

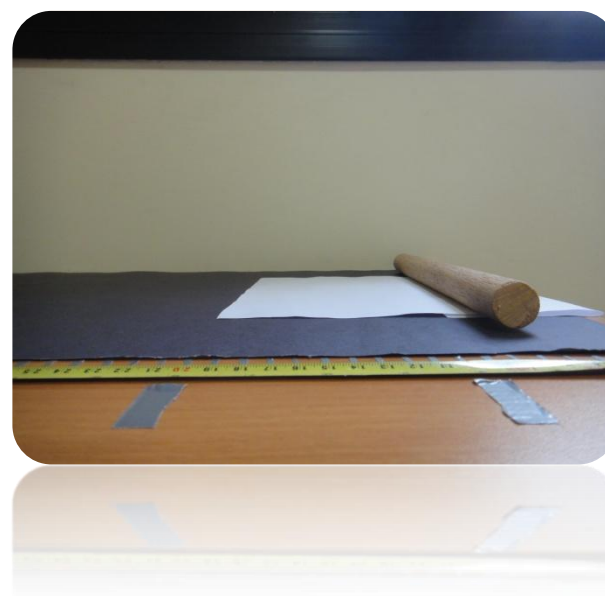
Brazilian team

## **Problem 14**

# **Moving Cylinder**

reporter:

Bárbara Cruvinel Santiago



# Problem 14

## Moving Cylinder

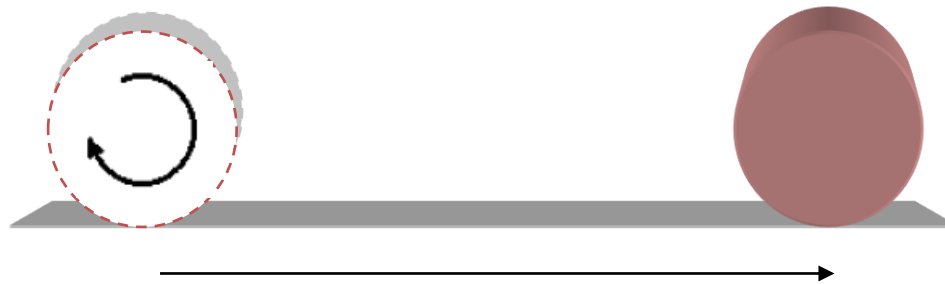
Place a sheet of paper on a horizontal table and put a cylindrical object (e.g. a pencil) on the paper. Pull the paper out. **Observe and investigate the motion** of the cylinder until it comes to rest.

### Contents

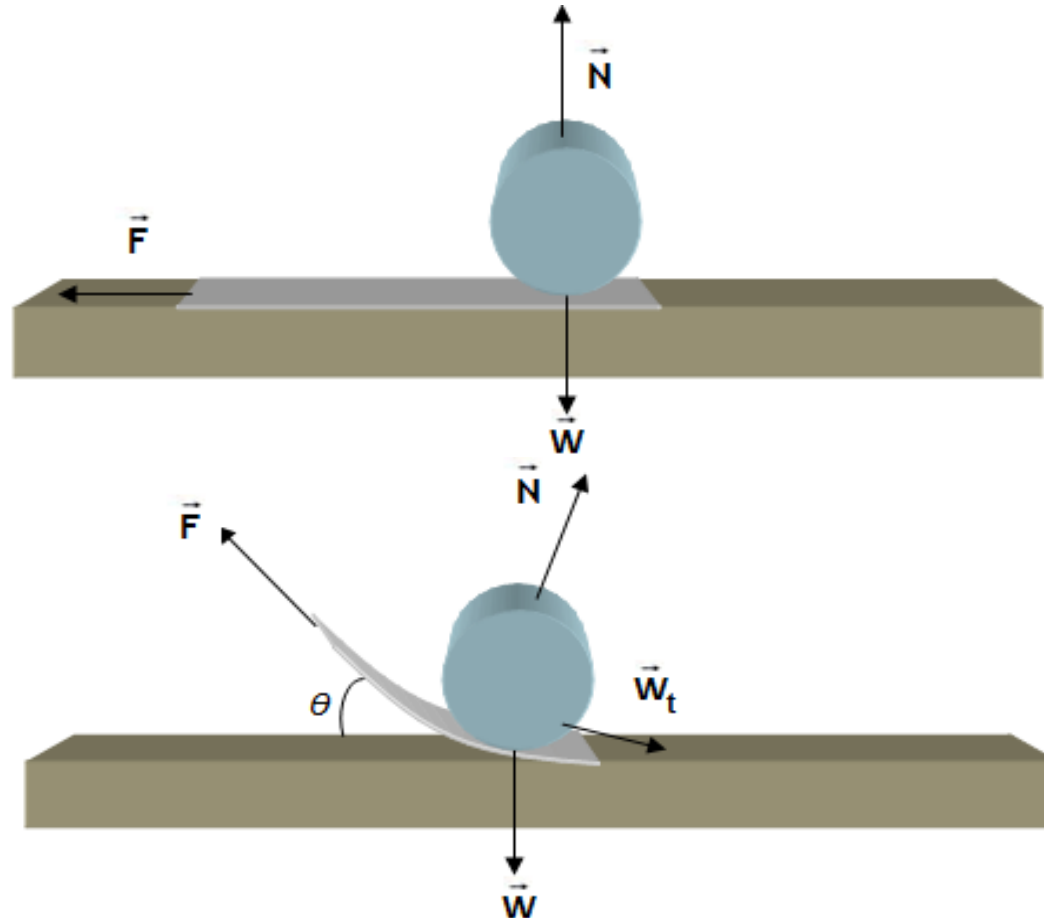
- Introduction
  - Definitions
  - Theory
  - Mathematical model
  - Computer simulation
- Experiments
  - Material
  - Experimental results
  - Analysis
- Conclusion

### Objective

- **Investigate** the cylinder **movement** (translation and rotation) and its **relevant parameters** till the moment the velocity turns to zero.



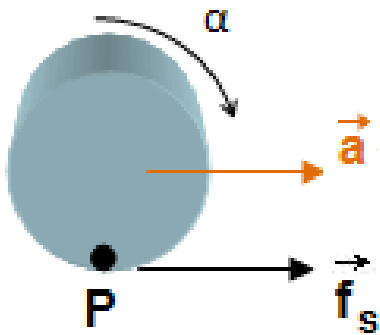
# Introduction: Forces acting over the cylinder



## Introduction: Forces acting over the cylinder

- Force  $F$  = friction force:
  - Static (without sliding);
  - Dynamic (with sliding).

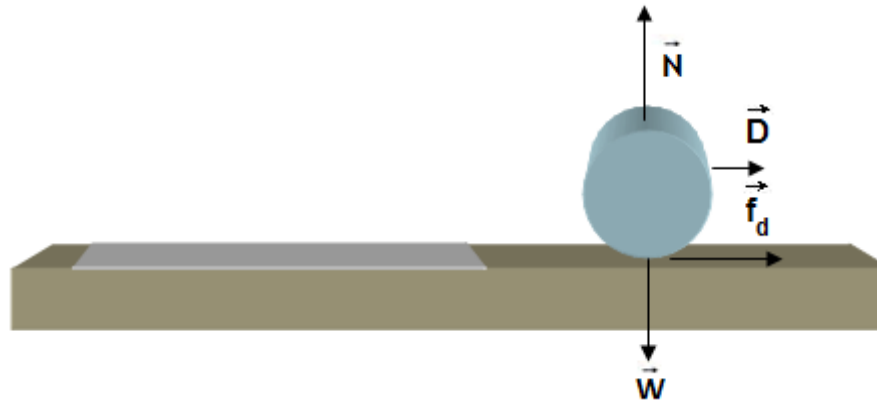
### Static friction



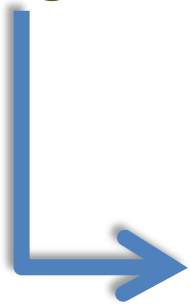
$$f_s = \mu_s N = \mu_s W = \mu_s \cdot mg$$

Valid only for horizontal surfaces.

## Introduction: Forces acting over the cylinder



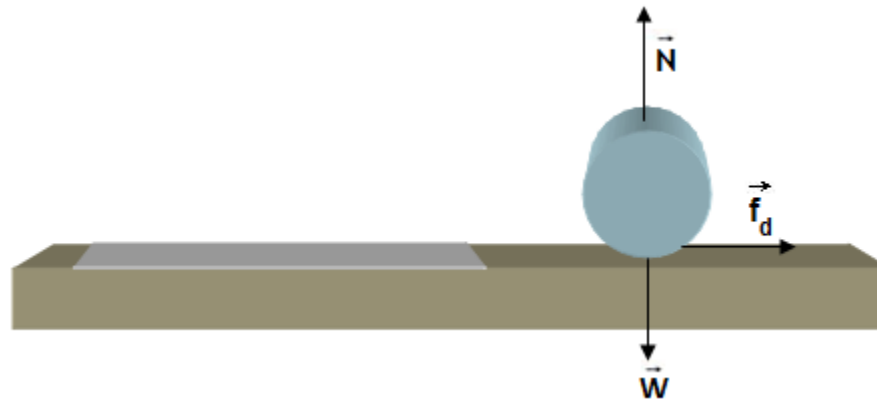
- Drag force:



Negligible

$$D = \frac{1}{2} \rho C_x A v^2$$

## Introduction: Forces acting over the cylinder



## Introduction: Forces acting over the cylinder

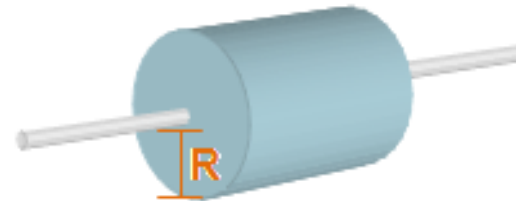


$$\tau = F \cdot r \cdot \sin\theta$$

$$\tau = f_d \cdot R$$

$$\tau = I \cdot \alpha$$

$$I = \frac{mR^2}{2}$$



### Introduction: General considerations

- During the sheet pulling:

$$F = m \cdot a$$

$$f = m \cdot a$$

$$a = \frac{f}{m}$$

$$v_{cm} = \frac{f}{m} t$$

$$\tau = I \cdot \alpha$$

$$f \cdot R = \frac{mR^2}{2} \cdot \alpha$$

$$\alpha = \frac{2f}{mR}$$

$$\omega = \frac{2f}{mR} t$$

### Introduction: General considerations

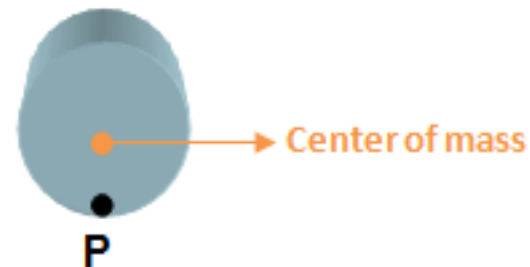
$$\mathbf{v}_P = \mathbf{v}_{cm} + \boldsymbol{\omega}R$$

$$\mathbf{v}_P = \frac{f}{m}t + \frac{2f}{m}t$$

$$\mathbf{v}_P = \frac{3f}{m}t$$



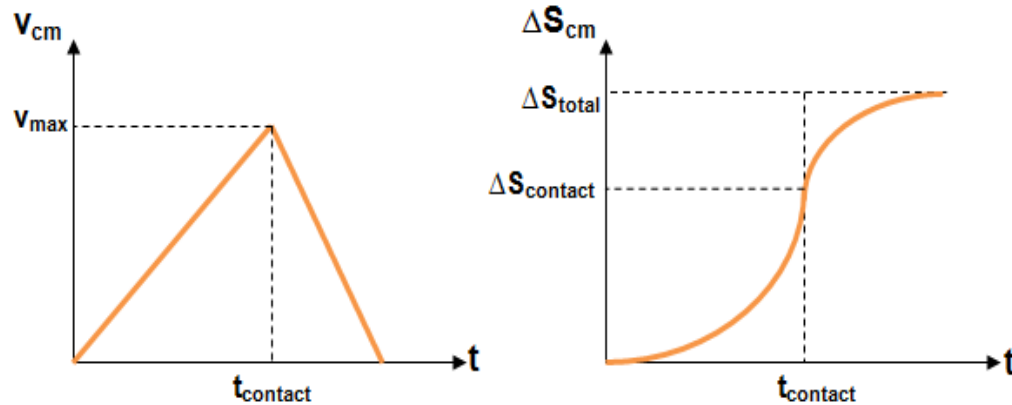
There's no influence of the radius.



$$\mathbf{v}_P = 3\mathbf{v}_{cm}$$

$$\mathbf{a} = \frac{3f}{m}$$

### General considerations



- After losing contact with the paper: dynamic friction
- While in contact with the paper:

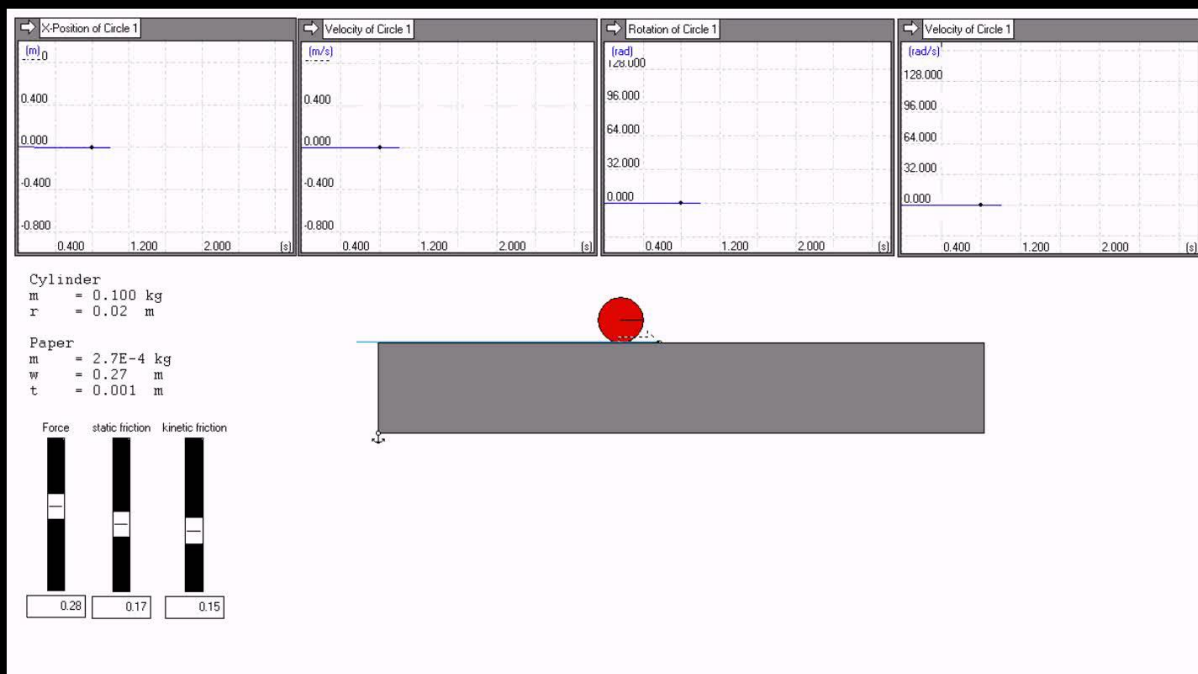
Static friction

$$a \leq \frac{3f_{max}}{m}$$

Dynamic friction

$$a > \frac{3f_{max}}{m}$$

### Introduction: Simulation for general considerations



## Introduction: Impulse and momentum

- Happens while pulling the paper.

$$I = \Delta p$$

$$f \cdot \Delta t = mv - mv_0$$

$$f \cdot \Delta t = m \cdot v$$

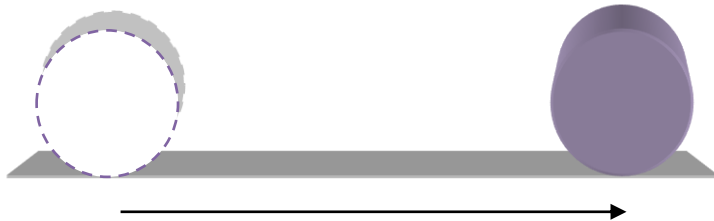
$$v_{max} = \frac{f \cdot \Delta t}{m}$$

- 
- Initial velocity after losing contact with the sheet: **maximum velocity.**

### Introduction: Translation and rotation

#### Translation

- Motion with a line as trajectory.



- Uniformly Varied Motion

#### Rotation

- Motion of the body points around an axis.



- Uniformly Varied Circular motion

Introduction: Translational motion

$$F = f$$

$$ma = mg\mu$$

$$a = g\mu$$

Static friction

$$S = S_0 + V_0 t + \frac{1}{2} \mu g t^2$$

$$V = V_0 + \mu g t$$

Dynamic friction

$$S = S_0 + V_0 t - \frac{1}{2} \mu g t^2$$

$$V = V_0 - \mu g t$$

### Introduction: Rotational motion

- Relating linear and angular quantities:

$$\begin{aligned}S &= \varphi R \\V &= \omega R \\a_t &= \alpha R\end{aligned}$$

$$\alpha = \frac{a_t}{R} \quad \alpha = \frac{\mu g}{R}$$

Static friction

$$\varphi = \varphi_0 + \omega_0 t + \frac{\mu g t^2}{2R}$$

$$\omega = \omega_0 + \frac{\mu g t}{2R}$$

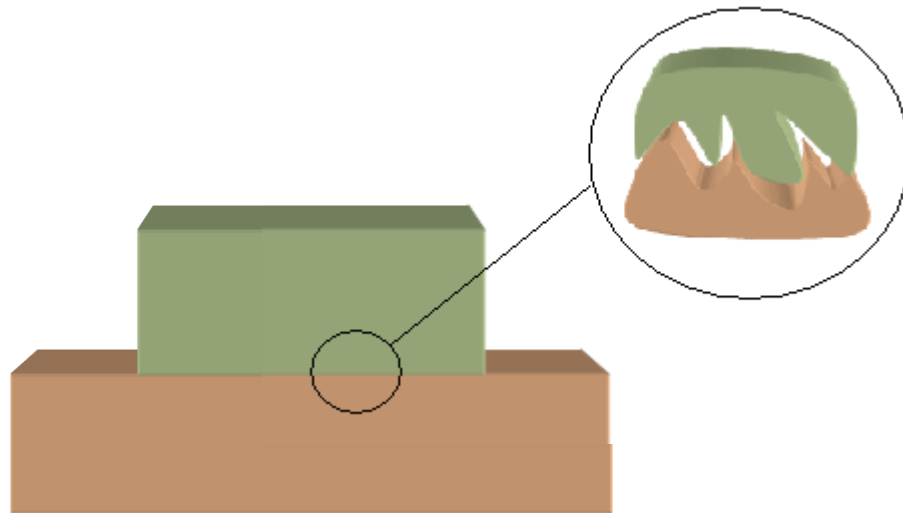
Dynamic friction

$$\varphi = \varphi_0 + \omega_0 t - \frac{\mu g t^2}{2R}$$

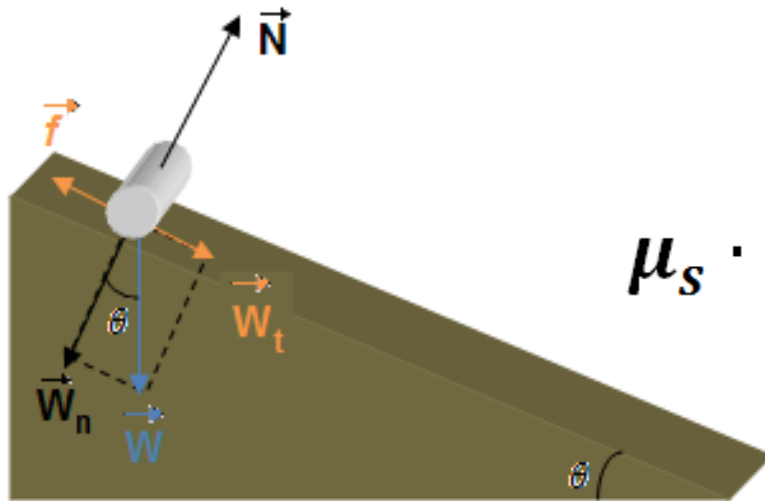
$$\omega = \omega_0 - \frac{\mu g t}{2R}$$

## Introduction: Cylinder material influence

- Friction is generated by the irregularities.



## Introduction: Coefficient of friction



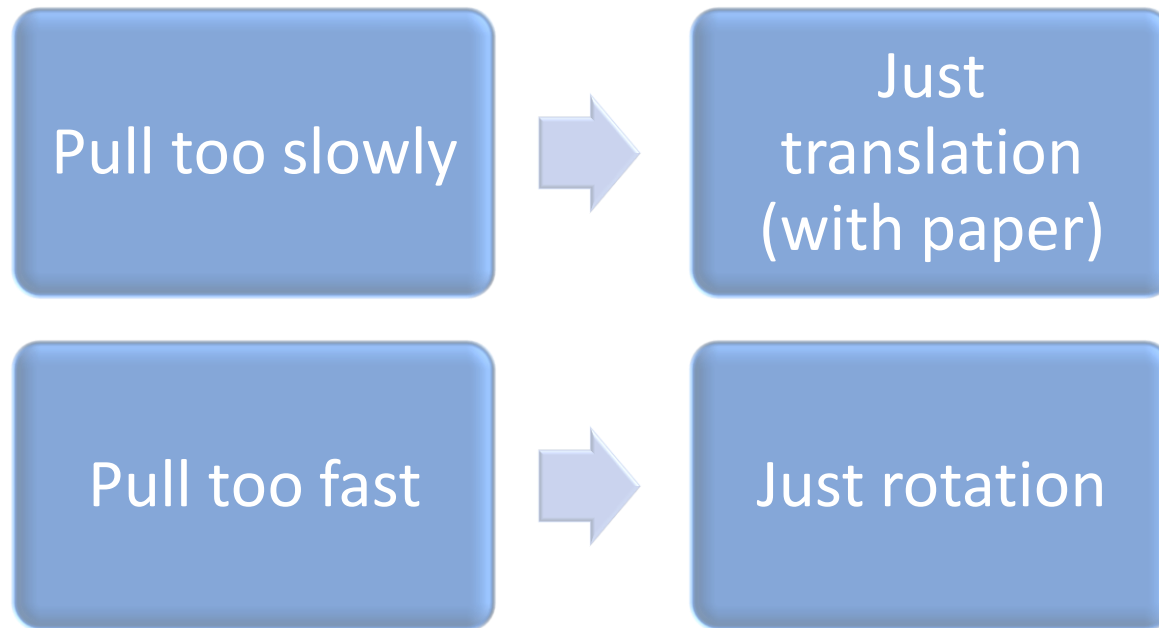
$$f_s = W_t$$

$$\mu_s \cdot mg \cdot \cos\theta = mg \cdot \sin\theta$$

$$\mu_s = \tan\theta$$

- $\mu_d \cong 10\%$  lower than  $\mu_s$

Introduction: Conditions of pulling for the cylinder motion

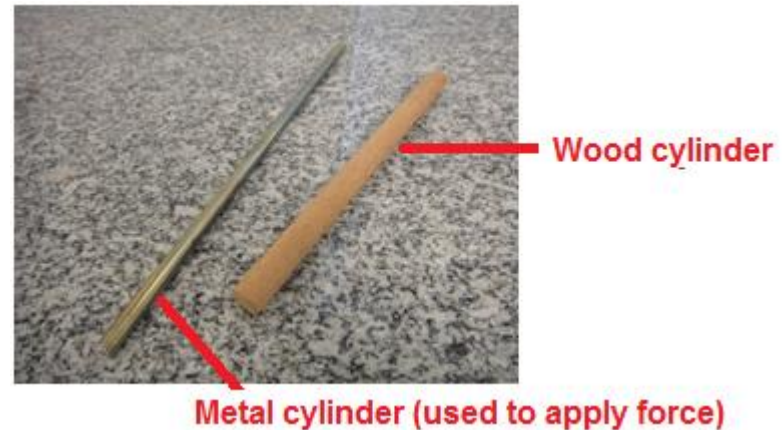


## Experiments: Experimental Description

- **Experiment 1:** find the coefficients of friction.
- **Experiment 2:** vary the surfaces.
- **Experiment 3:** conditions for the cylinder movement.
- **Make some computer simulations.**

### Experiments: Material

- Wood cylinder
- Wood pieces (same material of the cylinder)
- Metal cylinder (act like weights for pulling)
- Measuring tape with precision of 0.05cm
- Caliper rule with precision of 0.05mm
- EVA
- Cardboard
- Wood surface
- String
- Pencil



## Experiments: Experimental Data

- **Surface: EVA**
- **Maximum reached velocity: m/s**
- **Dynamic friction force: 3.84 N**
- **Maximum drag force:  $5 \cdot 10^{-3}$  N**

### Experiments: Experiment 1 – coefficient of friction

Find the friction coefficients

Video analysis of the velocity

Comparison with calculated data



Surface in horizontal position

The plane is inclined till it reaches the maximum angle of repose

Material	Coefficient of $f_s$	Coefficient of $f_d$
Cardboard	0.52	0.47
EVA	1.04	0.94
Wood	0.31	0.28

## Experiments: Experiment 2 – vary surfaces

Find the  
friction  
coefficients

Video analysis  
of the velocity

Comparison  
with  
calculated  
data



Wood



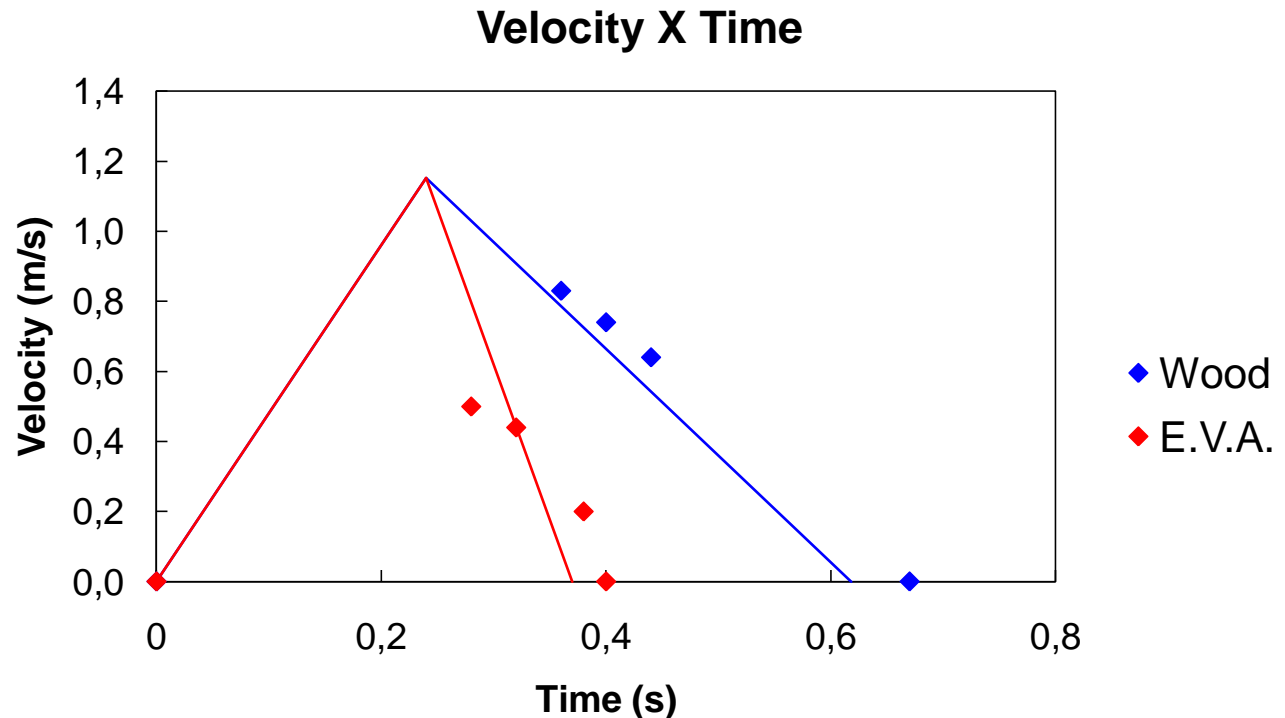
E.V.A.

### Experiments: Experiment 2 – vary surfaces

Find the friction coefficients

Video analysis of the velocity

Comparison with calculated data



### Experiments: Experiment 2 – vary surface

Find the  
friction  
coefficients

Video analysis  
of the velocity

Comparison  
with  
calculated  
data

Higher friction

Stops faster

Experiment  
agrees with  
theory

The motion  
proposed showed  
to be real

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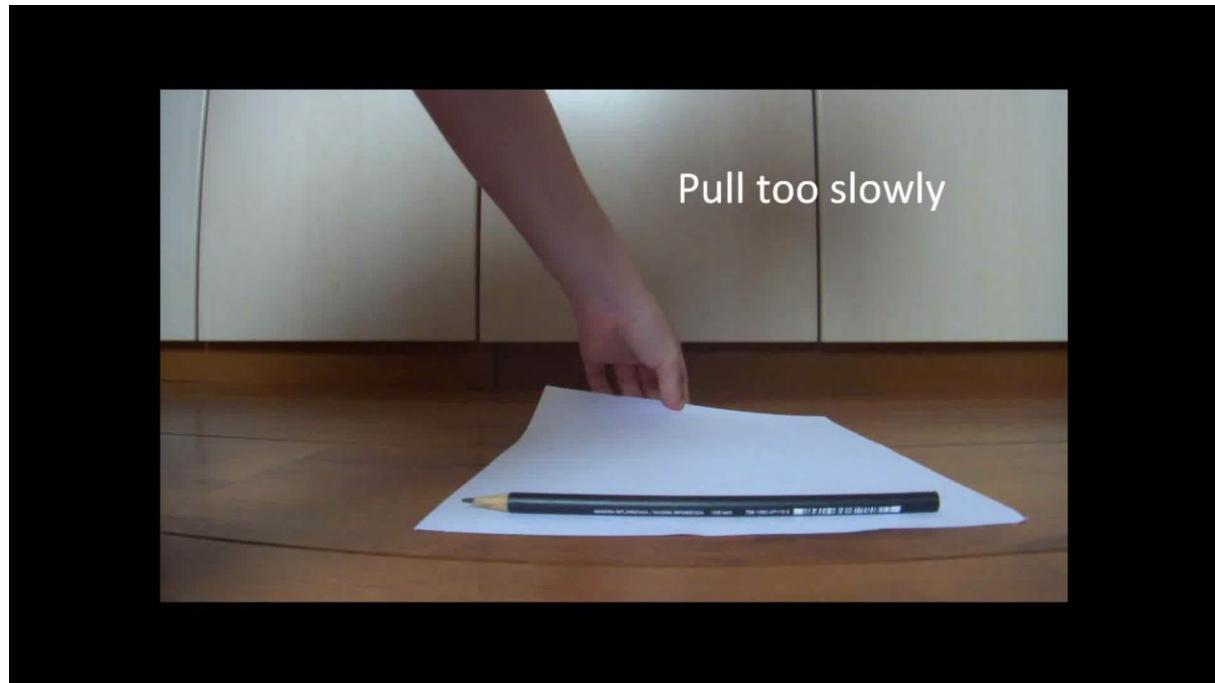
## Problem 14: Moving Cylinder

### Experiments: Experiment 3 – conditions for the cylinder movement

Find the friction coefficients

Video analysis of the velocity

Comparison with calculated data



- Pull too slowly  $\rightarrow$  translation
- Pull too fast  $\rightarrow$  rotation

## Conclusion

- Most relevant parameter: friction force
- Possible to consider Uniformly Varied Motion.
- Influence:
  - Cylinder and surface's material;
  - Moment of inertia.
- No influence or negligible:
  - Drag force;
  - Radius.
- Some conditions for the existence of rotational and translational motion (pulling conditions).

### References

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- Victor Ando
- Lucas Frenay



### Other shapes: Massive Sphere

$$I = \frac{2mR^2}{5}$$

$$F = m \cdot a$$

$$\tau = I \cdot \alpha$$

$$f = m \cdot a$$

$$f \cdot R = \frac{2mR^2}{5} \cdot \alpha$$

$$a = \frac{f}{m}$$

$$\alpha = \frac{5f}{2mR}$$

$$v_{cm} = \frac{f}{m} t$$

$$\omega = \frac{5f}{2mR} t$$

$$v_P = v_{cm} + \omega R$$

$$v_P = \frac{f}{m} t + \frac{5f}{2m} t$$

$$v_P = \frac{7f}{2m} t$$

$$v_P = 3.5 v_{cm}$$

$$a = \frac{7f}{2m}$$

### Other Shapes: Hollow Sphere

$$I = \frac{2mR^2}{3}$$

$$F = m \cdot a$$

$$\tau = I \cdot \alpha$$

$$f = m \cdot a$$

$$f \cdot R = \frac{2mR^2}{3} \cdot \alpha$$

$$a = \frac{f}{m}$$

$$\alpha = \frac{3f}{2mR}$$

$$v_{cm} = \frac{f}{m} t$$

$$\omega = \frac{3f}{2mR} t$$

$$v_P = v_{cm} + \omega R$$

$$v_P = \frac{f}{m} t + \frac{3f}{2m} t$$

$$v_P = \frac{5f}{2m} t$$

$$v_P = 2.5v_{cm}$$

$$a = \frac{5f}{2m}$$

Other Shapes: Ring or Hollow Cylinder (thin bark)

$$I = mR^2$$

$$F = m \cdot a$$

$$\tau = I \cdot \alpha$$

$$f = m \cdot a$$

$$f \cdot R = mR^2 \cdot \alpha$$

$$a = \frac{f}{m}$$

$$\alpha = \frac{f}{mR}$$

$$v_{cm} = \frac{f}{m} t$$

$$\omega = \frac{f}{mR} t$$

$$v_P = v_{cm} + \omega R$$

$$v_P = \frac{f}{m} t + \frac{f}{m} t$$

$$v_P = \frac{2f}{m} t$$

$$v_P = 2v_{cm}$$

$$a = \frac{2f}{m}$$

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## Problem 14: Moving Cylinder

### Table for experiment 1

	Cardboard			EVA			Wood		
	Height (cm)	Horizontal distance (cm)	$\tan\theta$	Height (cm)	Horizontal distance (cm)	$\tan\theta$	Height (cm)	Horizontal distance (cm)	$\tan\theta$
	18.2	34.2	0.53	30.4	27.1	1.12	52.2	169	0.31
	18.1	34.3	0.53	27.3	27.1	1.01	54.2	165	0.33
	17.8	35.3	0.50	26.5	26.7	0.99	51.2	172.8	0.30
<b>Average</b>			<b>0.52</b>			<b>1.04</b>			<b>0.31</b>
<b>Standard deviation</b>			0.01			0.07			0.02

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### Problem 14: Moving Cylinder

## Videos for cylinder motion

