Team Belarus presents...

Flying colours
Flying colors

Why do flags flutter in the wind?
Investigate experimentally the airflow pattern around a flag.
Describe this behaviour.
Animation
Number of whirls with fixed parameters

\[ d^2N = C \, N \, d\omega \, dR \]

N - general number of whirls
\( \omega \) - angular velocity of whirl
R - whirl’s radius
C - constant of proportionality
The energy of whirls with fixed parameters

\[ d^2 E = \beta E \, d^2 N \]

E - average kinetic energy of whirls
Full energy of one whirl

\[ E = \frac{mv^2}{2} + \frac{I\omega^2}{2} \]

\[ \nu = \omega R \]

\[ I = \frac{2}{5}mR^2 \]
The equation for the energy of whirl

\[ E = \frac{3}{4} \pi \rho l_2 \omega^2 R^4 \]

Here:

\[ m = l_2 \pi R^2 \rho \]

\( l_2 \) - flag's height
Equation for coefficient C

\[ \int_{0}^{R_0} \int_{0}^{\omega_0} d^2N = N \]

\[ \int_{0}^{R_0} \int_{0}^{\omega_0} CNd\omega dR = N \]

\[ C = \frac{1}{\omega_0 R_0} \]

\( \omega_0, R_0 \) - limit cyclic frequency and radius of the whirl nearby the flag
Full energy of whirls flying on the flag

\[ E_{II} = \int_{0}^{R_0} \int_{0}^{\omega_0} d^2E = \frac{1}{20} \pi \rho l_2 R_0^4 \omega_0^2 N \]
Average values

\[ \langle R \rangle = \frac{1}{R_0} \int_0^{R_0} RdR = \frac{R_0}{2} \]

\[ \langle \omega \rangle = \frac{1}{\omega_0} \int_0^{\omega_0} \omega d\omega = \frac{\omega_0}{2} \]
Restoring force

\[ F = C_1 \frac{\rho V^2}{2} S_1 \]

- \( C_1 \) - drag coefficient
- \( V \) - air’s speed
- \( \rho \) - air’s density
- \( x \) - flag’s deflection

\[ S_1 = x \cdot l_2 \]

\[ < x > = \frac{\alpha^2}{16} R_0 \]
Wave’s length

\[ \frac{\lambda}{2} \approx \frac{R_0}{2} \alpha \]

\[ \lambda \approx R_0 \alpha \]
Cyclic frequency of flag oscillation

\[ \omega_f = \frac{2\pi u}{\lambda} \]

u - speed of wave propagation
The energy of sinusoidal wave

\[ E_1 = \frac{1}{2} \rho_f \langle x \rangle^2 \omega_f^2 V_f \]

Here \( V_f = S \Delta \)
\( \Delta \) - flag’s thick
Cyclic frequency

\[ \omega_f = A \sqrt{\frac{kN}{\Delta S \rho_f}} \]

\[ A = \sqrt{\frac{64 \rho}{15}} \]

\[ k = C_1 \frac{\rho V^2}{2} l_2 \]
Final equation for flag cyclic frequency

\[ \omega_f = A' \sqrt{\frac{\rho N}{\Delta l_1 \rho_f}} V \]
Experimental setup
Conclusion:

This problem hasn’t exact solution because of random air whirl. But we have constructed experimental setup, which helped us to investigate the airflow pattern around the flag. Full description of flag’s behaviour was presented by theoretical and experimental findings.