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Problem No.5 “Falling ball”

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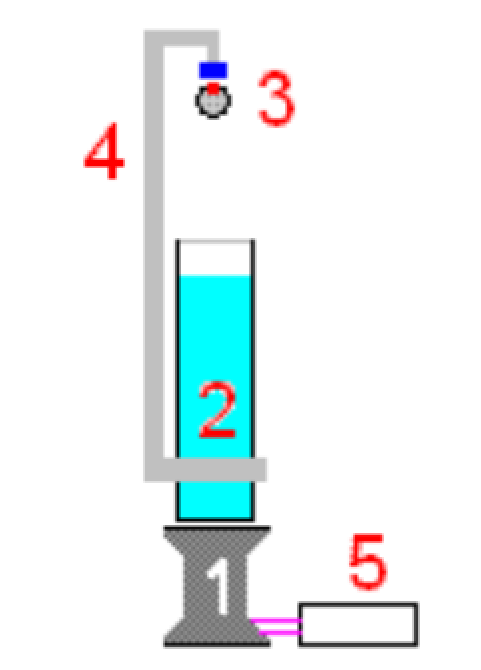
Problem Review

An electronic balance (1) is connected to a PC (5) in order to record the time dependence of the measured weight. A light frame (4) is mounted on a tall beaker (2) filled with water. The frame has a holder (3) allowing controlled release of a small ball such that it falls into the water. The beaker is placed on the balance as depicted in the Figure. Investigate how the readings of the balance reflect the different phases of the motion of the ball.



Outline

PART I Experiment

1.Equipment & Set up

2.The process of the experiments

3.Graphs

PART II Theoretical analysis

PART III Error analysis

PART IV References

I. Experiment

We observed the motion of the ball as well as the readings on the balance.

“Splash”: a dramatic increase of the reading

“Travelling in the water”: the reading tend to get smaller until the ball reaches the bottom.

When the balance is connected to a computer, it couldn’t “record the time dependence of the measured weight”.

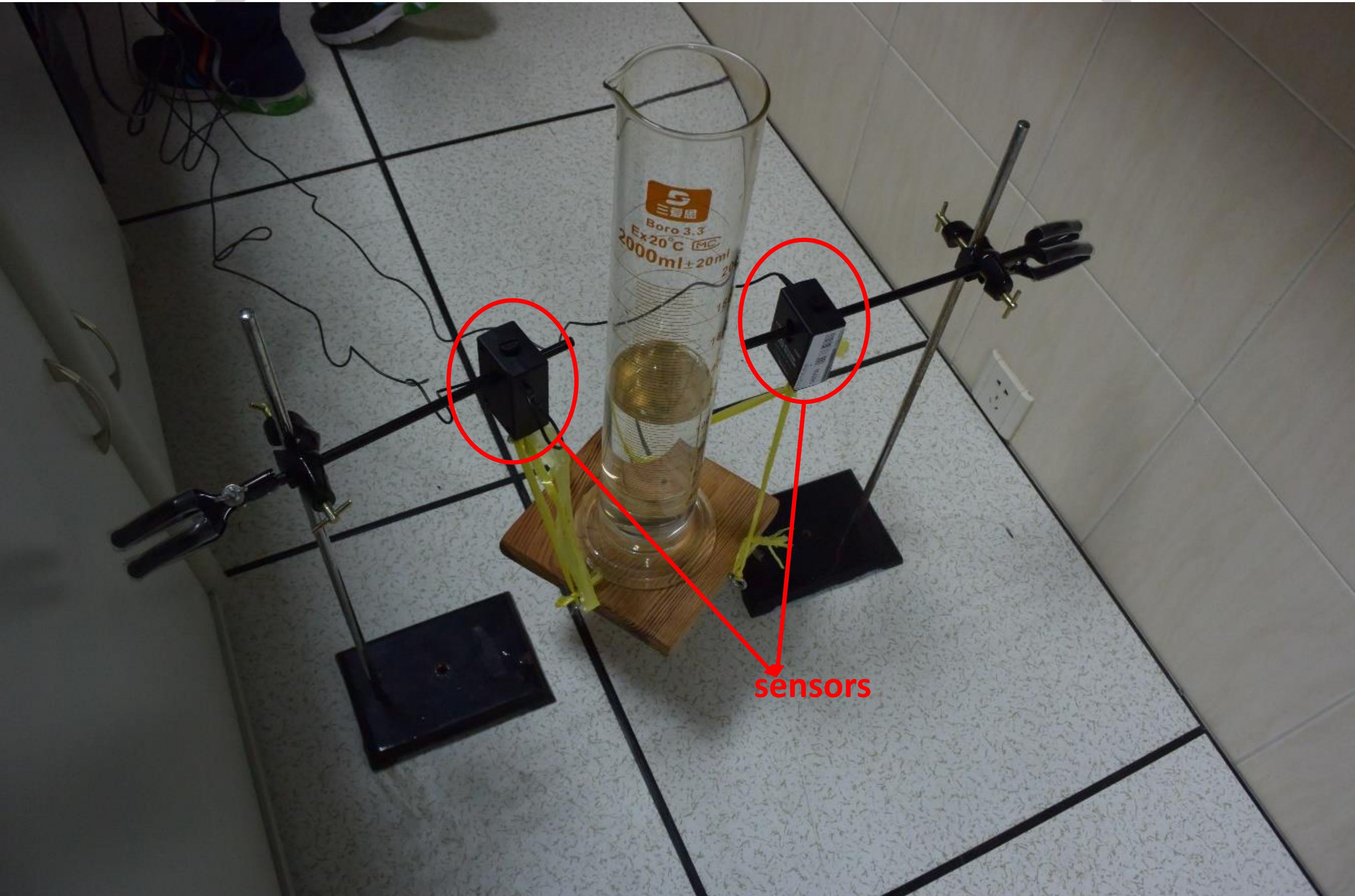
Replace the balance with force sensors:

1. Equipment & Set up

Electric balance: replaced by sensors for better accuracy

Camera: record the motion of the ball

Software(*Tracker*): analyze the motion of the ball


Boro 3.3
2000ml±20ml
sensors

The ball we used for the analyzed experiment is an iron ball with a density of $7.9 \times 10^3 kg/m^3$ and radius of 1.25cm. Consider the density of water $1 \times 10^3 kg/m^3$.

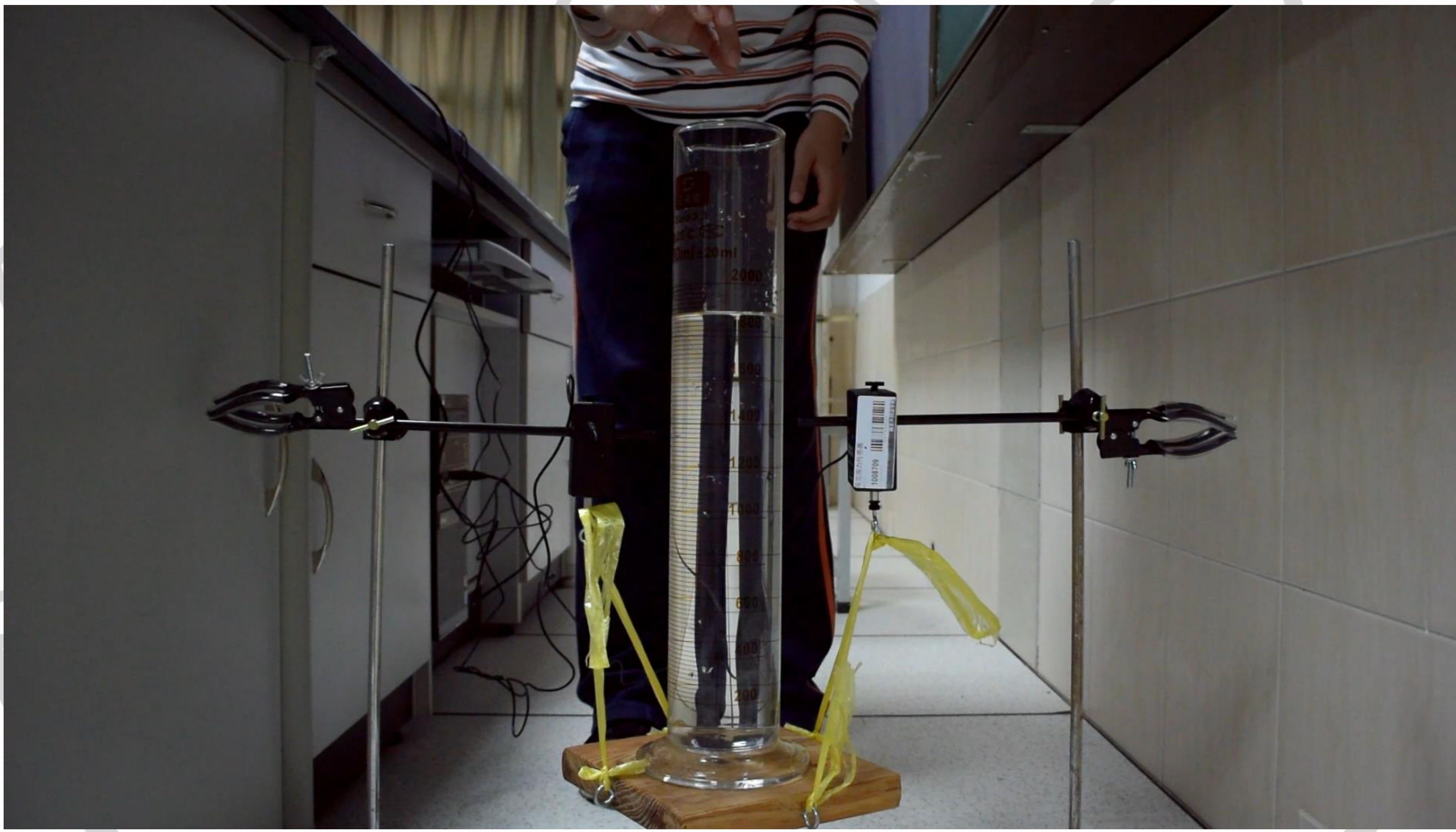
2.Process of the experiments

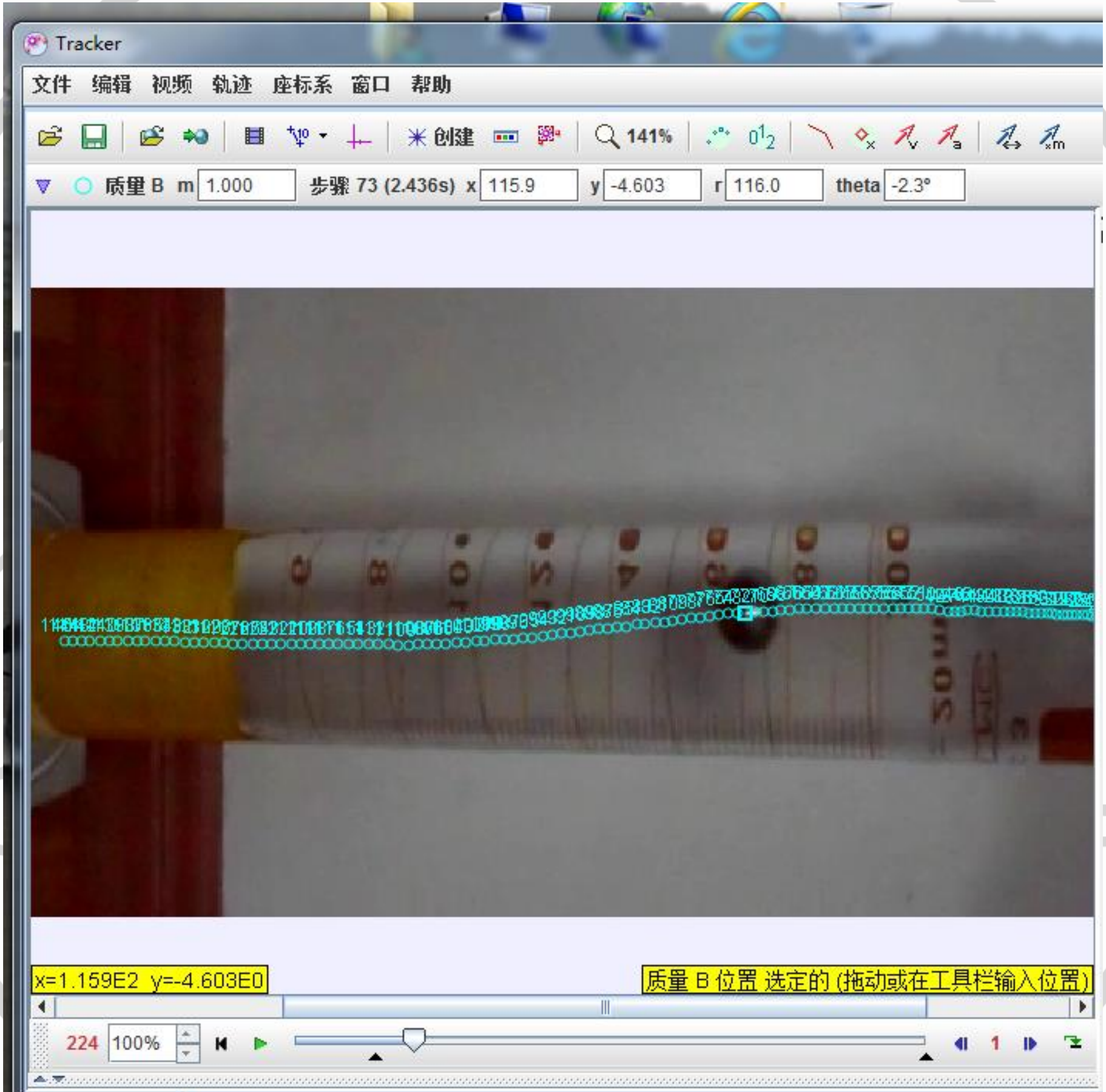
1.To simplify, we still choose to release by hand.

2.Use the camera to record the motion(400 fps).

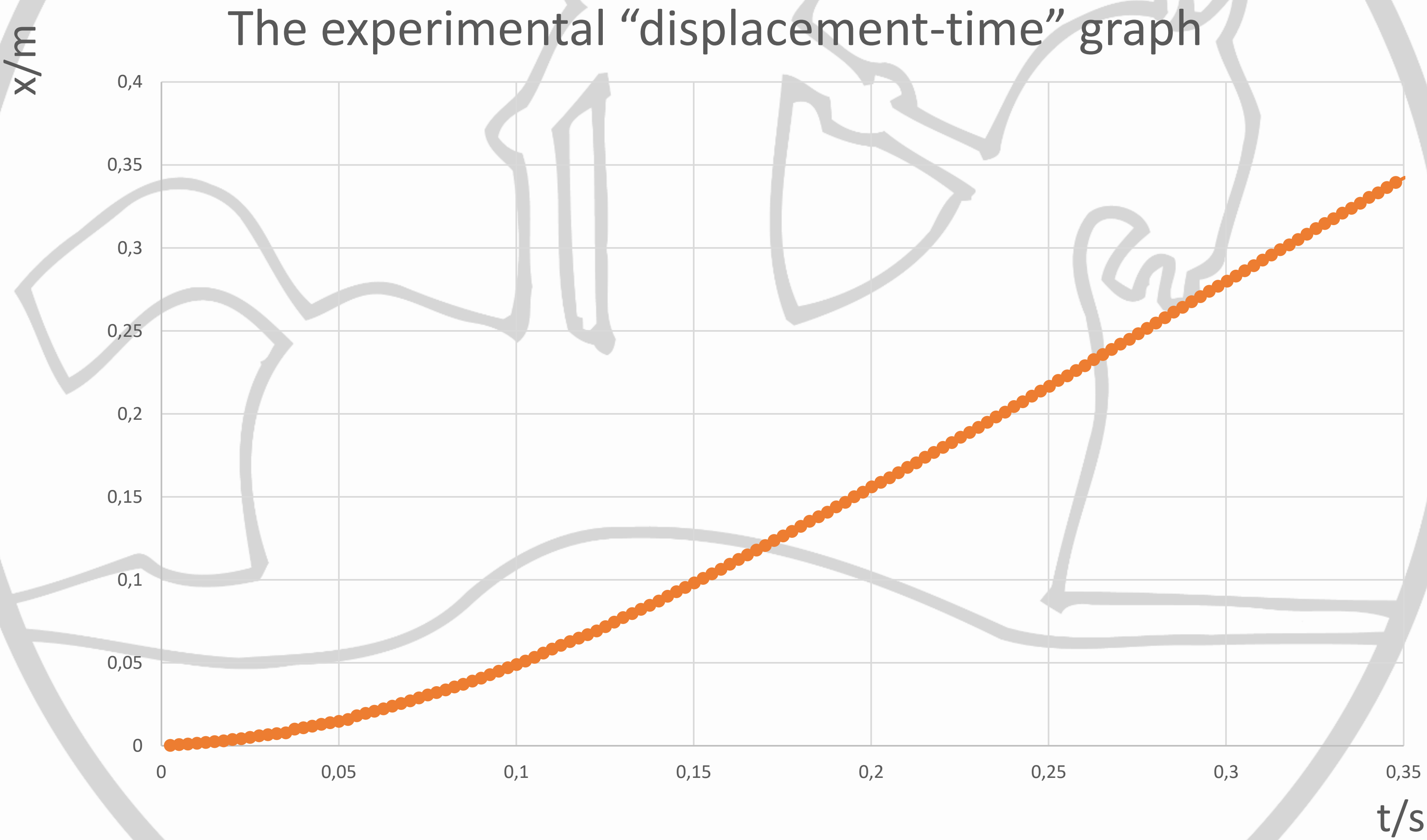
3.Use the sensors can record the time dependence of the force.

4.Export the data to Excel to draw the figure.



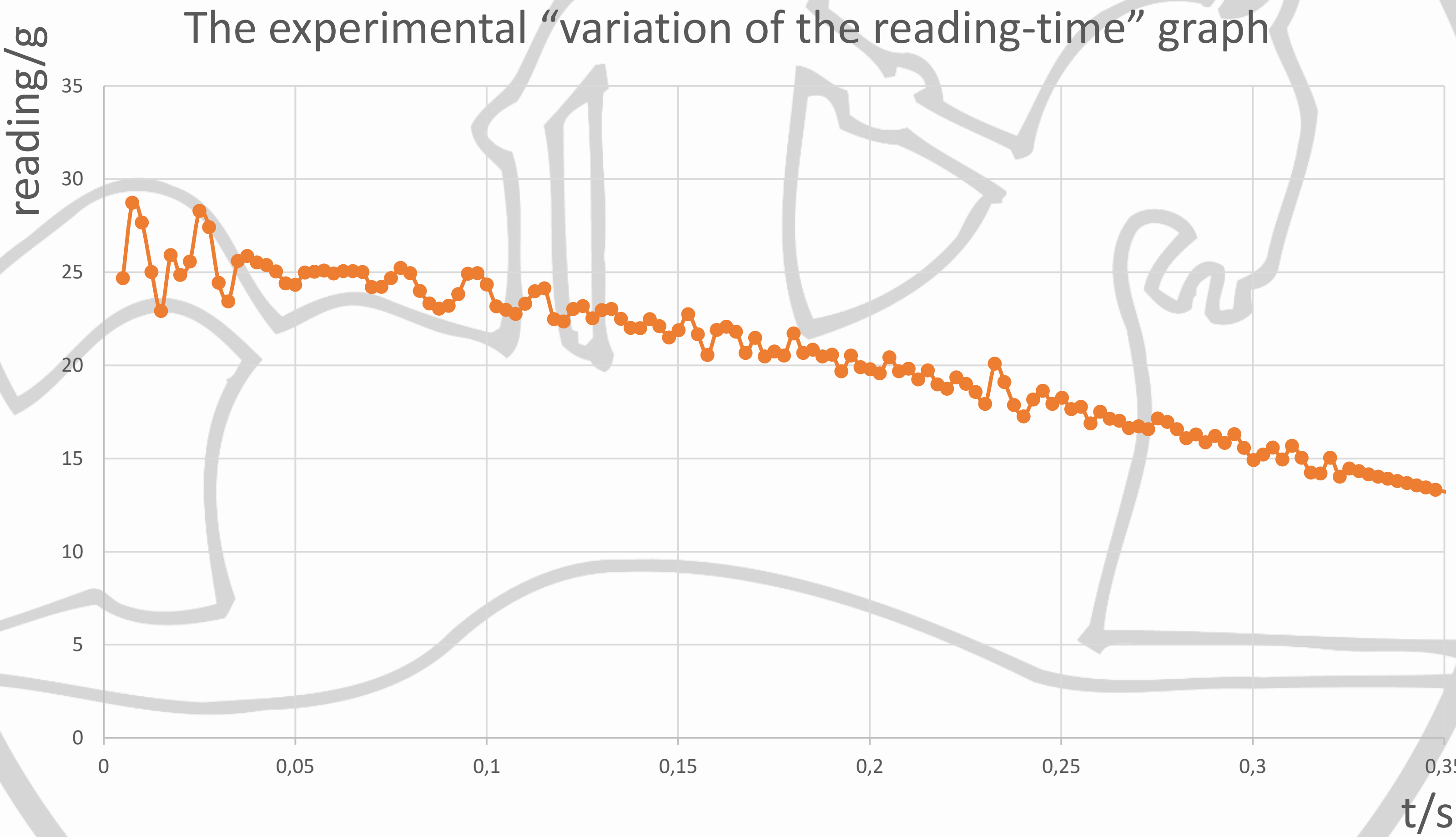

Tracker
文件 编辑 视频 轨迹 座标系 窗口 帮助
创建 141%
质量 B m 1.000 步骤 73 (2.436s) x 115.9 y -4.603 r 116.0 theta -2.3°
x=1.159E2 y=-4.603E0
质量 B 位置 选定的 (拖动或在工具栏输入位置)
224 100% 1

3.Graphs



The experimental “displacement-time” graph
x/m
0,4
0,35
0,3
0,25
0,2
0,15
0,1
0,05
0
0
0,05
0,1
0,15
0,2
0,25
0,3
0,35
t/s

3.Graphs



The experimental "variation of the reading-time" graph
reading/g
35
30
25
20
15
10
5
0
0
0,05
0,1
0,15
0,2
0,25
0,3
0,35
t/s

II. Theoretical analysis

From our experiments, we find three phases of motion:

1) Above the water;
2) Entering the water;
3) Travelling in the water, which consists of acceleration and uniform motion.

The phase of ‘entering the water’ is reflected by a dramatic increase with fluctuation in the reading. Because the time interval of this phase is short, it’s hard to do a quantitative analysis.

The next phase is the longest and most well-reflected. As observed, the reading of the balance is positive and tend to decrease during this phase. In the rest of the research, we focused on this phase of motion.

Theoretical Analysis

To begin a strict qualitative analysis of the last phase, we define some parameters:

ρ_L The density of water

ρ_B The density of the ball

r The radius of the ball

t The time interval after the set starting point

x The displacement of the ball from the starting point

v The velocity of the ball

Theoretical Analysis

Forces on the ball:

$$mg = \rho_B \cdot \frac{4}{3}\pi r^3 \cdot g$$

$$F_B = \rho_L \cdot \frac{4}{3}\pi r^3 \cdot g$$

$$F_D = \frac{1}{2} c \rho_L \pi r^2 v^2$$

$$c \approx 0.47$$

Theoretical Analysis

Newton's Second Law:

$$a = \frac{mg - F_B - F_D}{m}$$

$$mg = \rho_B \cdot \frac{4}{3}\pi r^3 \cdot g$$

$$F_B = \rho_L \cdot \frac{4}{3}\pi r^3 \cdot g$$

$$F_D = \frac{1}{2} c \rho_L \pi r^2 v^2$$

In differential form:

$$v = \frac{dx}{dt} \qquad a = \frac{d^2 x}{dt^2}$$

Theoretical Analysis

Substitution: $v = \frac{dx}{dt}$ $a = \frac{d^2x}{dt^2}$

$$\frac{d^2x}{dt^2} = \frac{\rho_B \cdot \frac{4}{3}\pi r^3 \cdot g - \rho_L \cdot \frac{4}{3}\pi r^3 \cdot g - \frac{1}{2}c\pi r^2 \left(\frac{dx}{dt}\right)^2}{\rho_B \cdot \frac{4}{3}\pi r^3}$$

Now we have finished analyzing the motion of the ball, we are moving on to make a connection between the motion and the reading of the balance.

Considering the system of the beaker, the water in it and the ball.

According to the Impulse-momentum theorem,

$$J = F\Delta t = p = m\Delta v$$

$$F = m\frac{\Delta v}{\Delta t} = ma$$

However, the reading of the balance doesn't directly show the force on it. Transfer the force to the reading in mass(g):

$$reading = \frac{1000}{g}\cdot F = \frac{1000}{g}\cdot ma$$

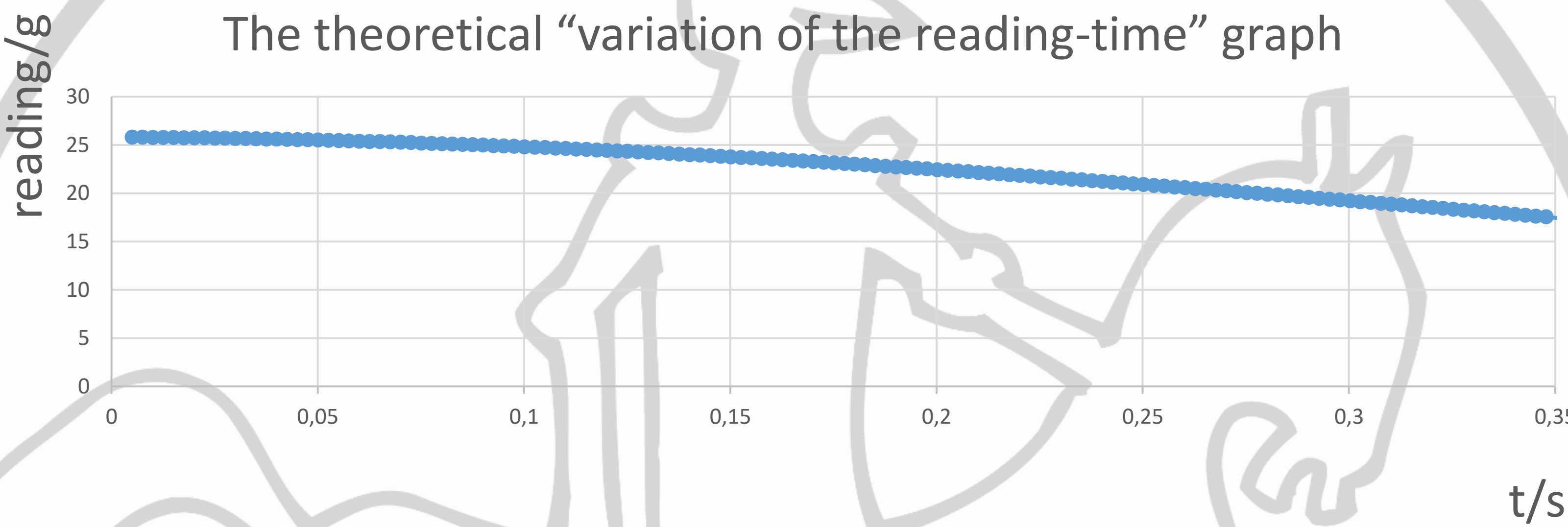
Solve the equations and let

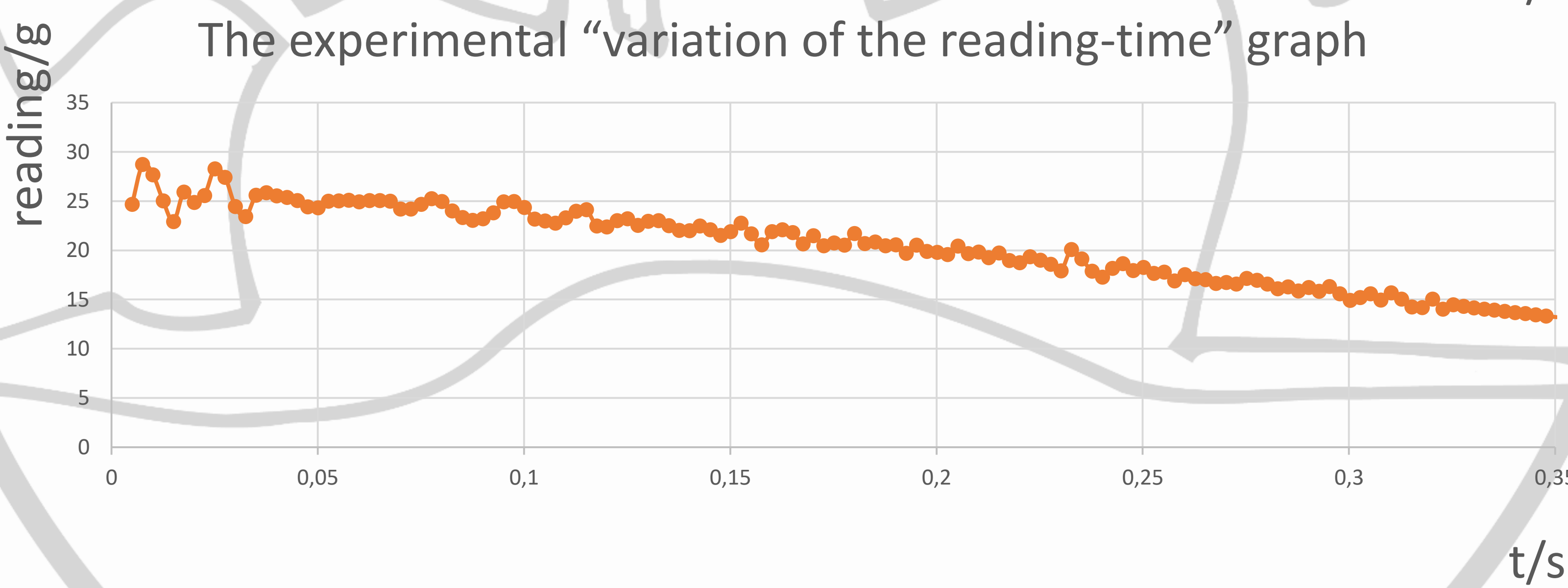
$$\rho_L = 1 \times 10^3 kg / m^3$$

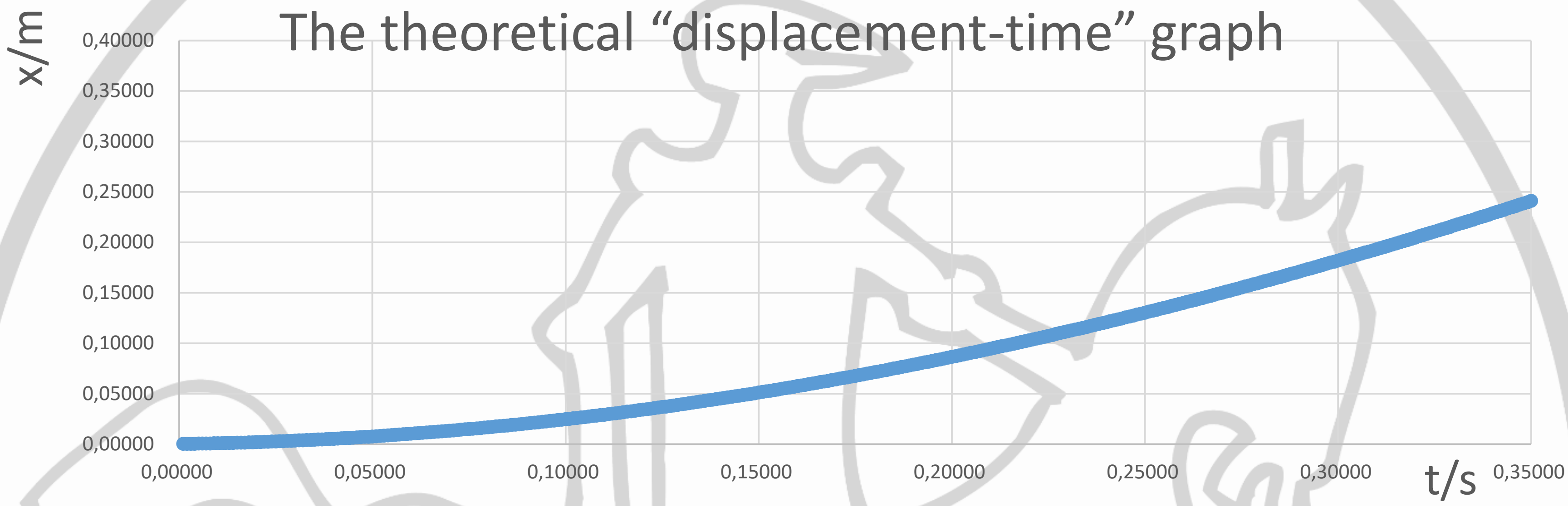
$$\rho_B = 7.9 \times 10^3 kg / m^3$$

$$r = 1.25 \times 10^{-2} m$$

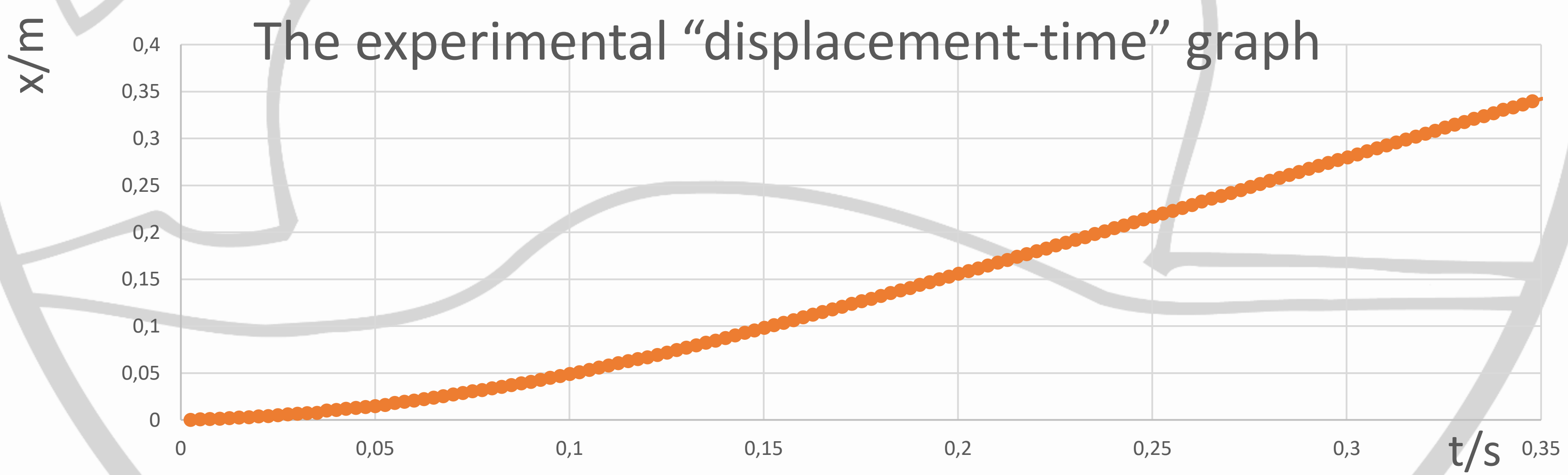
we can get the theoretical graphs:


The theoretical “variation of the reading-time” graph
reading/g
30
25
20
15
10
5
0
0
0,05
0,1
0,15
0,2
0,25
0,3
0,35
t/s


The experimental “variation of the reading-time” graph
reading/g
35
30
25
20
15
10
5
0
0
0,05
0,1
0,15
0,2
0,25
0,3
0,35
t/s



The theoretical “displacement-time” graph
x/m
0,40000
0,35000
0,30000
0,25000
0,20000
0,15000
0,10000
0,05000
0,00000
0,00000
0,05000
0,10000
0,15000
0,20000
0,25000
0,30000
0,35000
t/s



The experimental “displacement-time” graph
x/m
0,4
0,35
0,3
0,25
0,2
0,15
0,1
0,05
0
0
0,05
0,1
0,15
0,2
0,25
0,3
0,35
t/s

Error analysis

1. The inaccuracy of some parameters, e.g. the density of the ball and water, the value of current C.
2. The inaccuracy of tracking the ball with software.
3. The instability of the experiment set up may cause some shaking, thus the readings have a fluctuating graph.

IV. Conclusion

Conclusion

1. The different phases of the ball are reflected by evident changes in the reading of the balance.
2. The motion of the ball in the last phase can be well predicted with force analysis.
3. The reading of the balance in the last phase can be well predicted with the Impact-momentum theorem.

References

- *Wikipedia* Drag (physics) https://en.wikipedia.org/wiki/Drag_%28physics%29
- *Wikipedia* Drag equation https://en.wikipedia.org/wiki/Drag_equation
- *Wikipedia* Drag coefficient https://en.wikipedia.org/wiki/Drag_coefficient
- *Wikipedia* Reynold's number https://en.wikipedia.org/wiki/Reynolds_number
- *Wikipedia* Viscosity https://en.wikipedia.org/wiki/Viscosity

Thank you for you attention!