rotation

ω_0 – initial angular velocity τ – characteristic time

Mathematic model (laminar flow)



Elementary force (Newton's theory):

$$dF = \mu \cdot dS \cdot \frac{dv}{dz}$$

$$\frac{dv}{dz} - ?$$

Gradient of liquid's velocity on z direction

Mathematic model (prediction)

The Couette flow

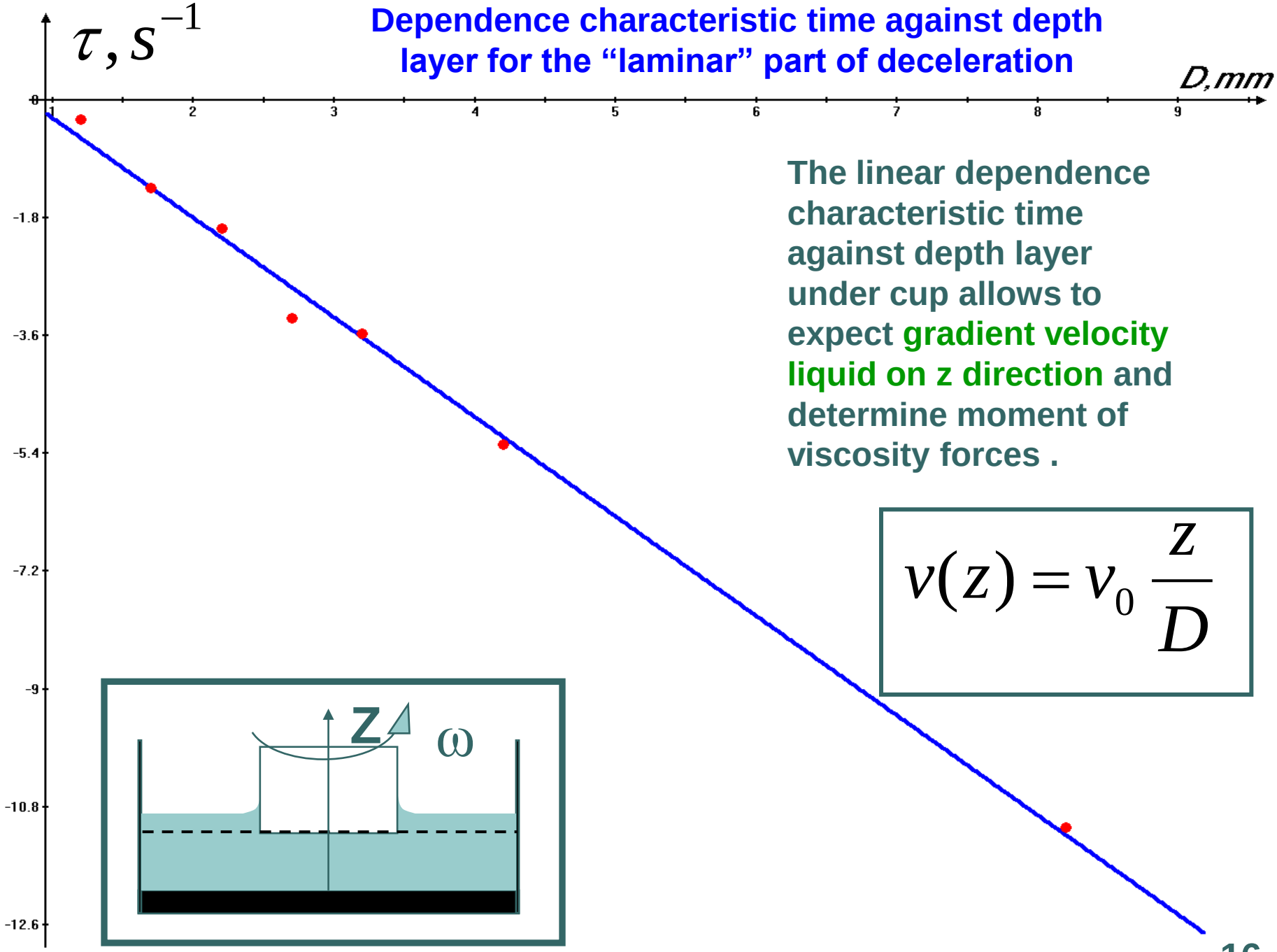
Liquid's velocity on z direction

$$v(z) = v_0 \frac{z}{D}$$

Moment of viscosity friction inversely layer's depth therefore characteristic time must be proportional D

$$M \sim \frac{1}{D} \quad \Rightarrow \quad \tau \sim D$$

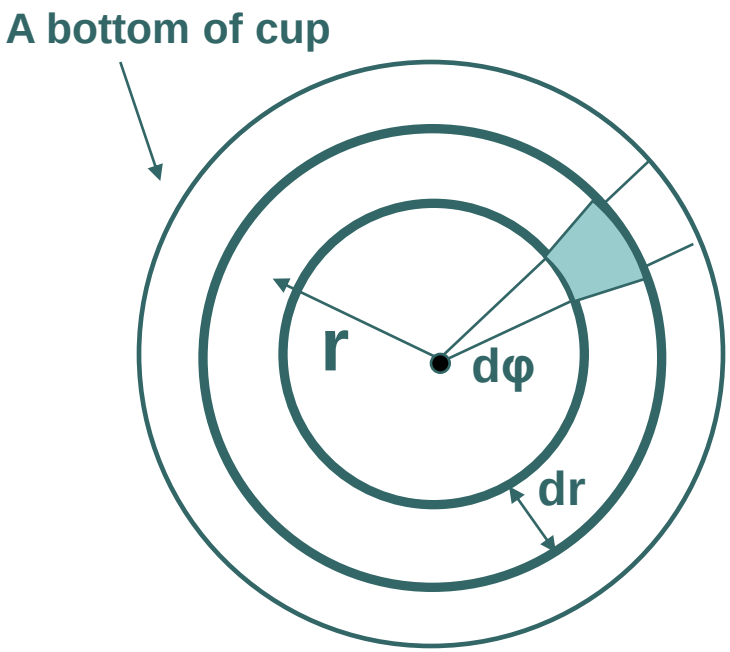
Dependence characteristic time against depth layer for the “laminar” part of deceleration



The linear dependence characteristic time against depth layer under cup allows to expect **gradient velocity liquid on z direction** and determine moment of viscosity forces .

$$v(z) = v_0 \frac{z}{D}$$

Moment of viscosity forces



Velocity of liquid under cup on z direction:

$$v(z) = v_0 \frac{z}{D}$$

Elementary force:

$$dF = \mu \frac{dv}{dz} \cdot dS = \mu \frac{\omega \cdot r}{D} \cdot d\varphi \cdot r \cdot dr$$

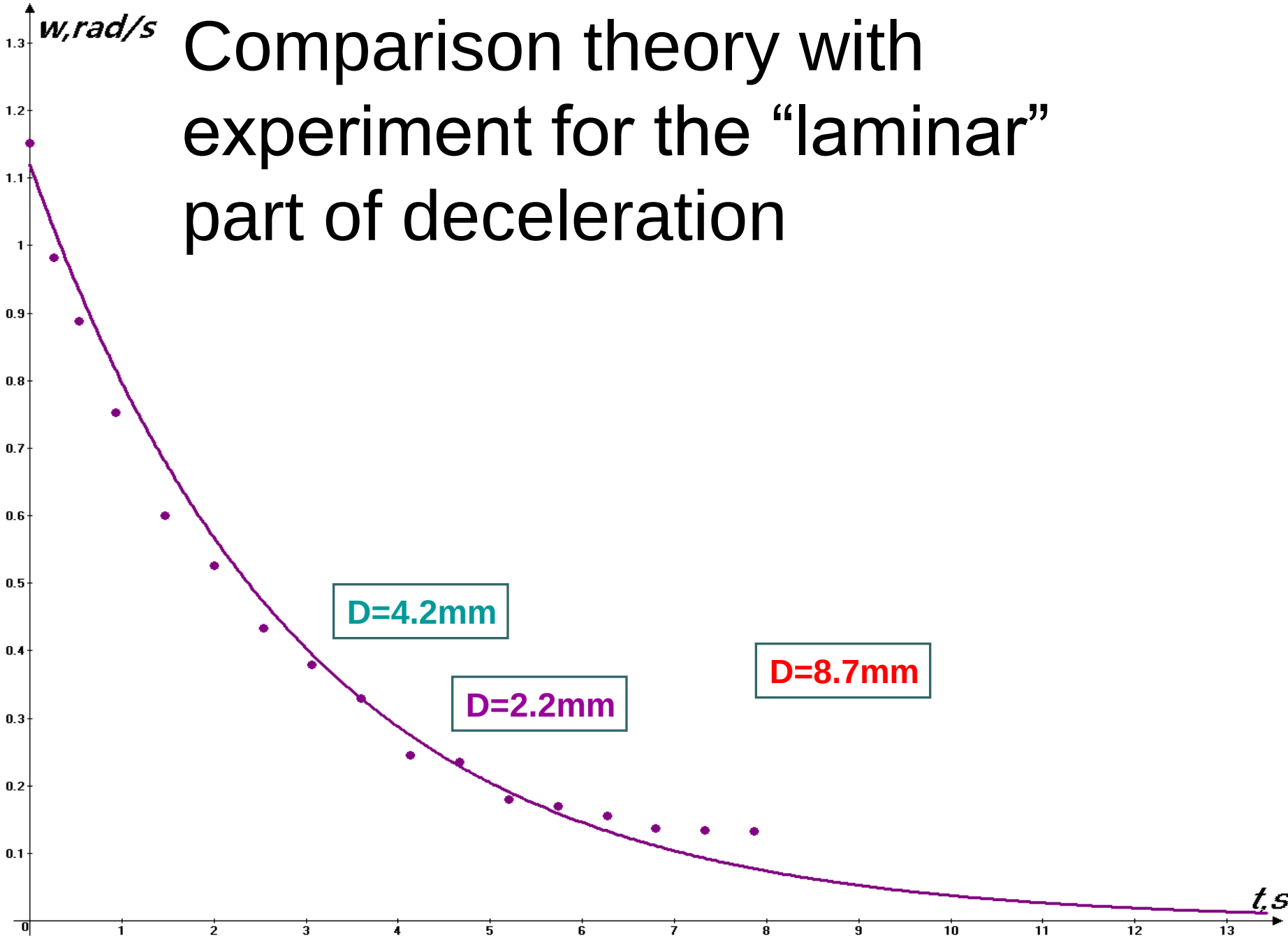
$$M = \int_0^{2\pi} \int_0^{R} dF \cdot r = \frac{\pi \mu R^4 \omega}{2D}$$

The law of cup's motion

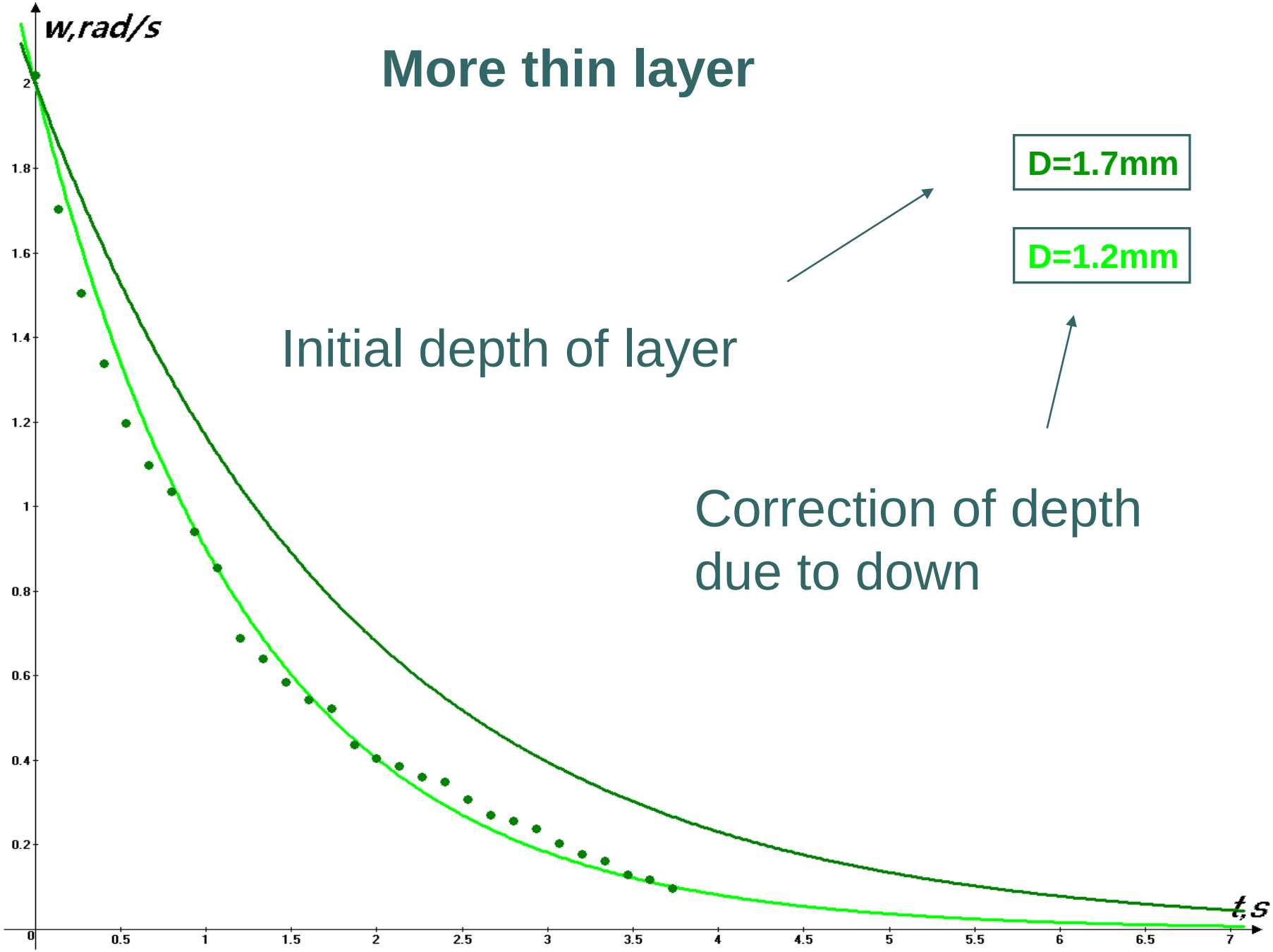
$$\omega = \omega_0 \exp\left(-\frac{\pi \mu R^4}{2DJ} \cdot t\right)$$

ω – angular velocity of cup	t – time of rotation
ω_0 – initial angular velocity	τ – characteristic time
J – moment of inertia	R – cup's radius
D – layer's depth of liquid	μ - viscosity

Comparison theory with experiment for the “laminar” part of deceleration



More thin layer



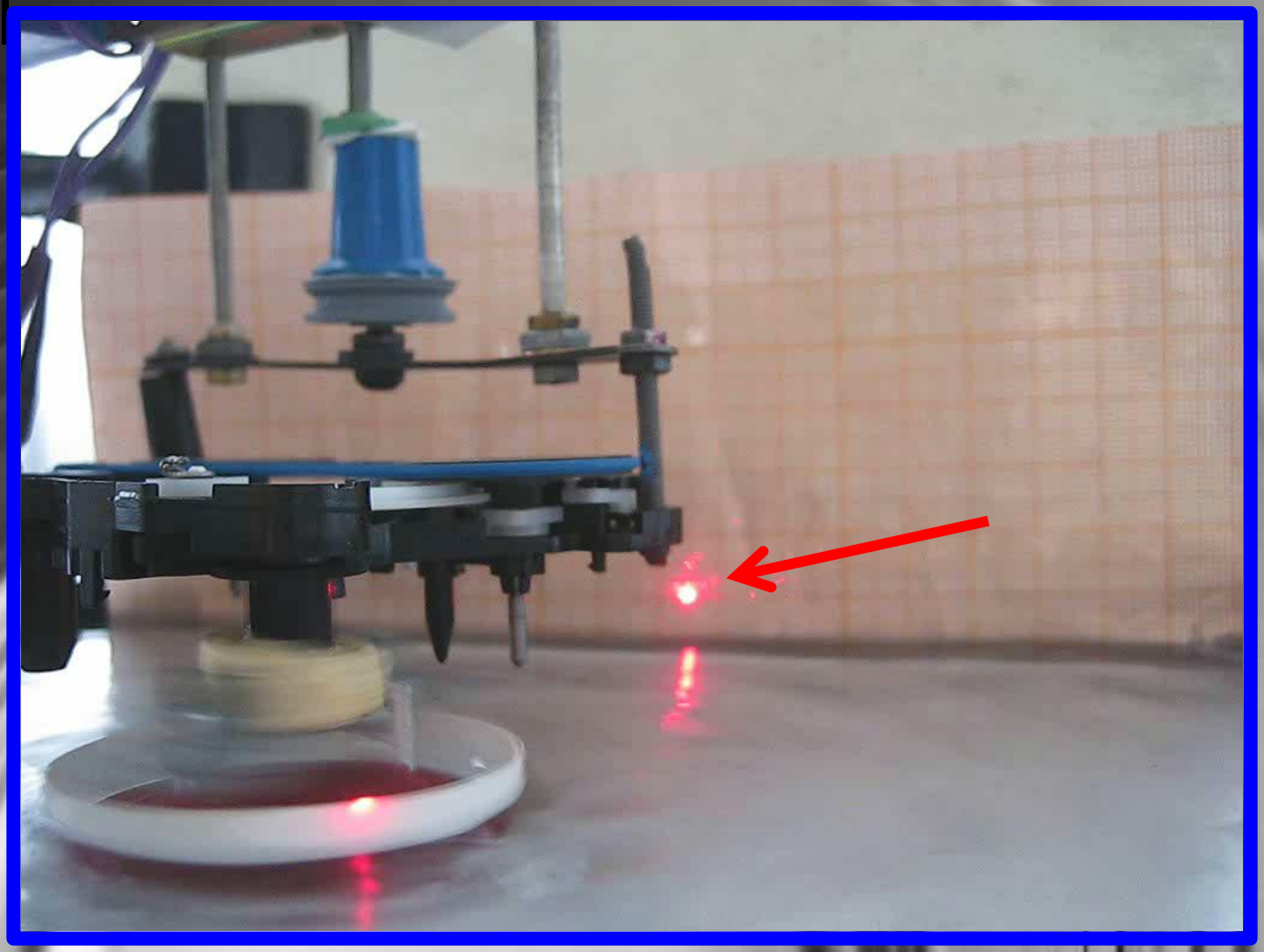
D=1.7mm

D=1.2mm

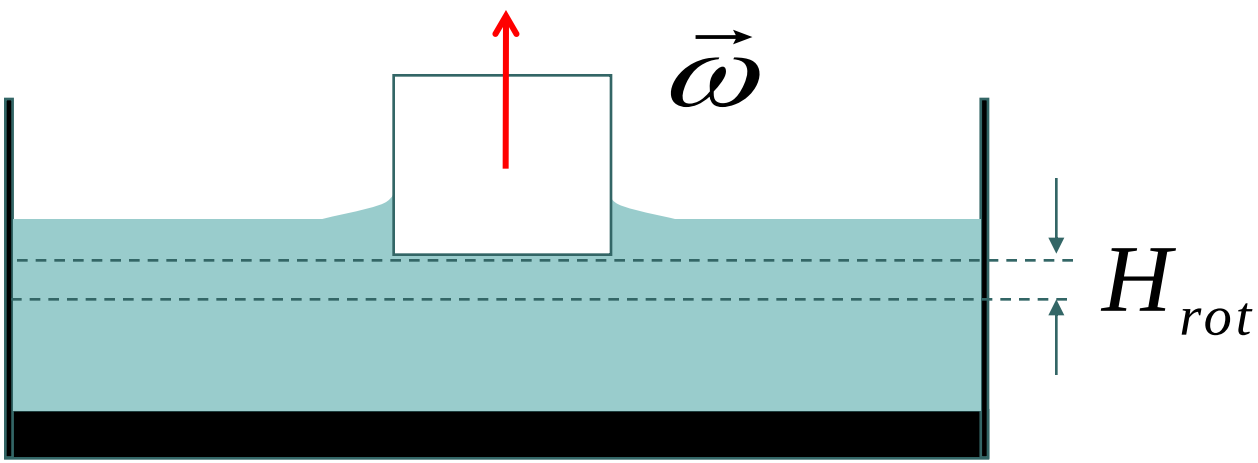
Initial depth of layer

Correction of depth due to down

Movie with down



Subsidence due to the rotation

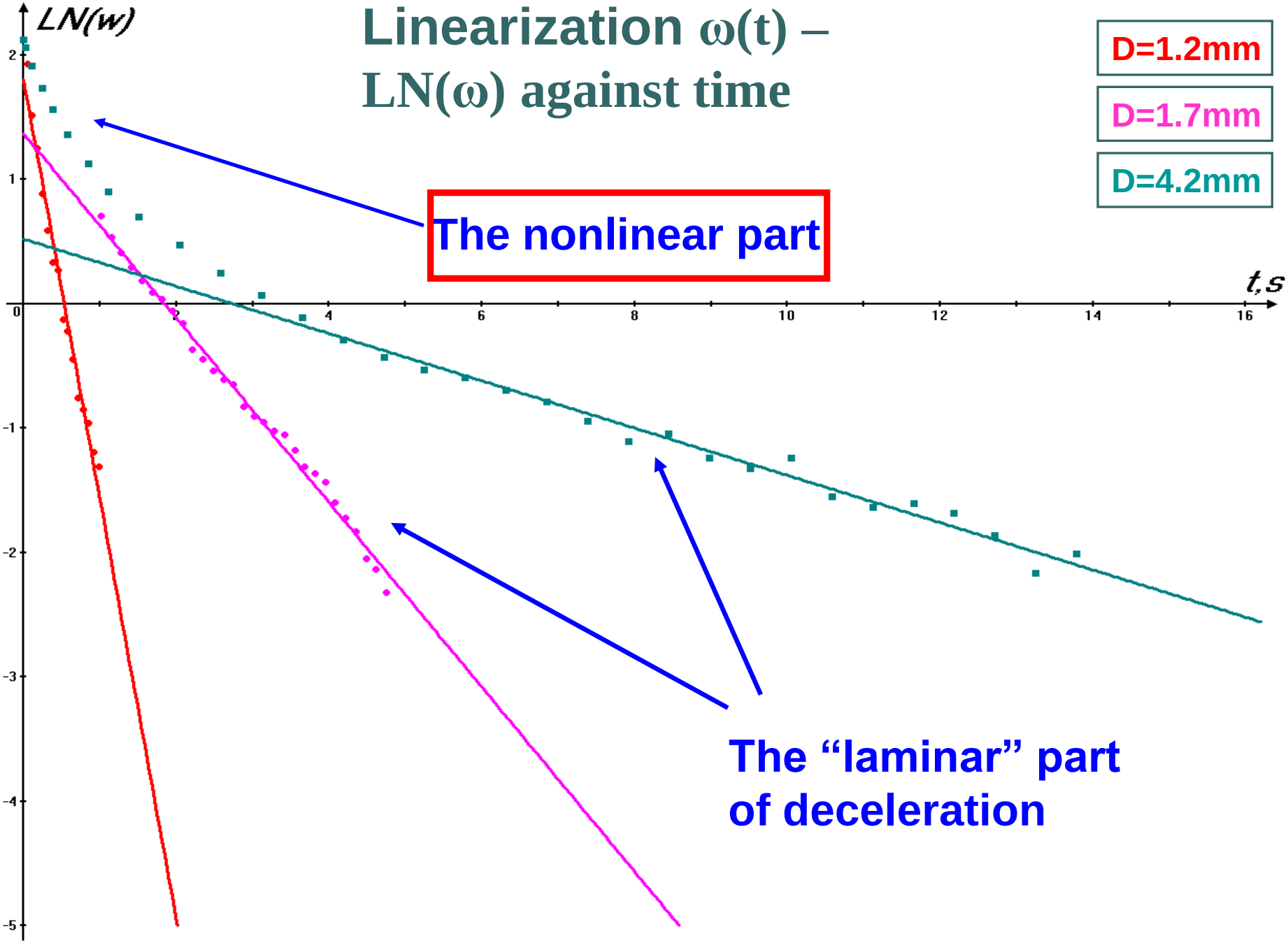


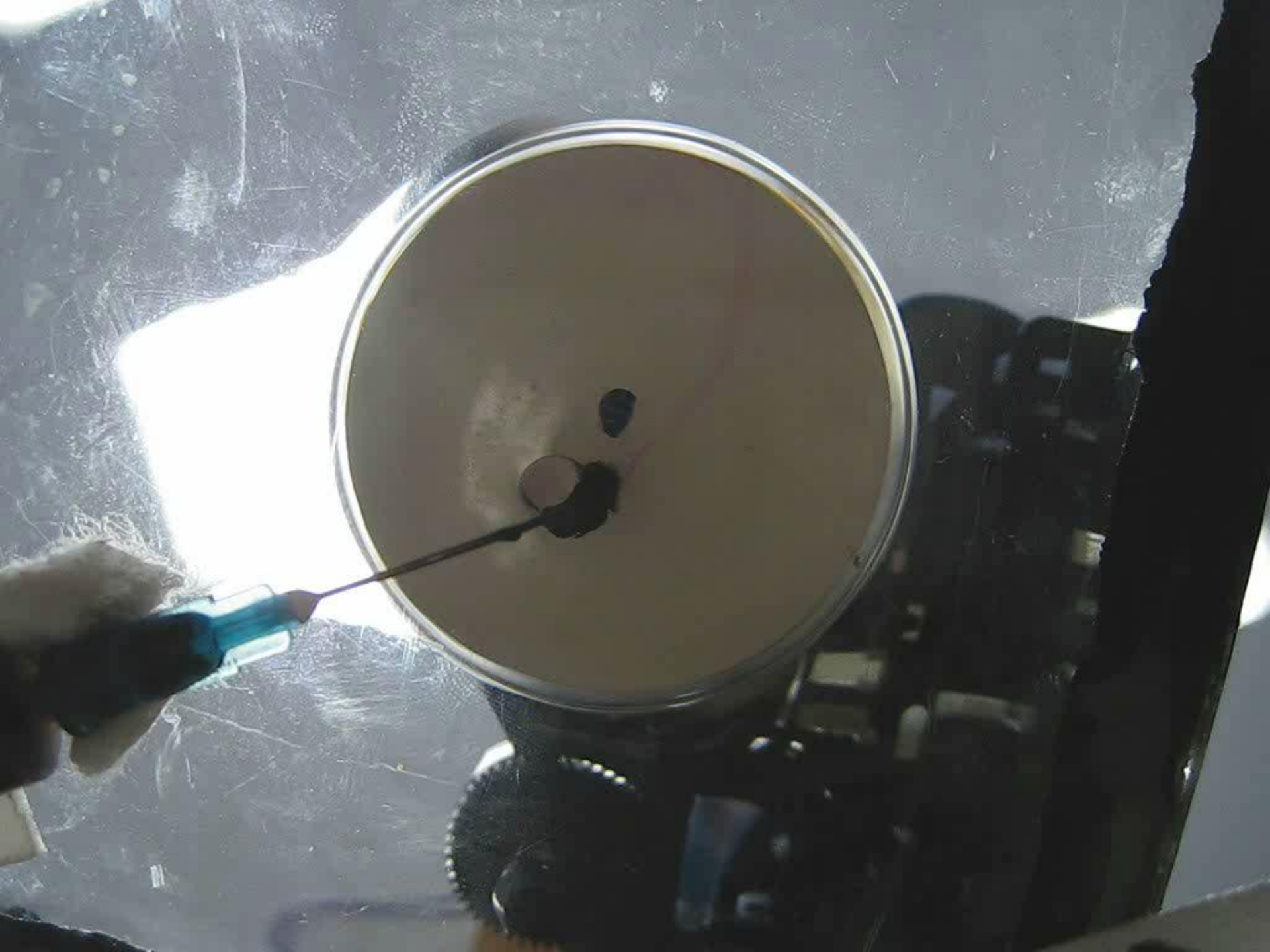
Condition for H_{rot}:

Forces of inertia = Hydrostatic forces

$$H_{rot} = \frac{\omega_0^2 \cdot R^2}{9 \cdot g} = 0{,}4mm$$

Linearization ω(t) – LN(ω) against time





The linearization(turbulence)

When flow under cup is turbulence moment of viscous forces is proportional a square angular velocity.

$$M \sim \omega^2$$

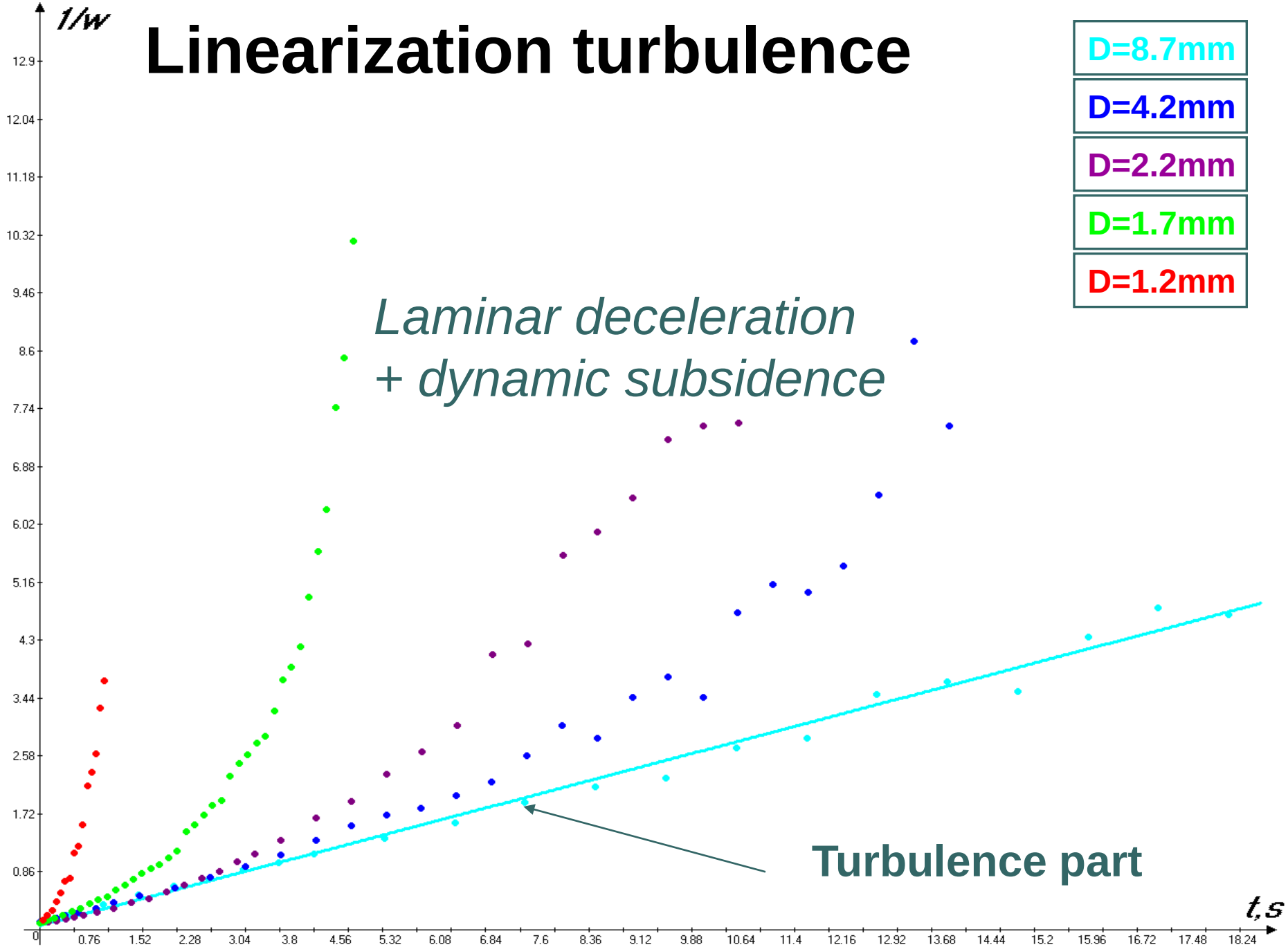
The law of cup's motion :

$$\omega = \frac{1}{\frac{1}{\omega_0} + \frac{t}{\tau}}$$

ω – angular velocity of cup t – time of rotation

ω_0 – initial angular velocity τ – characteristic time

Linearization turbulence



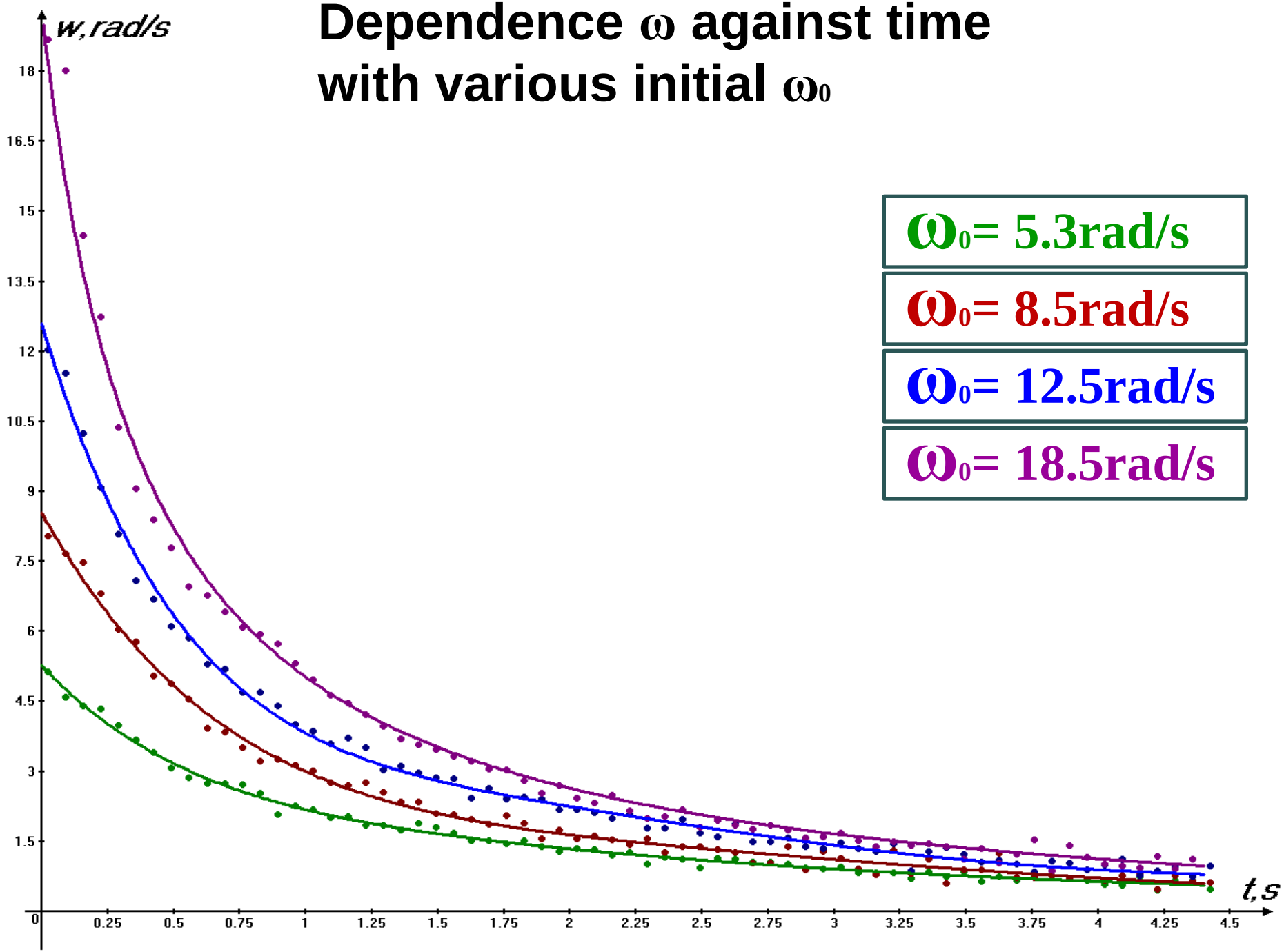
The kinds of deceleration:

- 1)Gluing
- 2)M~w , laminar flow
- 3)M~w^2 , turbulence

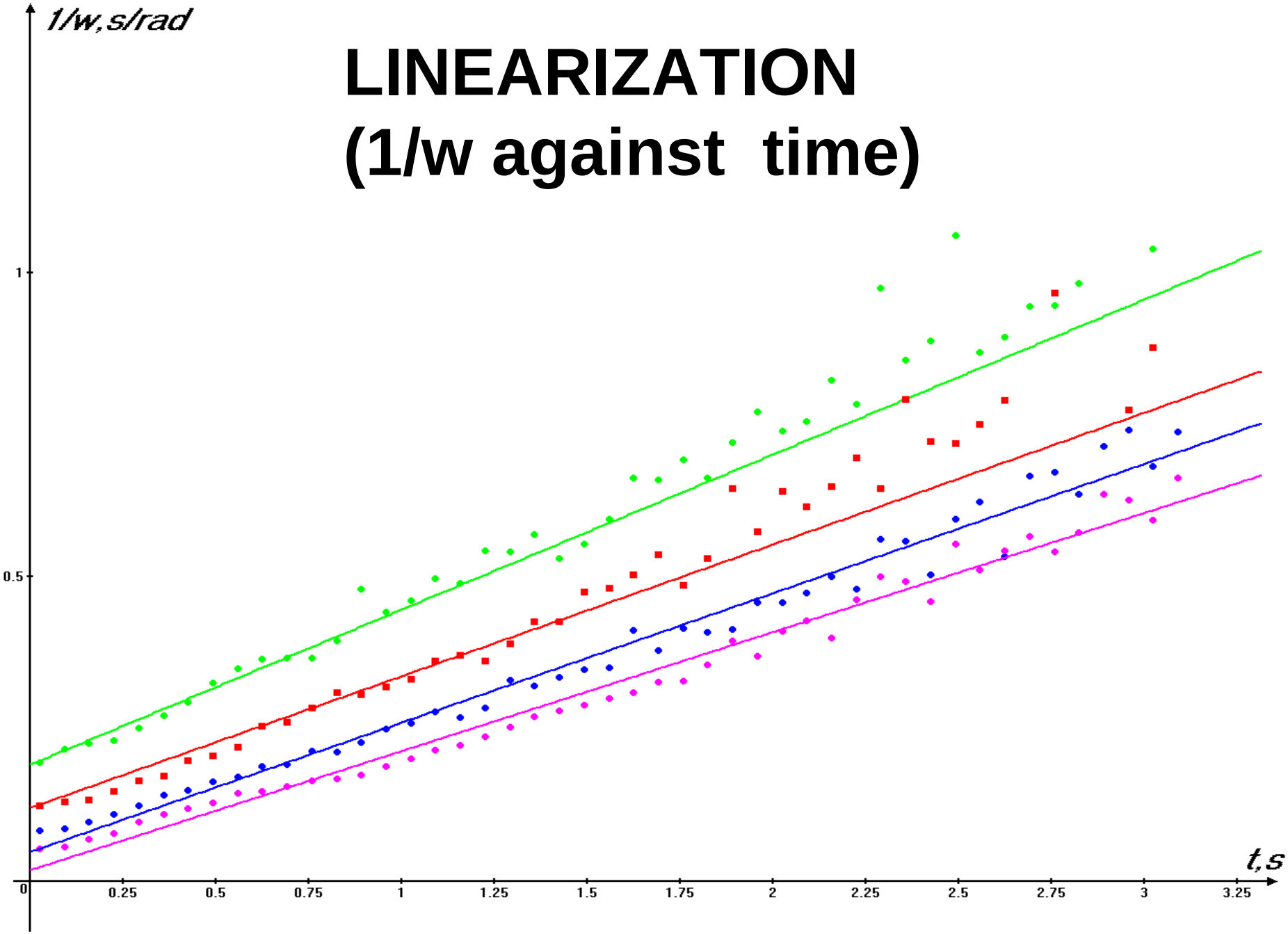
III. Parameters of racing

- 1) Initial angular velocity
- 2)Time of racing

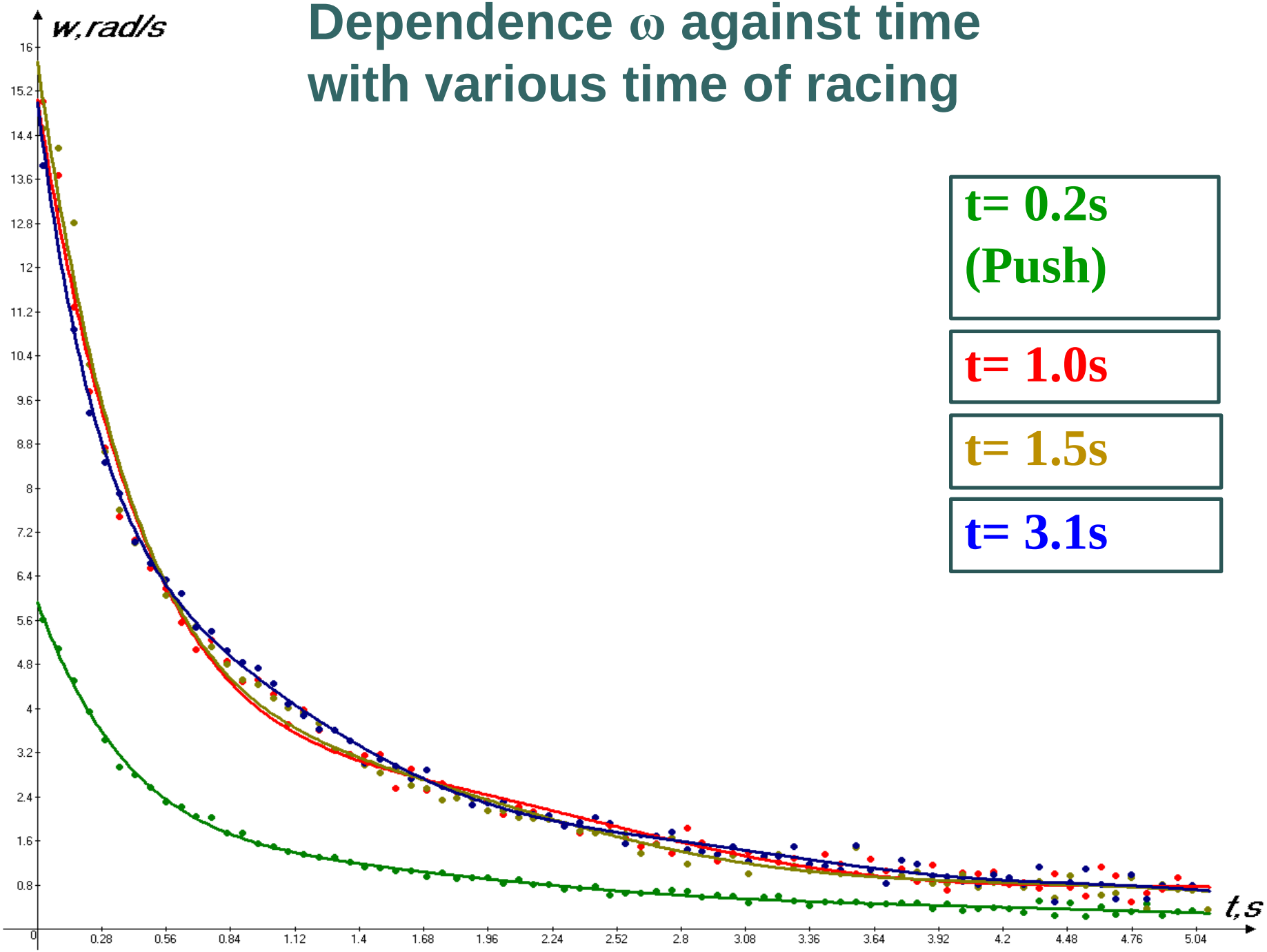
Dependence ω against time with various initial ω_0



LINEARIZATION
(1/w against time)



Dependence ω against time with various time of racing



IV. Parameters of cup

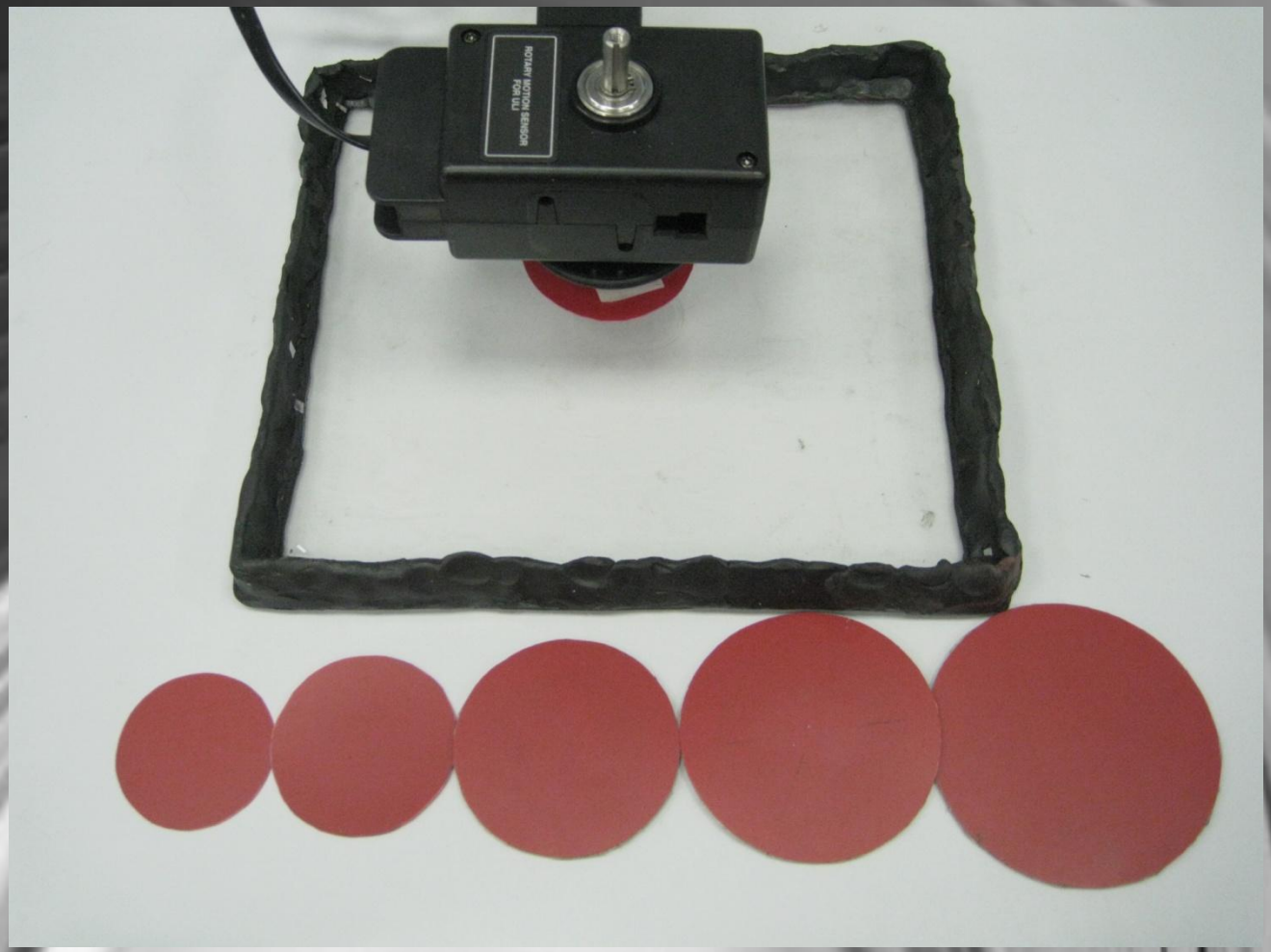
1)Mass

2)Radius(square of bottom)

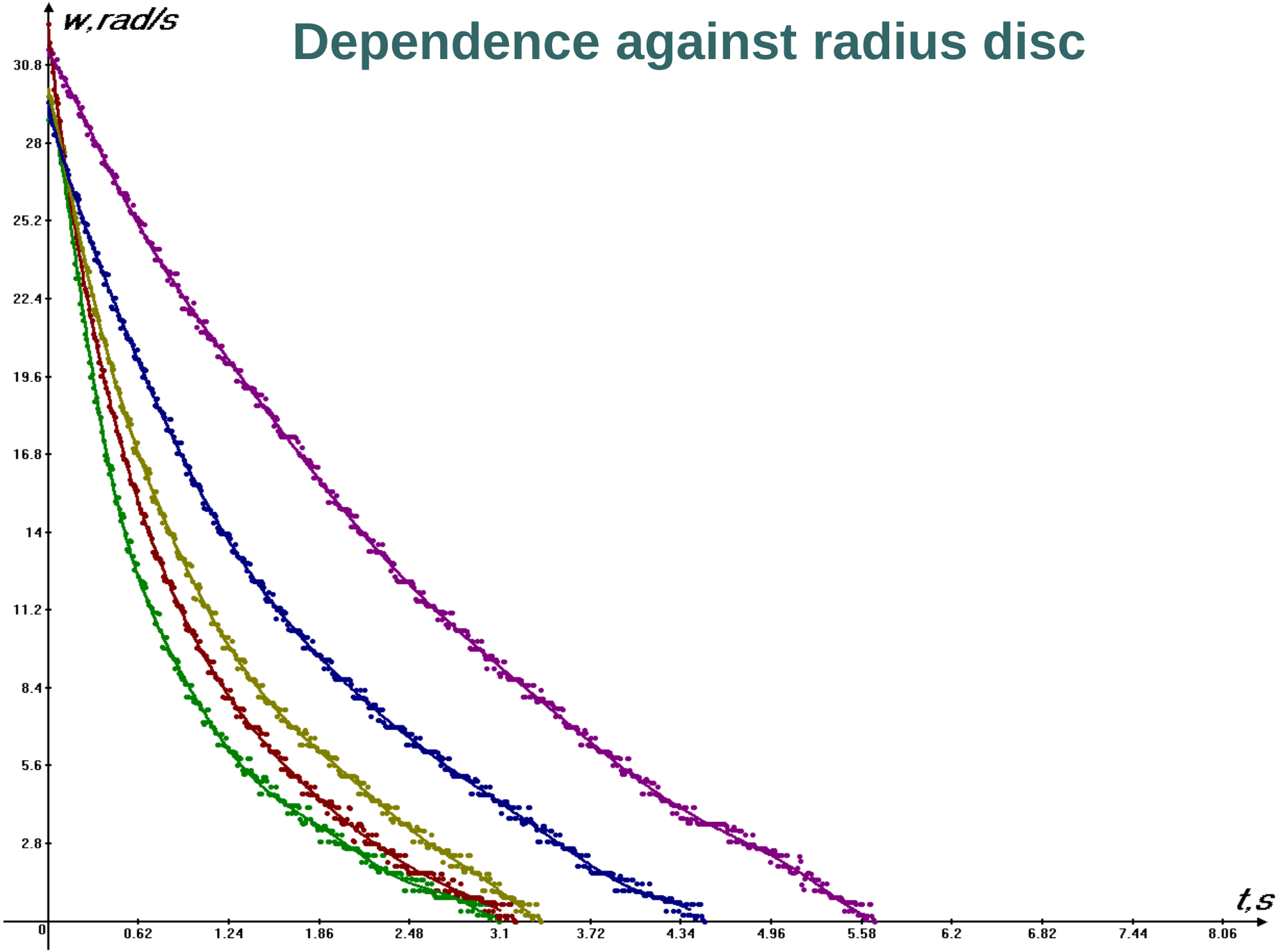
3)Moment of inertia

4)Angle of contact

Experiment with radius disc



Dependence against radius disc



V. Parameters of liquid

- 1) Viscosity
- 2) Density
- 3)Surface tension

Parameters of the roundabout

1) Depth of liquid under cup
2) Initial angular velocity
3) Duration of racing
4) A cup:
   a) Mass of cup
   b) Radius of cup
   c) Moment of inertia
   d) Contact’s angle
5) A liquid:
   a) Viscosity
   b) Density
   c) Surface tension

Conclusions:

- 1)Depth of liquid layer determines character of deceleration
- 2)There are 3 different modes of cup deceleration:
  - a) gluing
  - b) laminar flows
  - c) turbulence
- 3)Turbulent and laminar deceleration may be obtained during a single experiment.
- 4)Laminar mode is explained by model of Couette flow
- 5)The characteristic time doesn’t depend on depth of liquid layer for turbulence mode.

Summary:

- The experimental base of deceleration was built for explanation phenomenon.
- The most important parameters were marked.
- The experimental investigation some

References:

Wikipedia: Couette flow.

A. D. Glinkin and V. A. Rukavishnikov. Laminar flow of a viscous incompressible liquid over the surface of solids of revolution. J. Eng. Phys. Thermophys., 39, 1 (1980)

Slezkin. Hydrodynamics.

THANK YOU FOR ATTENTION!

The conditions for laminar flow

$$\mathrm{Re} = \frac{\rho \cdot v \cdot l}{\mu}$$

ρ – density of liquid μ – viscosity

***v* – characteristic velocity (ω*R)**

***l* – characteristic size (D)**

Boundary value in our experiments:

$$\mathrm{Re} \approx 100$$