Belarusian Team Presents

Roundabout

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6. Roundabout
Put a plastic cup on a thin layer of liquid on a flat solid surface.
Make the cup rotate.
On what parameters does the rotational deceleration of the cup depend?
Contents:

1) The main forces of deceleration
2) Determination of the depth of liquid under cup
   A. Research of flows under cup
   B. Mathematical model of laminar mode
   C. Turbulence mode
3) Parameters of racing
   A. Initial angular velocity
   B. Duration of racing
4) Properties of cup and liquid

Conclusions
I. The forces of deceleration

\[ F_v \sim A \cdot \mu \]

Viscosity of water and air:

\[ \mu_{water} = 1.00 \times 10^{-3} \, Pa \cdot s \]
\[ \mu_{air} = 17.4 \times 10^{-6} \, Pa \cdot s \]

\[ F_{air} \approx 8.7 \times 10^{-8} \]
\[ F_{water} \approx 4.5 \times 10^{-6} \]
The main forces of deceleration
Subsidence of static cup

\[ H_{st} = \frac{m}{\pi \cdot \rho \cdot R^2} + \frac{2 \cdot \sigma}{\rho \cdot g \cdot R} \cdot \cos \theta \]

- \( m \) – mass of the cup
- \( R \) – radius of cup
- \( \rho \) – density of liquid
- \( \theta \) – angle of contact
- \( \sigma \) – surface tension of liquid
Experimental setup

Device

Surface

“Roundabout”
II. Dependence of deceleration against depth of layer of liquid
When cup’s angular velocity is enough high liquid can outflow from under “roundabout”. Herewith the cup will down and fast decelerate.

\[ D = 1 \text{mm} \]
Dependence of the angular velocity against time. The approximating curves for different values of D are shown:

- D = 1mm
- D = 1.2mm
- D = 1.7mm
- D = 2.2mm
- D = 2.7mm
- D = 4.2mm
- D = 8.7mm

The graph illustrates how the angular velocity changes with time for each value of D.
If flow under cup is laminar moment of viscous forces is proportional to the angular velocity.

The basic equation of rotational motion’s dynamics:

\[ J \frac{d\omega}{dt} = -\alpha \cdot \omega \]

\[ \omega = \omega_0 \exp\left(-\frac{t}{\tau}\right) \]

- \(\omega\) – angular velocity of cup
- \(t\) – time of rotation
- \(\omega_0\) – initial angular velocity
- \(\tau\) – characteristic time
Mathematic model (laminar flow)

Elementary force (Newton’s theory):

\[ dF = \mu \cdot dS \cdot \frac{dv}{dz} \]

Gradient of liquid’s velocity on z direction
Mathematic model (prediction)

The Couette flow

Liquid’s velocity on z direction

\[ v(z) = v_0 \frac{z}{D} \]

Moment of viscosity friction inversely layer’s depth therefore characteristic time must be proportional D

\[ M \sim \frac{1}{D} \implies \tau \sim D \]
The linear dependence characteristic time against depth layer under cup allows to expect gradient velocity liquid on z direction and determine moment of viscosity forces.

\[ v(z) = v_0 \frac{z}{D} \]
Moment of viscosity forces

Velocity of liquid under cup on z direction:

\[ v(z) = v_0 \frac{z}{D} \]

Elementary force:

\[ dF = \mu \frac{d\nu}{dz} \cdot dS = \mu \frac{\omega \cdot r}{D} \cdot d\phi \cdot r \cdot dr \]

Moment:

\[ M = \int \int dF \cdot r = \frac{\pi \mu R^4 \omega}{2D} \]
The law of cup’s motion

\[ \omega = \omega_0 \exp \left( -\frac{\pi \mu R^4}{2DJ} \cdot t \right) \]

- \( \omega \) – angular velocity of cup
- \( \omega_0 \) – initial angular velocity
- \( t \) – time of rotation
- \( \tau \) – characteristic time
- \( J \) – moment of inertia
- \( R \) – cup’s radius
- \( D \) – layer’s depth of liquid
- \( \mu \) – viscosity
Comparison theory with experiment for the “laminar” part of deceleration.
More thin layer

Initial depth of layer

Correction of depth due to down

$D=1.7\text{mm}$

$D=1.2\text{mm}$
Movie with down
Subsidence due to the rotation

Condition for $H_{rot}$:

Forces of inertia = Hydrostatic forces

$$H_{rot} = \frac{\omega_0^2 \cdot R^2}{9 \cdot g} = 0,4\,mm$$
Linearization $\omega(t)$ – $\ln(\omega)$ against time

The "laminar" part of deceleration

The nonlinear part
The linearization (turbulence)

When flow under cup is turbulence moment of viscous forces is proportional a square angular velocity.

The law of cup’s motion:

\[ M \sim \omega^2 \]

\[ \omega = \frac{1}{\frac{1}{\omega_0} + \frac{t}{\tau}} \]

\( \omega \) – angular velocity of cup \( t \) – time of rotation

\( \omega_0 \) – initial angular velocity \( \tau \) – characteristic time
Linearization turbulence

Laminar deceleration + dynamic subsidence

Turbulence part
The kinds of deceleration:

1) Gluing
2) $M \sim w$, laminar flow
3) $M \sim w^2$, turbulence
III. Parameters of racing

- 1) Initial angular velocity
- 2) Time of racing
Dependence $\omega$ against time with various initial $\omega_0$

- $\omega_0 = 5.3\text{rad/s}$
- $\omega_0 = 8.5\text{rad/s}$
- $\omega_0 = 12.5\text{rad/s}$
- $\omega_0 = 18.5\text{rad/s}$
LINEARIZATION
(1/w against time)
Dependence $\omega$ against time with various time of racing:

- $t = 0.2s$ (Push)
- $t = 1.0s$
- $t = 1.5s$
- $t = 3.1s$
IV. Parameters of cup

1) Mass
2) Radius (square of bottom)
3) Moment of inertia
4) Angle of contact
Experiment with radius disc
Dependence against radius disc
V. Parameters of liquid

- 1) Viscosity
- 2) Density
- 3) Surface tension
Parameters of the roundabout

1) Depth of liquid under cup
2) Initial angular velocity
3) Duration of racing
4) A cup:
   a) Mass of cup
   b) Radius of cup
   c) Moment of inertia
   d) Contact’s angle
5) A liquid:
   a) Viscosity
   b) Density
   c) Surface tension
Conclusions:

1) Depth of liquid layer determines character of deceleration
2) There are 3 different modes of cup deceleration:
   a) gluing
   b) laminar flows
   c) turbulence
3) Turbulent and laminar deceleration may be obtained during a single experiment.
4) Laminar mode is explained by model of Couette flow
5) The characteristic time doesn’t depend on depth of liquid layer for turbulence mode.
Summary:

- The experimental base of deceleration was built for explanation phenomenon.
- The most important parameters were marked.
- The experimental investigation some
References:

Wikipedia: Couette flow.


Slezkin. Hydrodynamics.
THANK YOU FOR ATTENTION!
The conditions for laminar flow

\[ \text{Re} = \frac{\rho \cdot v \cdot l}{\mu} \]

- \( \rho \) – density of liquid
- \( \mu \) – viscosity
- \( v \) – characteristic velocity (\( \omega \cdot R \))
- \( l \) – characteristic size (\( D \))

Boundary value in our experiments:

\[ \text{Re} \approx 100 \]