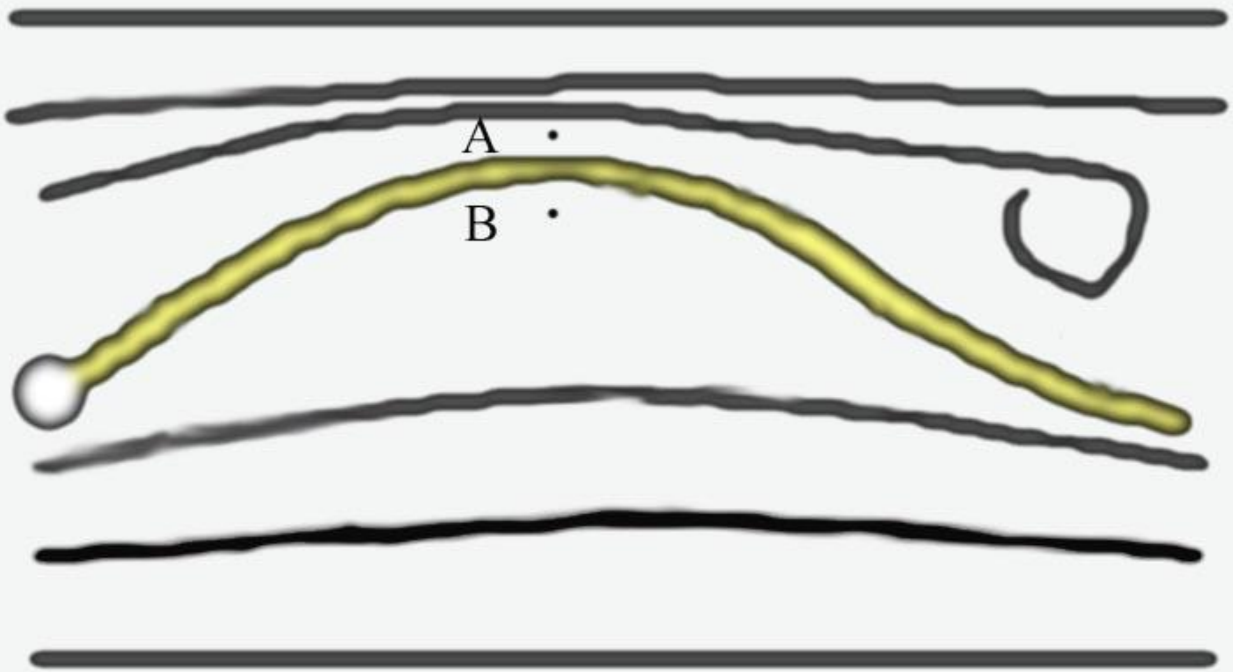

Team Belarus
presents...
Flying colours

Flying colors

Why do flags flutter in the wind? Investigate experimentally the airflow pattern around a flag.
Describe this behaviour.


A
B

Number of whirls with fixed parameters

$$d^2N = C\ N\ d\omega\ dR$$

N - general number of whirls
ω - angular velocity of whirl
R - whirl's radius
C - constant of proportionality

The energy of whirls with fixed parameters

$$d^2E = \beta E\, d^2N$$

E - average kinetic energy of whirls

Full energy of one whirl

$$E = \frac{m\upsilon^2}{2} + \frac{I\omega^2}{2}$$

$$\upsilon = \omega R \qquad I = \frac{2}{5} m R^2$$

The equation for the energy of whirl

$$E = \frac{3}{4}\pi\rho l_2 \omega^2 R^4$$

$$m = l_2 \pi R^2 \rho$$

Here:

l_2 - flag's height

Equation for coefficient C

$$\int_0^{R_0}\int_0^{\omega_0} d^2N = N$$

$$\int_0^{R_0}\int_0^{\omega_0} CN\,d\omega\,dR = N$$

$$C = \frac{1}{\omega_0 R_0}$$

ω_0, R_0 - limit cyclic frequency and radius of the whirl nearby the flag

Full energy of whirls flying on the flag

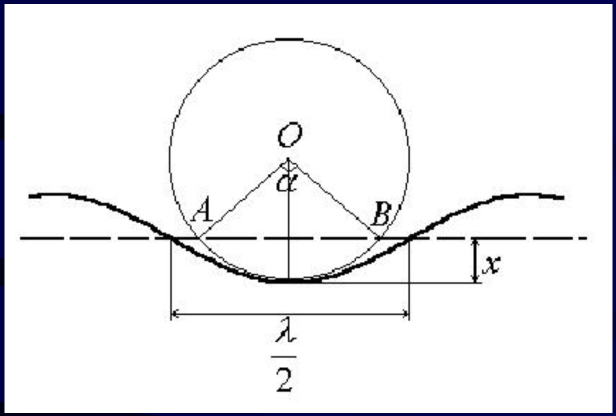
$$E_{\Pi} = \int_0^{R_0}\int_0^{\omega_0} d^2E = \frac{1}{20}\pi\rho l_2 R_0^4 \omega_0^2 N$$

Average values

$$< \mathrm{R} >= \frac{1}{R_0} \int_0^{R_0} R dR = \frac{R_0}{2}$$

$$< \omega >= \frac{1}{\omega_0} \int_0^{\omega_0} \omega d\omega = \frac{\omega_0}{2}$$

Restoring force



$$F = C_1 \frac{\rho V^2}{2} S_1$$

$$S_1 = x \cdot l_2$$

$$< x >= \frac{\alpha^2}{16} R_0$$

C_1 - drag coefficient
V - air’s speed
ρ - air’s density
x - flag’s deflection

Wave's length

$$\frac{\lambda}{2} \approx \frac{R_0}{2}\alpha$$

$$\lambda \approx R_0\alpha$$

Cyclic frequency of flag oscillation

$$\omega_f = \frac{2\pi u}{\lambda}$$

u - speed of wave propagation

The energy of sinusoidal wave

$$E_1 = \frac{1}{2}\rho_f < \mathrm{x} >^2 \omega_f^2 V_f$$

Here $V_f = S\,\Delta$

Δ - flag's thick

Cyclic frequency

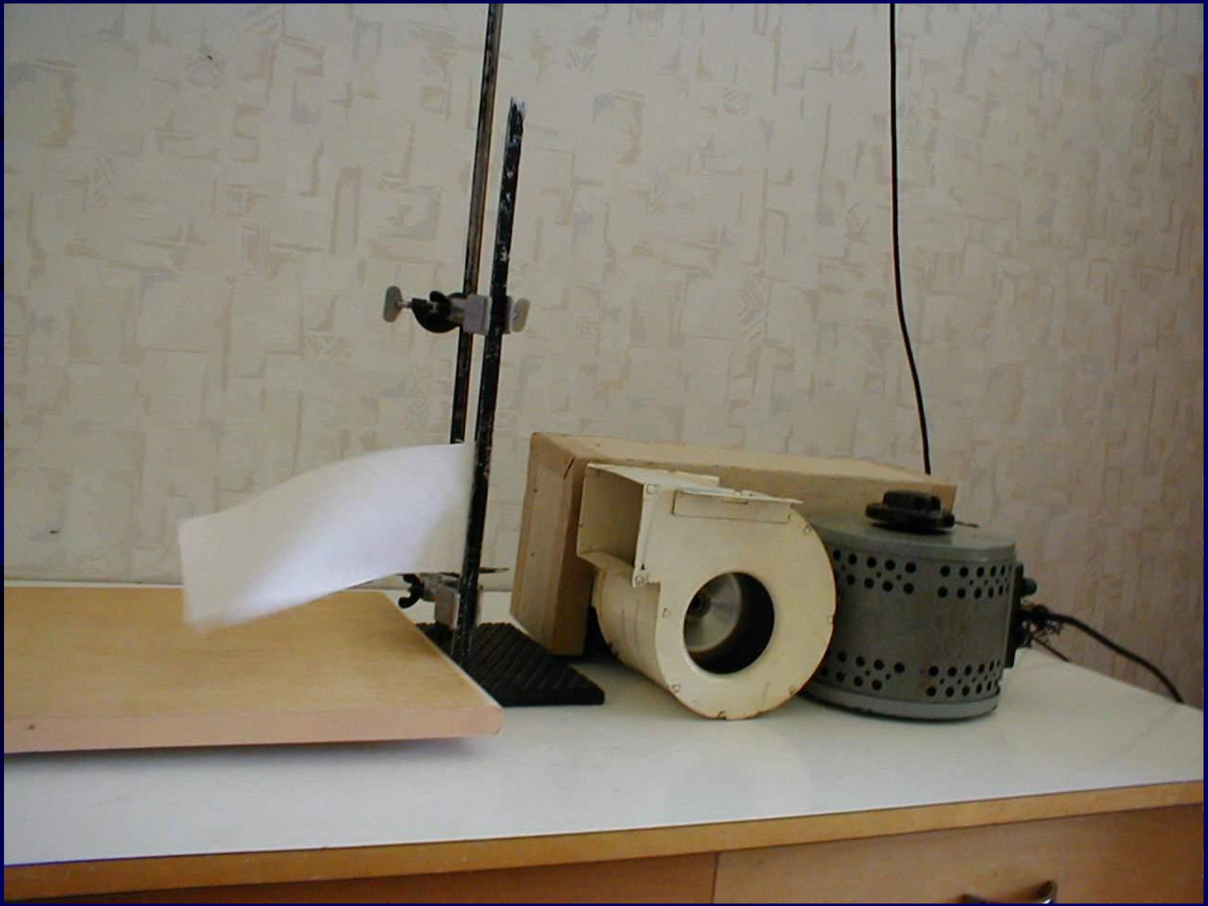
$$\omega_f = A\sqrt{\frac{kN}{\Delta S \rho_f}}$$

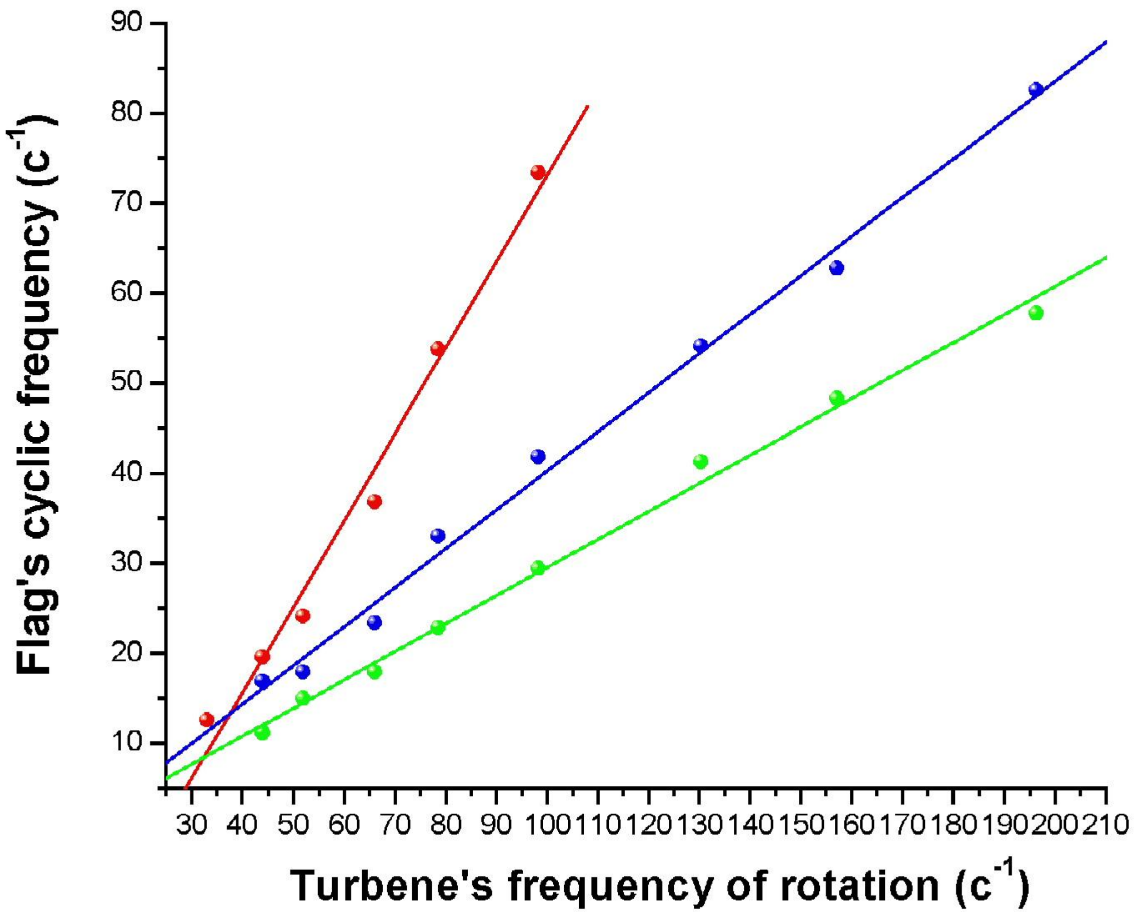
$$A = \sqrt{\frac{64\rho}{15}} \qquad k = C_1 \frac{\rho V^2}{2} l_2$$

Final equation for flag cyclic frequency

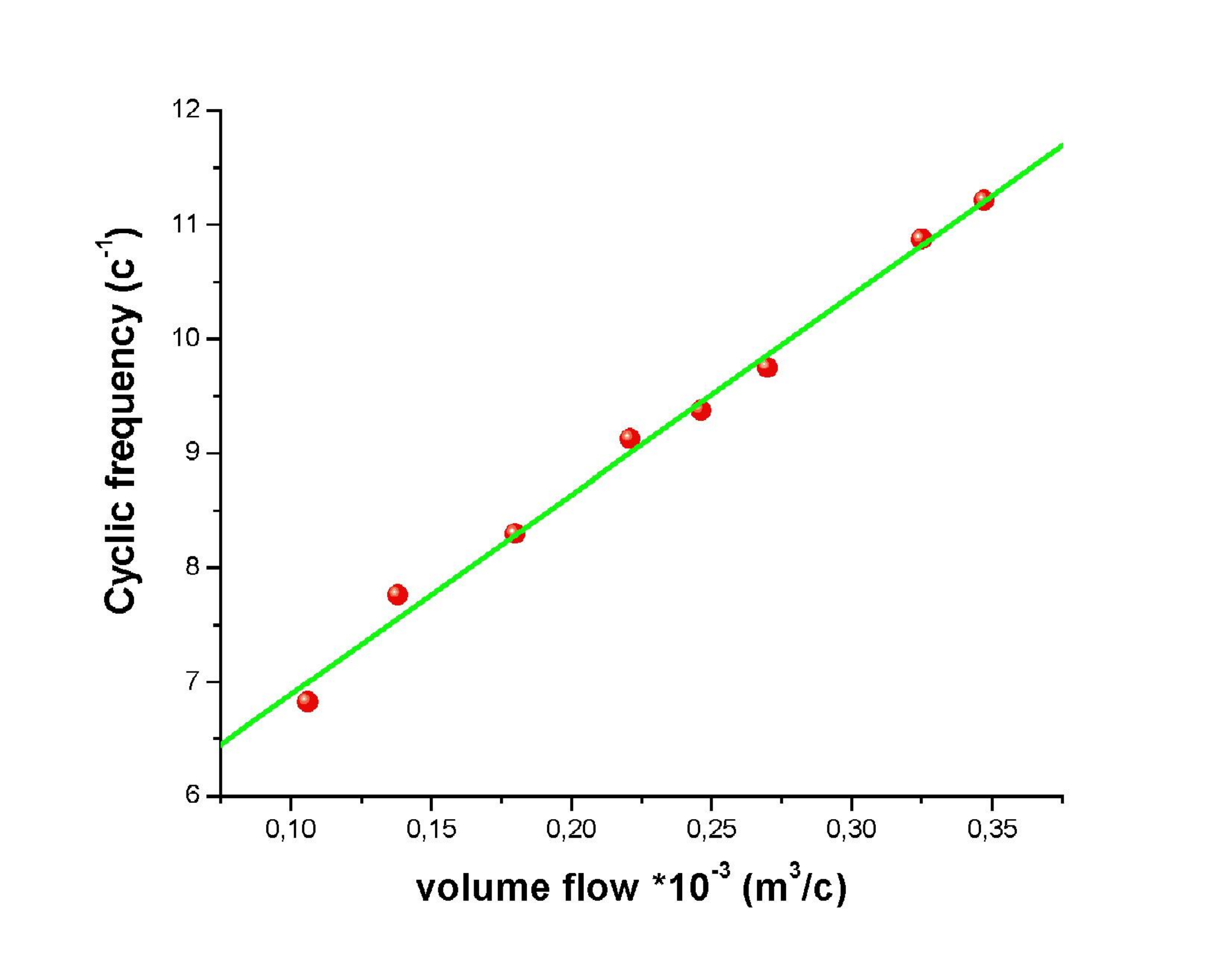
$$\omega_f = A' \sqrt{\frac{\rho N}{\Delta l_1 \rho_f}}\, V$$

Experimental setup

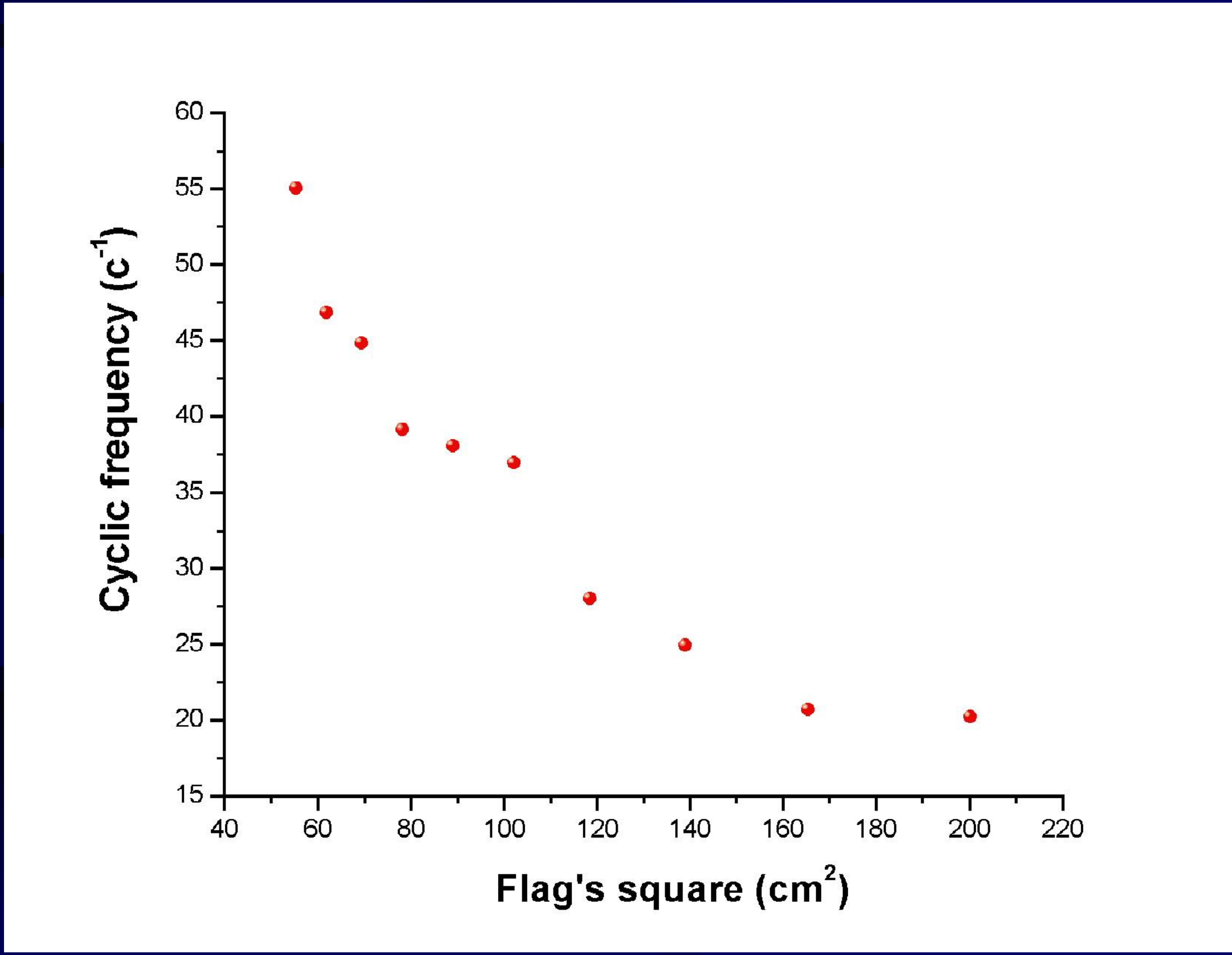


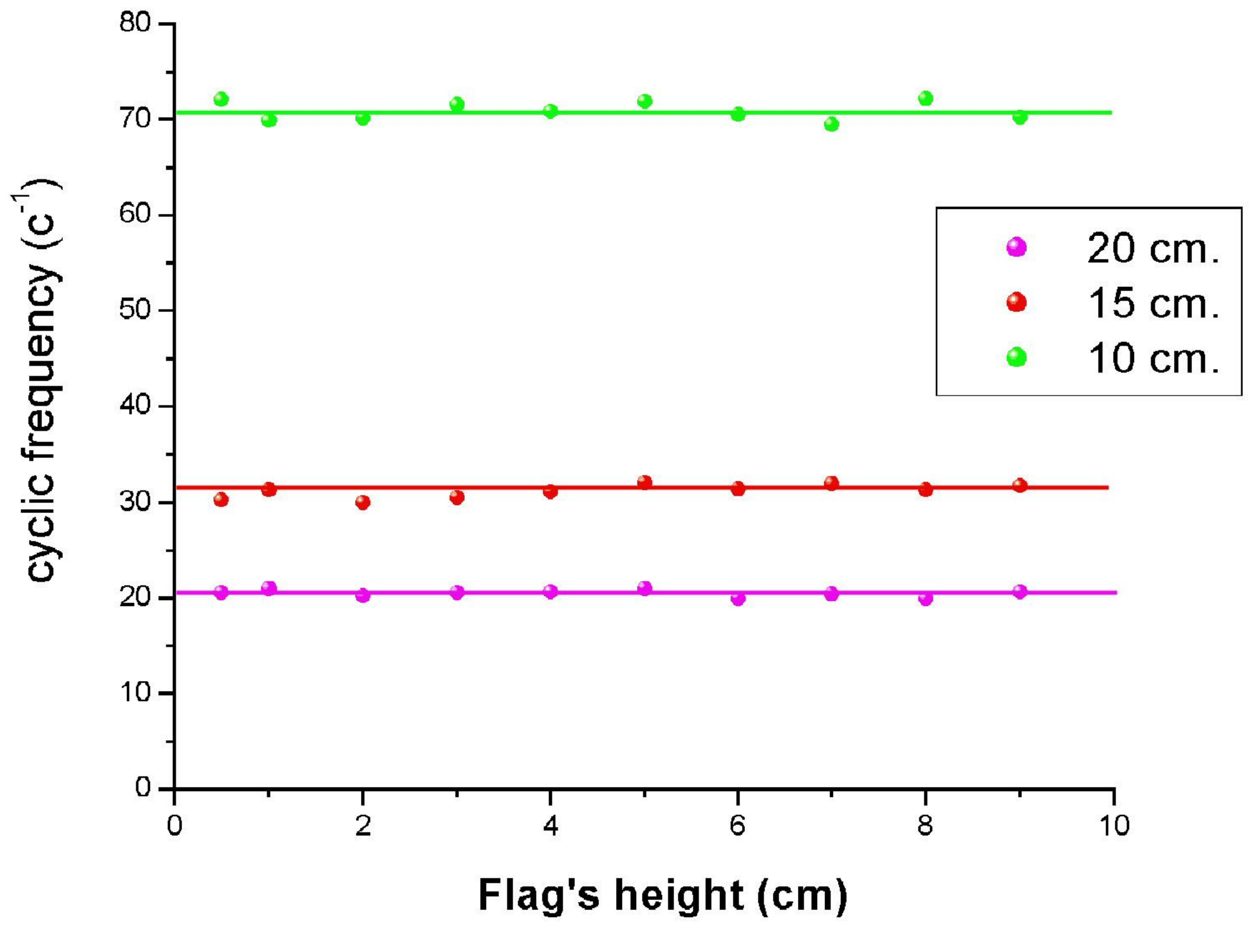


Flag's cyclic frequency (c-1)
Turbene's frequency of rotation (c-1)
10
20
30
40
50
60
70
80
90
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210



Cyclic frequency (c-1)
12
11
10
9
8
7
6
0,10
0,15
0,20
0,25
0,30
0,35
volume flow *10-3 (m3/c)


Cyclic frequency (c-1)
60
55
50
45
40
35
30
25
20
15
40
60
80
100
120
140
160
180
200
220
Flag's square (cm2)



80
70
60
50
40
30
20
10
0
cyclic frequency (c⁻¹)
0
2
4
6
8
10
Flag's height (cm)
20 cm.
15 cm.
10 cm.

Conclusion:

This problem hasn’t exact solution
because of random air whirl.
But we have constructed experimental setup,
which helped us to investigate
the airflow pattern around the flag.
Full description of flag’s behaviour
was presented by theoretical and
experimental findings.