Problem: Spring thread

Good day ladies and gentlemen! My name is Darija Kuleskaya and today I will represent report on problem 'spring thread'. Firstly, let me remind you a task:

Pull a thread through the button holes as shown in the picture. The button can be put into rotating motion by pulling the thread. One can feel some elasticity of the thread. Explain the elastic properties of such a system.

Now, I'll determine purposes of our work:

- ✓ to describe investigated system qualitatively
- ✓ to compare our system with others
- ✓ to give a definition of elasticity and it's physical interpretation in this problem
- ✓ to evaluate dependence of speculative coefficient of elasticity on systems parameter
- ✓ to estimate dependence experimentally.

This system is an example of unusual oscillating system, and for describing its properties we should compare it with more usual ones. For this purpose we decided to choose well-known system 'spring-bob'. For simplicity we denoted it as system №1, and our system 'thread-button' as system №2.

In the first system, motion is forward and described by coordinate x, in the second system motion is rotational and described by angle of rotation φ. Potential energy in the first system is saved in resilience of spring and in the second system potential energy is saved in deformation of thread. The role of restoring force in the first system plays elastic force, and in the second system this role plays moment of projection of tighting force.

<u>Now, let's examine oscillation of these systems:</u>

At first, we considered our system as self-contained (View slide). At first moment our systems possess maximum potential energy. Then in the s.№2 thread starts to untwist and potential energy turns into kinetic energy of rotation of button. At the same time in the s.№1 spring starts to relax and potential energy turns into kinetic energy of forward motion of bob. Then, when all potential energy have turned into kinetic energy, systems continue their motion in the same direction and kinetic energy begins to turn into potential. So, we have a cycle. Peak value of kinetic and potential energy in every period is constant in self-contained system. But in reality some part of energy disappears and without its pumping system will stop. In the picture you can see transformation of energy in both systems. Analogy is obvious.

However, in the s.№2 there is one distinctive feature: system parameters are not constant. Twisted thread is shorter then untwisted one, and we should change distance between arms to avoid whipping of thread.

<u>Why button starts rotation:</u>

Why the button starts rotation? The button starts to rotate when the forces acting on a button, because of the asymmetry, are not mutually destroyed, and create the rotary moment. Really, we shall consider a button during that moment when the thread is untwisted: on picture 1 position of threads during this moment is shown by pink color. Projections of forces of a tension to a button are equal to zero. When the string is twisted (picture 2) projections of forces of a tension to a button (are shown by yellow color) create rotary.

<u>Forces:</u>

Now, let's value the moment of forces. Arm of force is less or equal to radius of the hole of the button. Projection of force depends on angle of departure (α), α is approximately equal: (d- distance between the holes of the button, l- length of thread, l'- length of twisted part of thread). So, expression for moment of forces is equal:

$$M = -2F \cdot R \cdot \sin\alpha \approx -2F \cdot R \cdot \frac{4d}{l - 2l'} \text{ (2);}$$

Let's use linear approximation and write down the equations of motion for both systems:

$$m\ddot{x} = -kx;\ (1)$$

$$I\ddot{\varphi} = -k\varphi;\ (2)$$

(Why this linear approximation is correct, can be shown with the help of simple logic reasoning, and if this question will interest audience, we will answer it during debate). The widespread method of the description of such systems is construction of a phase plane. For systems without taking into account friction the phase trajectory represents a circle with the center in the beginning of coordinates (shown in the picture).

Let's consider our system as not self-contained. (View slide) Distinctive feature is that in the system 1 there is static friction and, as a result, area of stagnation. In this area, bob won't come into movement even when spring possesses potential energy if elastic force of a spring will be less, than the force of friction of rest. There is no area of stagnation for system 2, because there is no state friction in it.

Equations of motion for both systems in this case:

$$m\ddot{x} = -kx + \delta(\dot{x}) \quad (6);$$

$$I\ddot{\varphi} = -F \cdot r \cdot \frac{4d}{l - 2l'} + \xi(\dot{\varphi}) \quad (7);$$

We consider that friction is pro rata speed of movement (6), of rotation (7). Taking into account forces of friction the phase trajectory represents a spiral, twists to the beginning of coordinates, in case of when area of stagnation is not present and twisting in a point near the center of coordinates when the area of stagnation is. These phase planes are shown in the picture.

<u>Elasticity:</u>

'Elasticity'-system attribute to revert to the original state. For describing of our system we will insert artificial coefficient of elasticity. We will use it at calculations. We calculated it experimentally, because it was impossible for us, to calculate it theoretically. At first we will express coefficient of elasticity from equations of motion $k = \frac{M_{рук}}{\varphi}$ (11);, but in this expression there is unknown parameter – moment of force operating from hands. Let's write law of conservation of energy and express coefficient of elasticity.

$$\frac{k\varphi_{\max}^{2}}{2} = \frac{I\omega_{\max}^{2}}{2}$$

But this expression can be used only for self-contained system, but our system is not. But during one period of fluctuation the full work accomplished by hands is equal to zero. So, full energy is constant and expression (13) is correct.

$$k = \frac{I\omega_{max}^2}{\varphi_{max}^2} \quad (13)$$

Experiment:

For experimental calculation of coefficient of elasticity it is necessary for us to know such parameters, as the moment of inertia of a button, angle of rotation of thread φ and the maximal value of angular velocity. With the two first parameters we had no problems, but calculation of the maximal angular velocity was a real problem. So we decided to carry out rather interesting experiment. In the slide you can see photo of experimental assembly. As you can see, for convenience, we have replaced a button by CD with the holes drilled in it. (*Also, it is necessary to note, that in quiescent state threads are parallel to each other)* Then, we fixed symmetrically light plastic sticks on a disk. Then we placed a disk in such way, that during rotation, ray was periodically blocked by sticks. The oscillograph has been fixing voltage surges in system. They were equal to changes of angular velocity. But this experiment wasn't very convenient, so we decided to change oscillograph by set of sensors "Current" from the set "Lab pro". These sensors take off voltage surges and send data to computer, where special program 'Logger Pro' processes it. This program also allows making Furies expansion, which helps to estimate maximal angular velocity.

So, we have all parameters for estimating coefficient of elasticity. Example of it you can see at the slide.

Interesting phenomena:

1. Appearance of waves
2. Rotation of some buttons
3. Distinctive sound

Resume:

- System was described qualitatively

- ❖ Analogy with other systems was drawn
- ❖ Definition of elasticity and its physical interpretation in this problem were given
- ❖ Dependence of artificial coefficient of elasticity on systems parameter was described
- ❖ A number of experiments, which allowed us to estimate dependence exactly enough, were made
- ❖ Coefficient of elasticity isn't constant in this system
- ❖ Coefficient of elasticity is function of applied force
- ❖ We can estimate coefficient of elasticity from energy conservation law

That's all. Thank you for your attention!