Problem 14
Moving Cylinder

reporter:
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Problem 14: Moving Cylinder

Place a sheet of paper on a horizontal table and put a cylindrical object (e.g. a pencil) on the paper. Pull the paper out. Observe and investigate the motion of the cylinder until it comes to rest.
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Contents

• Introduction
  – Definitions
  – Theory
  – Mathematical model
  – Computer simulation

• Experiments
  – Material
  – Experimental results
  – Analysis

• Conclusion
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**Objective**

- Investigate the cylinder movement (translation and rotation) and its relevant parameters till the moment the velocity turns to zero.
Introduction: Forces acting over the cylinder
Introduction: Forces acting over the cylinder

- **Force F= friction force:**
  - Static (without sliding);
  - Dynamic (with sliding).

**Static friction**

\[ f_s = \mu_s N = \mu_s W = \mu_s \cdot mg \]

Valid only for horizontal surfaces.
Introduction: Forces acting over the cylinder

- Drag force:

\[ D = \frac{1}{2} \rho C_x A v^2 \]

Negligible
Introduction: Forces acting over the cylinder
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Introduction: Forces acting over the cylinder

\[ \tau = F \cdot r \cdot \sin \theta \]
\[ \tau = f_d \cdot R \]
\[ \tau = I \cdot \alpha \]
\[ I = \frac{mR^2}{2} \]
Introduction: General considerations

• During the sheet pulling:

\[ F = m \cdot a \]

\[ f = m \cdot \alpha \]

\[ \alpha = \frac{f}{m} \]

\[ \nu_{cm} = \frac{f}{m} t \]

\[ f \cdot R = \frac{mR^2}{2} \cdot \alpha \]

\[ \alpha = \frac{2f}{mR} \]

\[ \omega = \frac{2f}{mR} t \]
**Introduction: General considerations**

\[ v_P = v_{cm} + \omega R \]

\[ v_P = \frac{f}{m} t + \frac{2f}{m} t \]

\[ v_P = \frac{3f}{m} t \]

\[ v_P = 3v_{cm} \]

\[ a = \frac{3f}{m} \]

There’s no influence of the radius.
General considerations

- After losing contact with the paper: **dynamic friction**
- While in contact with the paper:
  
  **Static friction**
  
  \[ a \leq \frac{3f_{\text{max}}}{m} \]

  **Dynamic friction**
  
  \[ a > \frac{3f_{\text{max}}}{m} \]
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Introduction: Simulation for general considerations
Introduction: Impulse and momentum

- Happens while pulling the paper.

\[ I = \Delta p \]

\[ f \cdot \Delta t = m v - m v_0 \]

\[ f \cdot \Delta t = m \cdot v \]

\[ \nu_{max} = \frac{f \cdot \Delta t}{m} \]

• Initial velocity after losing contact with the sheet: maximum velocity.
Introduction: Translation and rotation

Translation

- Motion with a line as trajectory.
- Uniformly Varied Motion

Rotation

- Motion of the body points around an axis.
- Uniformly Varied Circular motion
Introduction: Translational motion

\[ F = f \]
\[ ma = mg\mu \]
\[ a = g\mu \]

\[ S = S_0 + V_0 t + \frac{1}{2} \mu gt^2 \]

\[ V = V_0 + \mu gt \]

\[ S = S_0 + V_0 t - \frac{1}{2} \mu gt^2 \]

\[ V = V_0 - \mu gt \]
Introduction: Rotational motion

- Relating linear and angular quantities:

\[
S = \varphi R \\
V = \omega R \\
a_t = \alpha R
\]

\[
\alpha = \frac{a_t}{R} \\
\alpha = \frac{\mu g}{R}
\]

Static friction

\[
\varphi = \varphi_0 + \omega_0 t + \frac{\mu g t^2}{2R}
\]

\[
\omega = \omega_0 + \frac{\mu g t}{2R}
\]

Dynamic friction

\[
\varphi = \varphi_0 + \omega_0 t - \frac{\mu g t^2}{2R}
\]

\[
\omega = \omega_0 - \frac{\mu g t}{2R}
\]
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Introduction: Cylinder material influence

• Friction is generated by the irregularities.

Smother material → Less friction → More time of motion
Introduction: Coefficient of friction

\[ f_s = W_t \]

\[ \mu_s \cdot mg \cdot \cos \theta = mg \cdot \sin \theta \]

\[ \mu_s = \tan \theta \]

- \( \mu_d \approx 10\% \) lower than \( \mu_s \)
Introduction: Conditions of pulling for the cylinder motion

Pull too slowly → Just translation (with paper)

Pull too fast → Just rotation
Experiments: Experimental Description

- **Experiment 1:** find the coefficients of friction.
- **Experiment 2:** vary the surfaces.
- **Experiment 3:** conditions for the cylinder movement.
- Make some computer simulations.
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Experiments: Material

• Wood cylinder
• Wood pieces (same material of the cylinder)
• Metal cylinder (act like weights for pulling)
• Measuring tape with precision of 0.05cm
• Caliper rule with precision of 0.05mm
• EVA
• Cardboard
• Wood surface
• String
• Pencil
Experiments: Experimental Data

- **Surface**: EVA
- **Maximum reached velocity**: m/s
- **Dynamic friction force**: 3.84 N
- **Maximum drag force**: $5 \cdot 10^{-3}$ N
Experiments: Experiment 1 – coefficient of friction

<table>
<thead>
<tr>
<th>Material</th>
<th>Coefficient of $f_s$</th>
<th>Coefficient of $f_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cardboard</td>
<td>0.52</td>
<td>0.47</td>
</tr>
<tr>
<td>EVA</td>
<td>1.04</td>
<td>0.94</td>
</tr>
<tr>
<td>Wood</td>
<td>0.31</td>
<td>0.28</td>
</tr>
</tbody>
</table>
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Experiments: Experiment 2 – vary surfaces

- Find the friction coefficients
- Video analysis of the velocity
- Comparison with calculated data

Wood vs. E.V.A.
Experiments: Experiment 2 – vary surfaces

Find the friction coefficients
Video analysis of the velocity
Comparison with calculated data

Velocity X Time

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Velocity (m/s)</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>0.6</td>
<td>0.2</td>
</tr>
<tr>
<td>0.8</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Wood
E.V.A.
Experiments: Experiment 2 – vary surface

- Find the friction coefficients
- Video analysis of the velocity
- Comparison with calculated data

1. Higher friction
2. Stops faster
3. Experiment agrees with theory
4. The motion proposed showed to be real

Higher friction stops the cylinder faster, which agrees with theoretical predictions.
Experiments: Experiment 3 – conditions for the cylinder movement

- Pull too slowly → translation
- Pull too fast → rotation
Conclusion

• Most relevant parameter: friction force
• Possible to consider Uniformly Varied Motion.
• Influence:
  – Cylinder and surface’s material;
  – Moment of inertia.
• No influence or negligible:
  – Drag force;
  – Radius.
• Some conditions for the existence of rotational and translational motion (pulling conditions).
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References

• 2. FOGO, Ronaldo; FIGUEIREDO, Eduardo – Apostila Objetivo 1ª série do Ensino Médio 3º Bimestre – Ed. Cered – São Paulo – 2010
Acknowledgements

- B8 Projetos
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Other shapes: Massive Sphere

\[ I = \frac{2mR^2}{5} \]
\[ F = m \cdot a \]
\[ f = m \cdot a \]
\[ a = \frac{f}{m} \]
\[ \nu_{cm} = \frac{f}{m} t \]
\[ \tau = I \cdot \alpha \]
\[ f \cdot R = \frac{2mR^2}{5} \cdot \alpha \]
\[ \alpha = \frac{5f}{2mR} \]
\[ \omega = \frac{5f}{2mR} t \]

\[ \nu_p = \nu_{cm} + \omega R \]
\[ \nu_p = \frac{f}{m} t + \frac{5f}{2m} t \]
\[ \nu_p = \frac{7f}{2m} t \]
\[ \nu_p = 3.5\nu_{cm} \]
\[ a = \frac{7f}{2m} \]
### Other Shapes: Hollow Sphere

\[
I = \frac{2mR^2}{3}
\]

\[
F = m \cdot a
\]

\[
f = m \cdot a
\]

\[
f \cdot R = \frac{2mR^2}{3} \cdot \alpha
\]

\[
a = \frac{f}{m}
\]

\[
\nu_{cm} = \frac{f}{m} \cdot t
\]

\[
\omega = \frac{3f}{2mR} \cdot t
\]

\[
\nu_P = \nu_{cm} + \omega R
\]

\[
\nu_P = \frac{f}{m} \cdot t + \frac{3f}{2m} \cdot t
\]

\[
\nu_P = \frac{5f}{2m} \cdot t
\]

\[
\nu_P = 2.5 \nu_{cm}
\]

\[
a = \frac{5f}{2m}
\]
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Other Shapes: Ring or Hollow Cylinder (thin bark)

\[ I = mR^2 \]
\[ F = m \cdot a \]
\[ \tau = I \cdot \alpha \]
\[ f = m \cdot a \]
\[ f \cdot R = mR^2 \cdot \alpha \]
\[ a = \frac{f}{m} \]
\[ \alpha = \frac{f}{mR} \]
\[ v_{cm} = \frac{f}{m} t \]
\[ \omega = \frac{f}{mR} t \]
\[ v_P = v_{cm} + \omega R \]
\[ v_P = \frac{f}{m} t + \frac{f}{m} t \]
\[ v_P = \frac{2f}{m} t \]
\[ v_P = 2v_{cm} \]
\[ a = \frac{2f}{m} \]
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Table for experiment 1

<table>
<thead>
<tr>
<th></th>
<th>Cardboard</th>
<th></th>
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<tbody>
<tr>
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<td>Horizontal distance (cm)</td>
<td>tanθ</td>
<td>Height (cm)</td>
<td>Horizontal distance (cm)</td>
<td>tanθ</td>
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<td>tanθ</td>
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</table>
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Videos for cylinder motion