24th International Young Physicists’ Tournament

Team of Belarus

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 №10
Formulation of the problem: FARADAY HEAPING

When a container filled with small spheres (e.g. mustard seeds) is vibrated vertically with a frequency between 1 – 10 Hz, so called Faraday heaping occurs. Explore this phenomenon.
Demonstration
Steps of solution the problem:

I. Explanation of the phenomenon

II. Experimental investigation

III. Theoretical model
I. Explanation of the phenomenon
One cycle of heap forming

1

Reasons of particle horizontal motion

Air flows

Static pressure differences

2

\[ \vec{a}; \quad a > g \]
Heaps merging
Evolution of Faraday Heaping
N(n), number of heaps Vs number of oscillations

Number of oscillations

Number of heaps

0 100 200 300 400 500

0 5 10 15 20 25

1
Air influence

\[ P << P_A \]
Conclusions of the first part

1. Air flows and static pressure differences are main reasons for heap forming

2. Particles circulation is observed in stationary heaps

3. After some time initial small heaps tend to merge into one big heap

4. Air is the key factor for heap forming
II. Experimental investigation
EXPERIMENTAL SETUP

Parameters of setup:

Input signal: sine

\[ \nu \in (1;50) \text{Hz} \]

\[ A_{\text{max}} = 7 \text{mm} \]
Particles used

Particle parameters

1. Shape
2. Dimension
3. Density
4. Elastic properties
5. Friction

- Sand
  - D≈0.2 mm

- Semolina
  - D≈0.6 mm

- Millet
  - D≈2.0 mm
Motion law of the vessel $v=8$ Hz
Harmonic oscillations

(1) \( x = A \sin(\omega t) \)

(2) \( x'' = -A \omega^2 \sin(\omega t) \)

(3) \( A \omega^2 > g \) - condition of particles separation

(4) \( A \leq 7 \text{mm} \) - limitation of experimental setup

(5) \( v \geq 6 \text{Hz} \) - theoretical frequency for observing phenomenon with amplitude of 7 mm
The acceleration of vessel Vs time

Time of separation

\[ t_{sep} \approx 10^{-3} \, s \] (6)
$H(h)$ height of heap Vs initial layer height

Initial layer height

Heap height

Initial layer height (mm)
$t(\nu)$, time of separation Vs frequency for ideal sine signal
$h(\nu)$, height of heap Vs frequency
Conclusions for the second part

1. Critical parameter for heap formation is time of separation

2. The certain optimal frequency exists for highest heaps formation

3. With increasing the initial height of layer maximum heap height tends to some constant value.
III. Theoretical model
Model assumptions

1. All sand movement is caused by unequal distribution of static air pressure.

2. The effect of directed air flows dragging particles is negligible.

3. Heap incline is limited by a certain critical angle attributed to particle type, rolling down intensity per unit area is constant.
Initial shape of heap profile

\[ h = \varphi(r) \] - line for cone

Structural elements for calculation
Vertical displacement of elements

Vertical acceleration of each element

\[ a(r) = \frac{(P_A - P)}{h(r) \rho} \approx 10^4 \text{ m/s}^2 \] (8)

Average vertical displacement

\[ \left\langle \Delta z_r \right\rangle = \frac{at_{sep}^2}{2} \approx 10^{-2} \text{ m} \] (9)
Pressure distribution in particles layer

\[ p(y) = P_A - (P_A - P) \frac{y}{h(r)} \]  

(10)
Motion of elements

\[ a(r) = \frac{F_R - F_L}{m} \sim \frac{\Delta P}{\rho} (\Delta z)'_r \]  

(11)

\[ a(r) \sim \frac{\Delta P^2}{\rho^2} t^2 \]  

(12)

\[ v(r) \sim \frac{\Delta P^2}{\rho^2} t^3 \]  

(13)
Growing of cylinder

\[ \Delta h = \Delta P^2 \frac{t_{sep}}{\rho^2} f(r) \]

\( \rho \) - substance density

\( t_{sep} \) - separation time
Heap increase per one cycle Vs radius

![Graph showing the relationship between heap increase per one cycle and radius. The x-axis represents radius (cm) ranging from 1 to 5, and the y-axis represents height increase per one cycle (cm) ranging from 0.010 to -0.002. The graph shows a decreasing trend as radius increases.]
Model calculation

Iteration number: 0
Key conclusions

1. Low pressure zone forms if the acceleration of the vessel more than $g$.

2. Horizontal particle motion occur by air flows and static air pressure differences.

3. After some time initial small heaps tend to merge into one big heap

4. Separation time and number of layers $\left( \frac{H}{d} \right)$ are main parameters of heaps formation.
Key conclusions

5. Frequency and amplitude determine separation time. In experiments with amplitude equals 7 mm and range of frequencies between 6 and 20 Hz Faraday Heaping is observed.

6. Heap growing could described by static pressure differences.

7. Horizontal displacement of particles is caused by unequal shift of layers under pressure difference.


D.V. Sivukhin «General Course of physics. Mechanics». 
Thank you for your attention!
Additional slides
II-nd EXPERIMENTAL SETUP

Parameters of setup:

\[ \nu \in (\cdot) \text{Hz} \]
\[ A = \text{mm} \]
Demonstration

usual row slow motion(4)