PROBLEM NO. 4
BREAKING SPAGHETTI

Find the conditions under which dry spaghetti falling on a hard floor does not break.

Overview

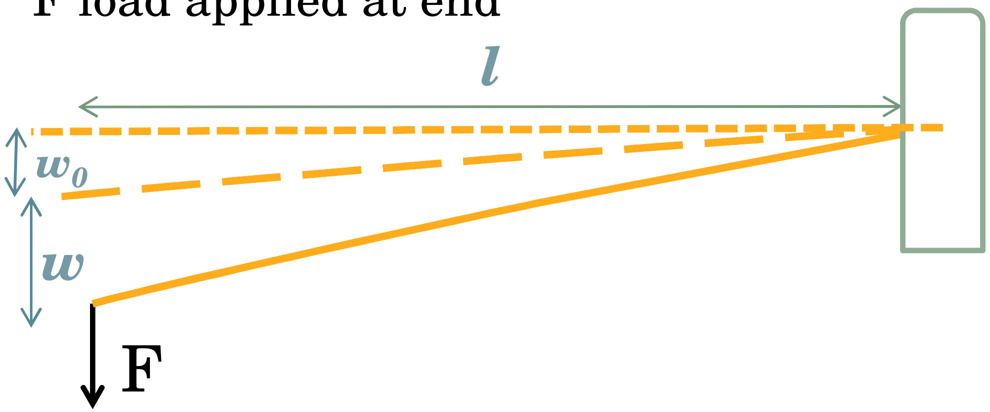
- mechanical properties
  - Young's modulus
- impact
- buckling
  - Euler's critical buckling load
  - modes
- simulation
  - fracture points
- experimental setup
  - tube, camera, debris
- results
  - weakest fracture force, various sizes
  - surface, number of spaghetti, angle dependence
  - comparison
- conclusion

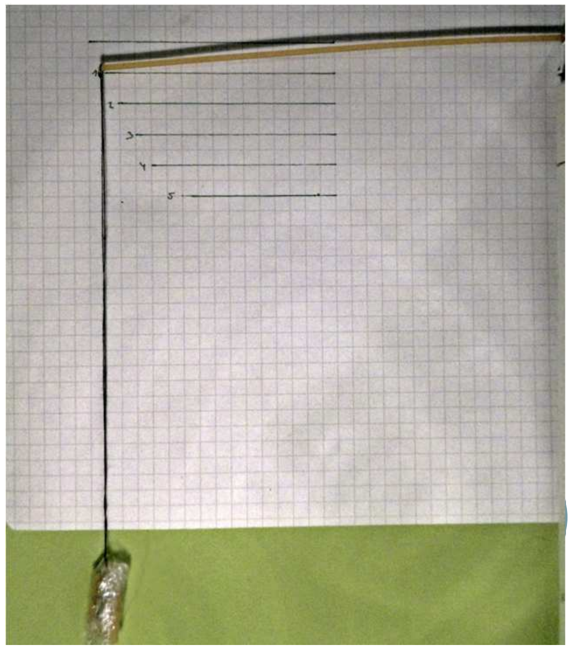
SPAGHETTI PROPERITES

- lenght = 25.5cm
- mass, density – five sizes

mass (g)	density (kg/m^3)
0.474	1515.474
0.627	1489.091
0.81	1486.303
0.980	1429.764
1.177	1399.133

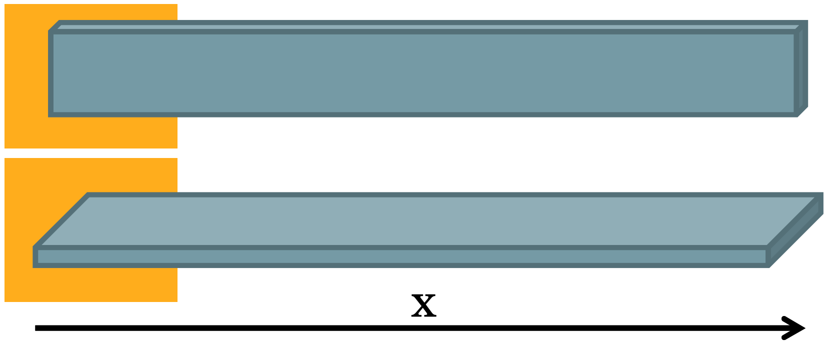
- <u>Young's modulus (E)</u>
  - stress/strain ratio
  - material characteristic
- measured from beam deflection
  - $w + w_0$, w_0 initial deflection (spaghetti mass)
  - F load applied at end

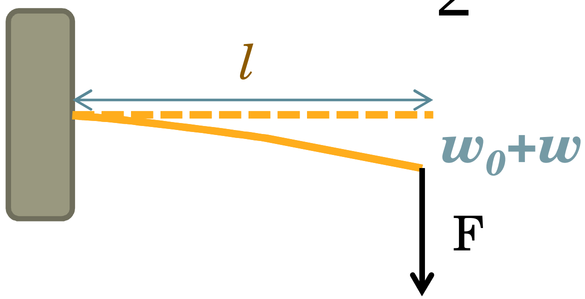




Spaghetii properties

- Young's modulus E – beam deflection
  - area moment of inertia - circular cross-section $I = \dfrac{r^4\pi}{2}$





  - applied load F
    - m spaghetti mass , l lenght, E Young's modulus
- deflection $for\ x = 0,\ probe\ beam\ deflection\ method$
  - *bending moment*

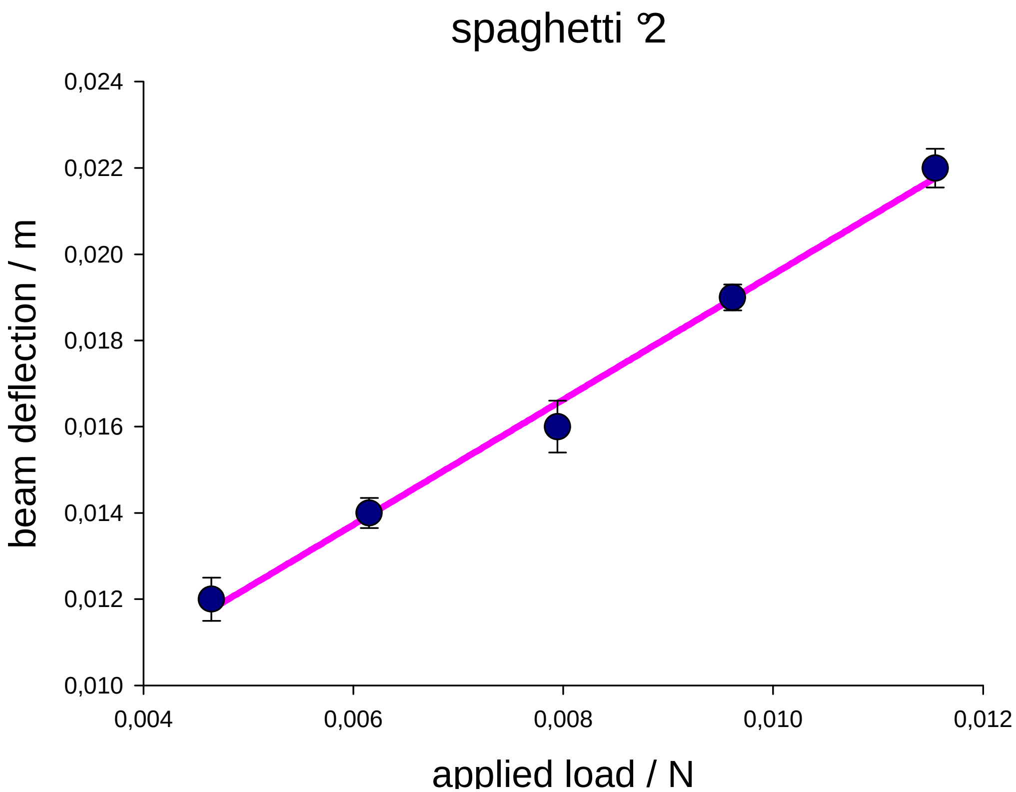
$$M = M_F = \frac{d^2 w}{dx^2} EI \qquad w = \frac{l^3}{EI}\frac{F}{3}$$

BEAM DEFLECTION – YOUNG'S MODULUS

- determined from the coefficient
  - different applied loads F
  - deflection measurement y_{max}

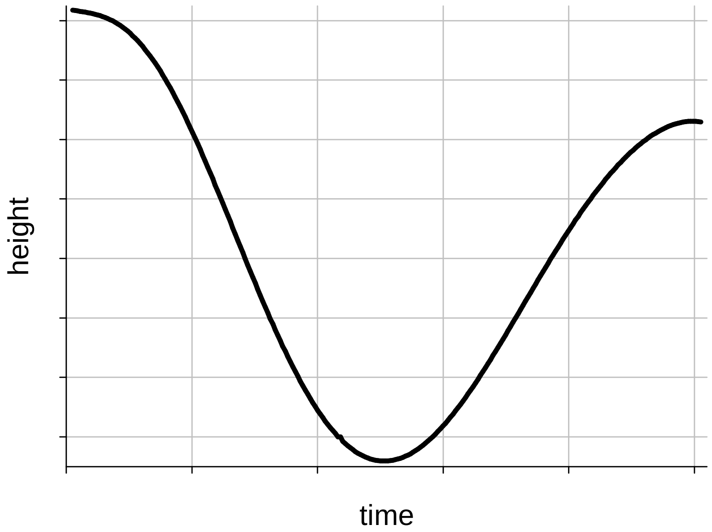
$$w = w_0 + F\frac{l^3}{3EI}$$

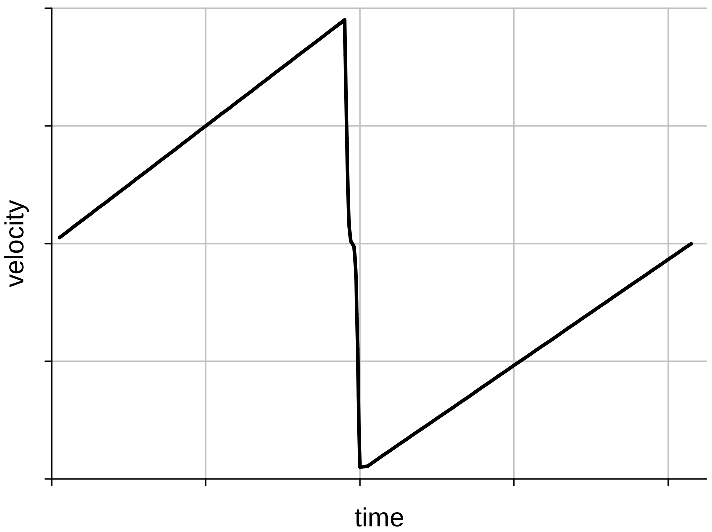
- °1
  - 1,81E+10 N/m^2
- °2
  - 1,76E+10 N/m^2
- °3
  - 1,31E+10 N/m^2
- °4
  - 1,04E+10 N/m^2
- °5
  - 9,13E+09 N/m^2



IMPACT

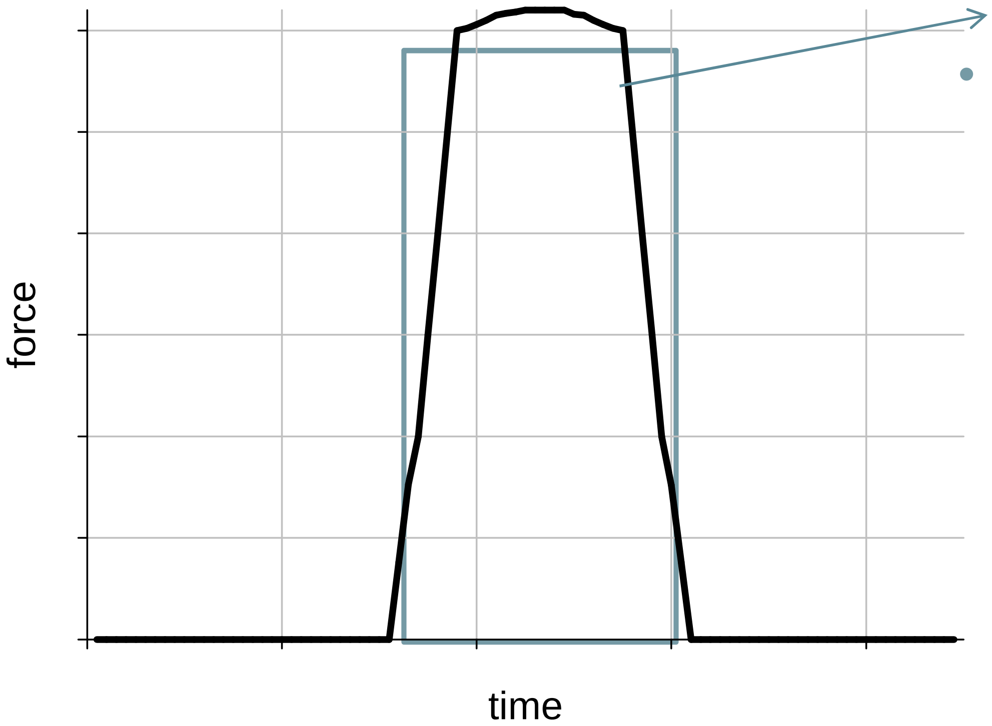
- elastic
- spaghetti fall accellerated *(g)*
  - *impact with the surface*
  - *both surface and spaghetti*
    - acting like springs that obey Hooke's law
    - force is proportional to the amount of deformations



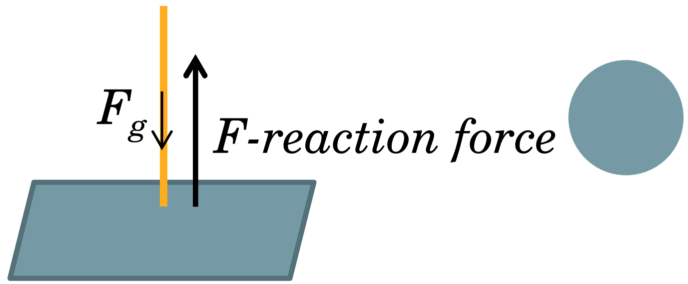


IMPACT

- momentum mdv is the surface force impulse Fdt
  - force is small at first
  - enlarges to a maximum when spaghetti reverses directions
  - drops down as it jumps-off

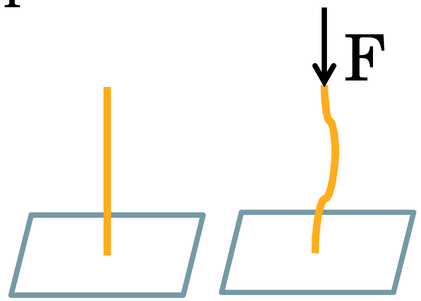


- approximated constant F
  - interested in maximum
- varies for different surfaces
  - $m\Delta v = F\Delta t$
  - Δv *and* Δt *evaluated from video*
  - $surface\ force - mg$
  - causes spaghetti do deform
    - break

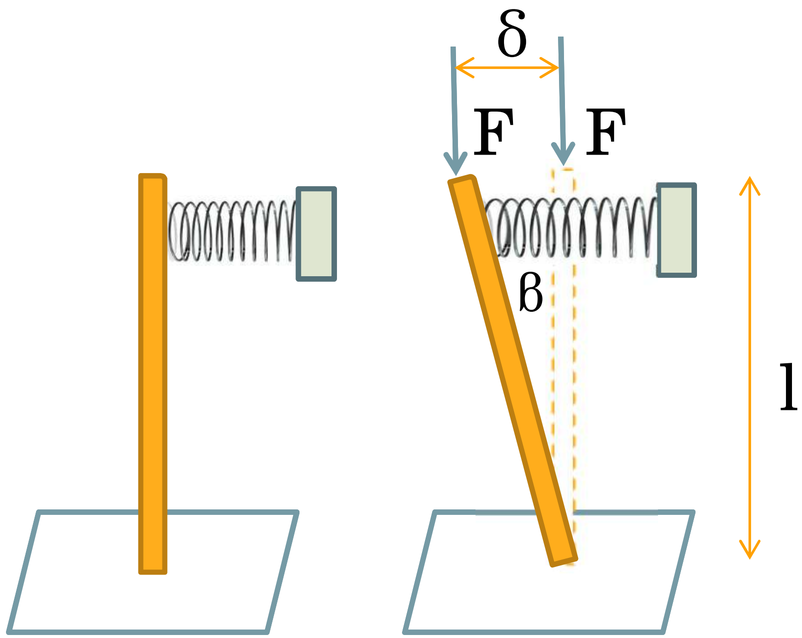


BUCKLING

- displacement of structure transverse to load



- buckling model (spring)



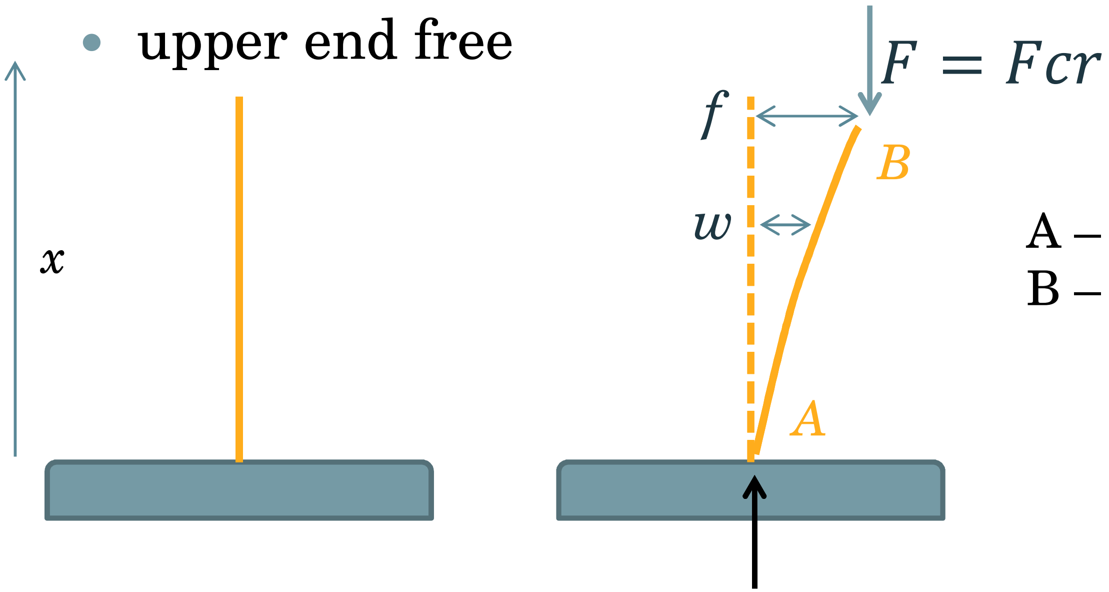
- elastic force moment
  - $M_{el} = Fe_{ll} = k\delta l$
    - k-spring constant
- load moment
  - $M = F\delta$

- $M < M_{el}$ stable equilibrium - beam returns to the initial position
- $M = M_{el}$ indifferent equilibrium – remains at δ: $F = kl$
  - *initial buckling occurs*
- $M > M_{el}$ unstable equilibrium – plastic deformations

BUCKLING

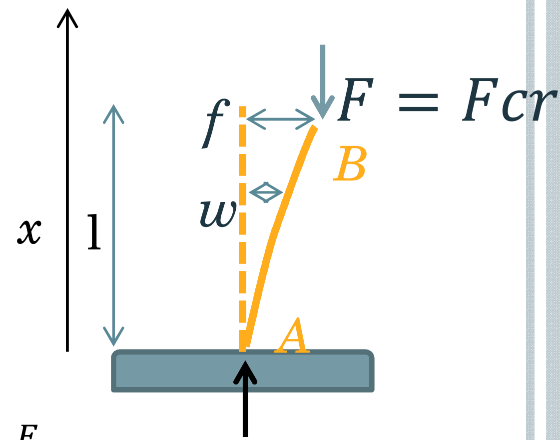
- at $M = M_{el}$ buckling occurs
  - critical condition
  - depends on the beam support type

- beam support
  - lower end simple (can rotate and slide)
  - upper end free



A – deflection at point A
B – deflection at point B (f)

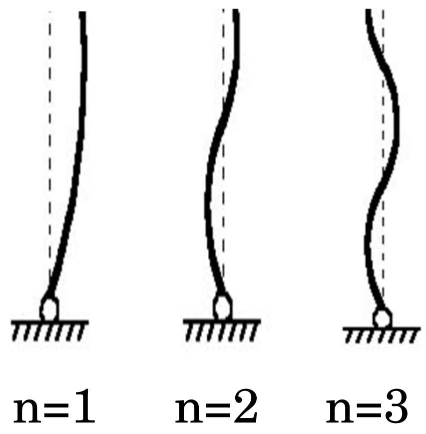
BUCKLING



- buckling moment, equation of the beam elastic line
- $M = -F(f - w) = -\frac{d^2w}{dx^2}EI$
  - $\frac{d^2w}{dx^2} = \frac{F}{EI_{min}}(f - w)$
  - $\frac{d^2w}{dx^2} = \frac{F}{EI_{min}}u$ *harmonic oscillator equation*, $\alpha^2 = \frac{F}{EI_{min}}$ to simplify calculations
- $w = A\sin(\alpha x) - B\cos(\alpha x) + f$, *integrated equation of the beam elastic line*
- boundary conditions at point A, $x = 0$
  - $w(0) = A\sin(\alpha 0) - B\cos(\alpha 0) + f = 0 \; B = -f$
  - $w'(0) = A\cos(\alpha 0) + B\sin(\alpha 0) = A\cos(\alpha 0)$
  - $\cos(\alpha l) = 0$ (critical states), $\alpha l = (2n - 1)\frac{\pi}{2}, n = 1,2,3, \ldots$
- minimal critical force n=1
  - $F_{cr} = \frac{\pi^2 EI}{4l^2}$

BUCKLING

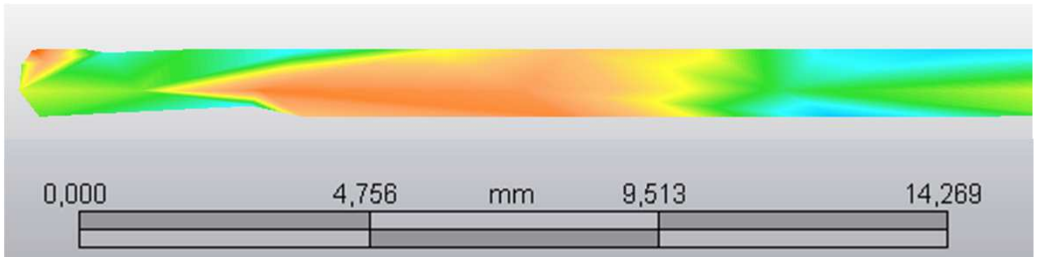
- buckling modes
  - if the force $F = \frac{\pi^2 EI}{4l^2}(2n-1)^2, n \in N$ related to $F = (2n-1)^2 F_{cr}$
    - spaghetti forms a sinusoidal line
    - depending on the relation – different buckling modes
    - greatest deflection – highest stress point



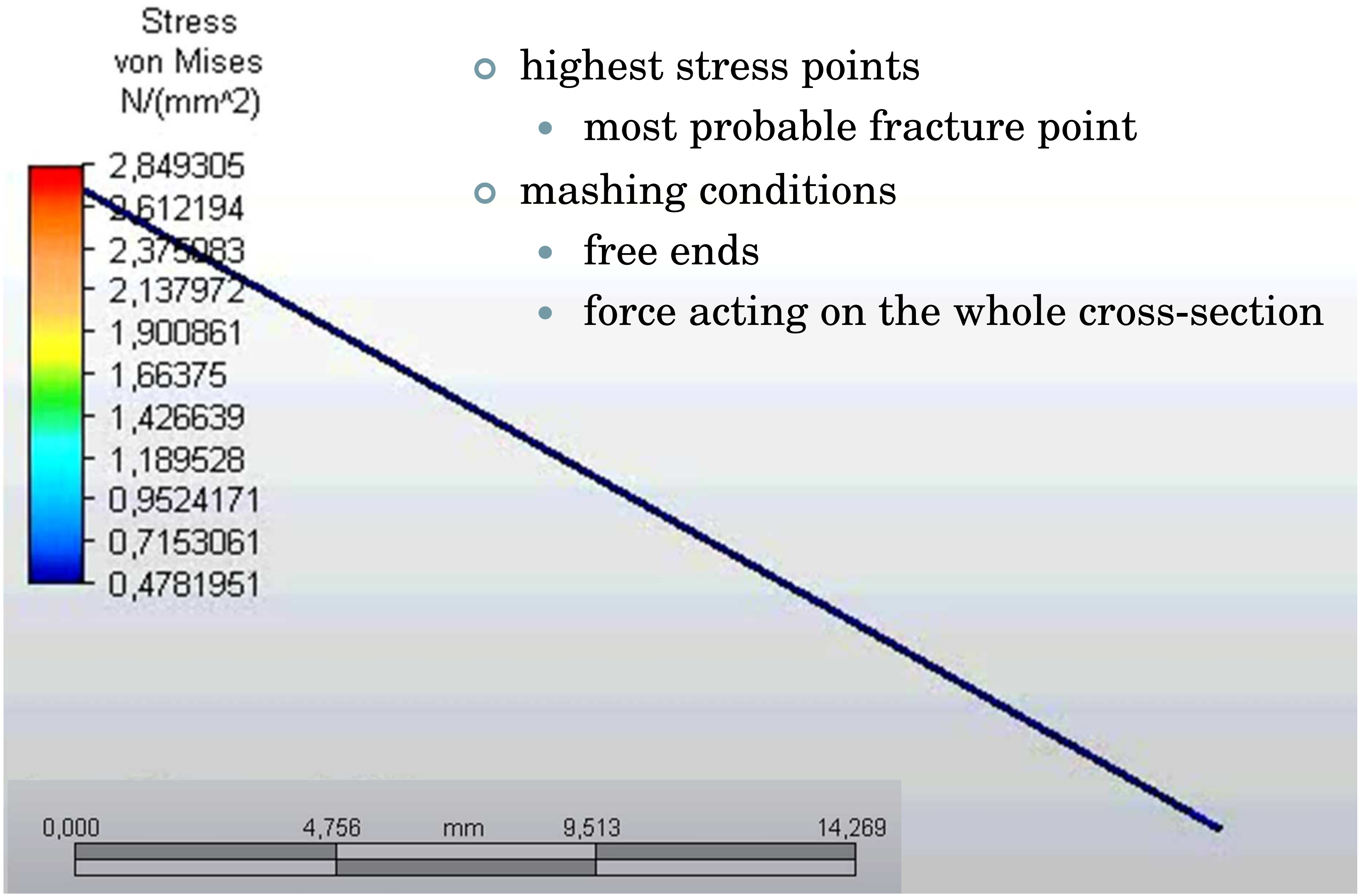
- critical buckling force
  - $F_{cr} = \frac{\pi^2 EI}{4l^2}$
    - °1 *0.33 N*, °2 *0.58 N*, °3 *0.72 N*, °4 *0.91 N*, °5 *1.20 N*
  - even the smallest impact forces exceed these values!
    - buckling deformation occurs
- since surface reaction force is not related to F_{cr}
  - IRREGULAR BUCKING MODE
    - greatest probability fracture points - simulation

FRACTURE POINT

- irregular buckling modes
- debris lenght measured
  - most probable values and simulation compared
- simulation
- AutoCAD, Autodesk simulation multiphysics
  - measured material properties and spaghetti dimension
  - force acting conditions
    - whole surface, directioned through spaghetti
  - ~gradual mesh
    - highest stress point
      - center



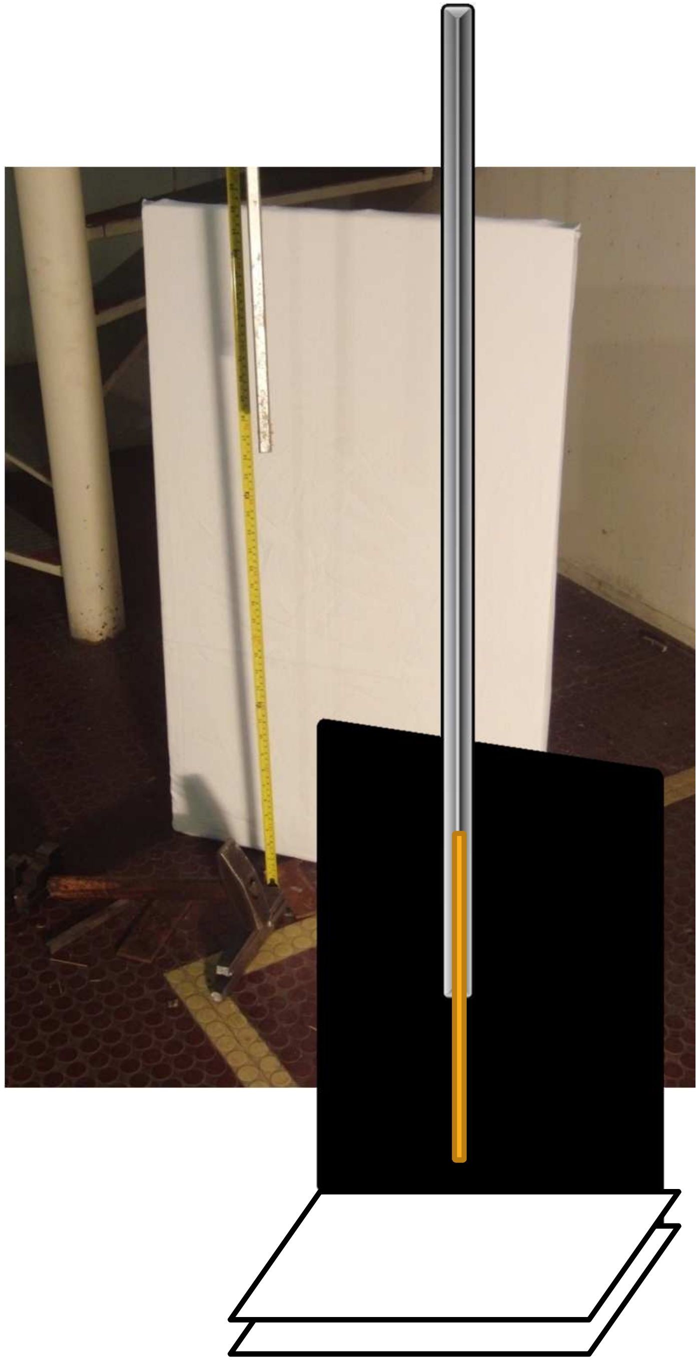
Fracture point



- highest stress points
  - most probable fracture point
- mashing conditions
  - free ends
  - force acting on the whole cross-section

Experimental setup

- directed through a long vertical pipe
- obtaining ~equal impact velocities
- recording the process camera
  - 120 fps
  - impact time and velocity evaluation
- debris measured
  - fracture point
  - probability of fracture

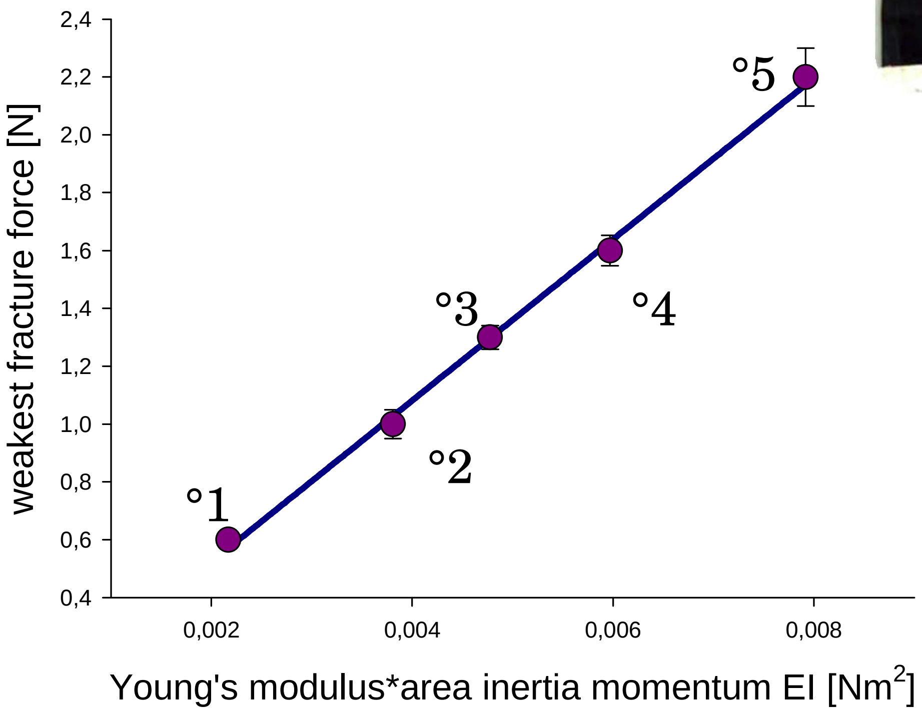


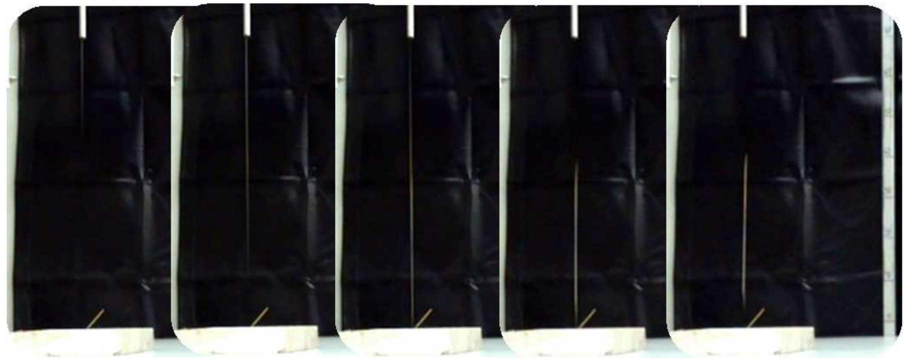
PARAMETERS

- weakest fracture force
- spaghetti size
  - Young’s modulus, area inertia moment, mass
- surface hardness
- impact angle
  - buckling and bending
- surface roughness
- number of spaghetti
  - interactions during the fall

Spaghetti size dependence
Young's modulus

- $F = a\frac{\pi^2 EI}{4l^2}$ *a - relation to critical buckling force*
- $F = m\frac{\Delta v}{\Delta t}$ - evaluated from video
- $F/Fcr \sim 2 = (2n-1)2$
  - buckling mode $\sim n = 1.21$

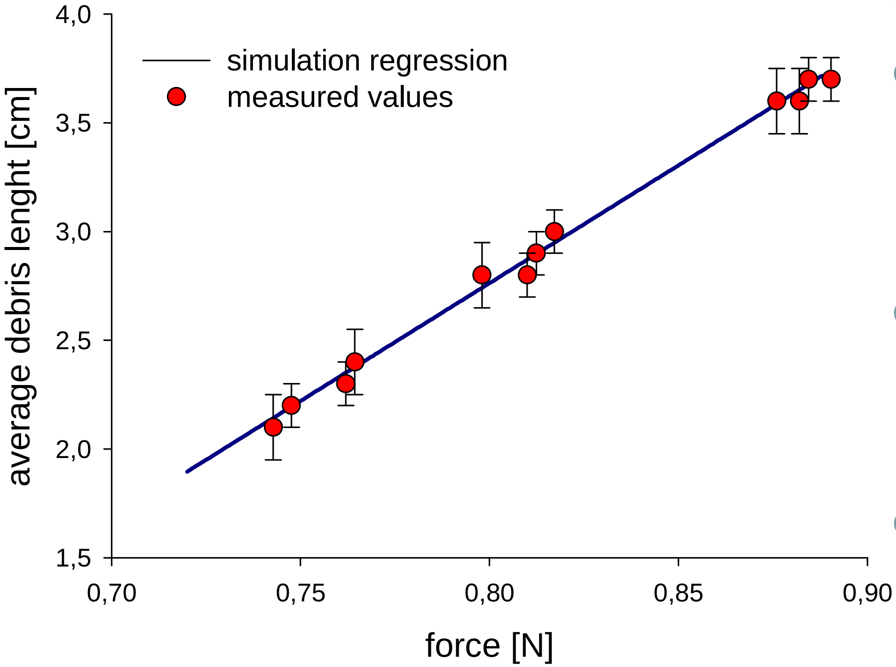




repeated measurements
marked spaghetti
image sequence observed

SPAGHETTI SIZE DEPENDENCE YOUNG'S MODULUS

- on a narrow force scale
  - smaller debris lenght is proportional to impact force
    - mode slightly changes
  - simulation and measured values agreement



- °1 spaghetti – 3 initial heights
- metal surface – steel
- debris lenght zero at
  - $F = 0.56$ N
  - estimated from the simulation
- experimenal value
  - $F = 0.59 \pm 0.03N$

SURFACE DEPENDENCE

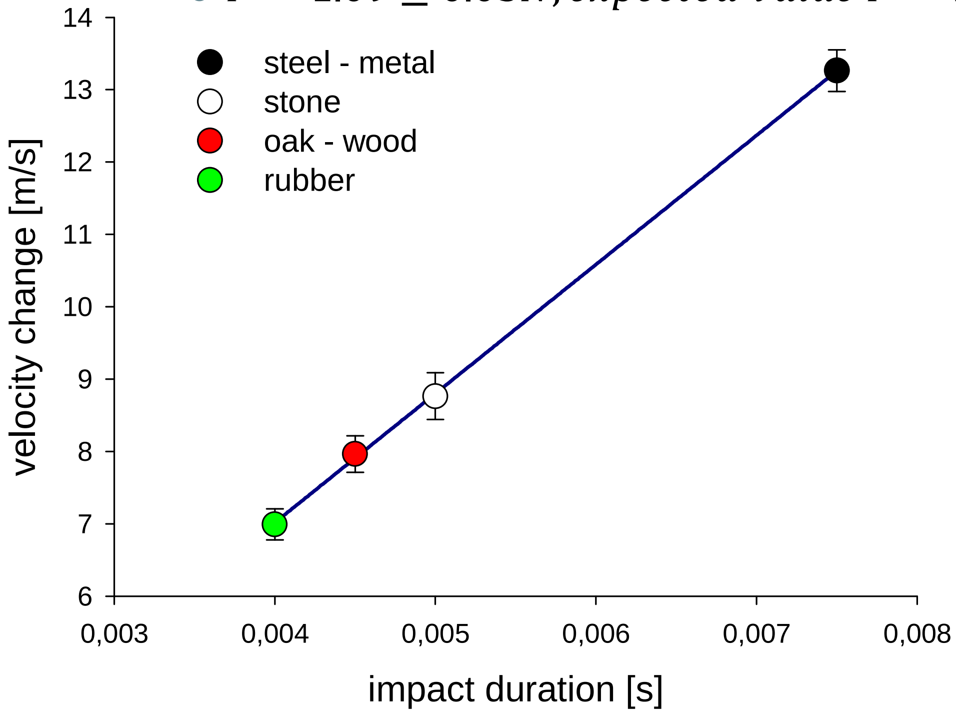
- HB – Brinell hardness
  - steel 120HB
  - (oak) wood 3.8HB
  - rubber not comparable
  - rough/smooth stone 35HB
- DIFFERENT SURFACE
  - impact duration
  - velocity after impact
    - losses due to surface deformation

Surface dependence hardnes

- necessary force remains the same

$$F = m\frac{\Delta v}{\Delta t} = m\frac{v_0 + v_1}{\Delta t}$$

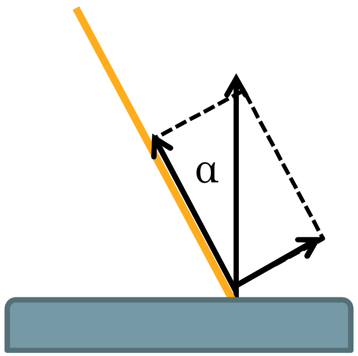
- v_0 velocity before impact ~shared, v_1 velocity after impact varies!
  - linear fit coefficient $= \frac{force}{mass}$
    - $F = 1.09 \pm 0.05N, expected\ value\ F = 1.10 \pm 0.02N$ *spaghetti °2*

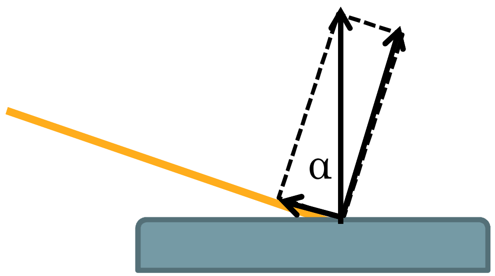


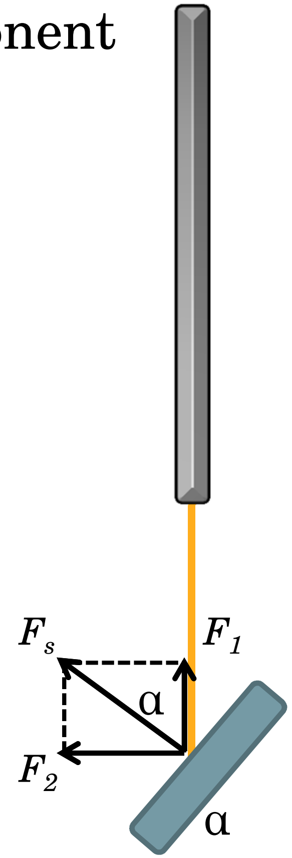
Impact angle dependence

- tube remains vertical
  - surface changes angle, smooth stone surface
- surface reaction force is vertical to the surface Fs
  - buckling $F_1 = Fscos\alpha$ and bending $F_2 = Fssin\alpha$ component

- as the impact angle α *increases*
  - bending force becomes more significant ($F_s sin\alpha$)
    - strucutures are more sensitive to bending displacements
  - *after a certain angle*
    - friction force is not great enough to keep the spaghetti steady
    - it slides of the surface – no fracture

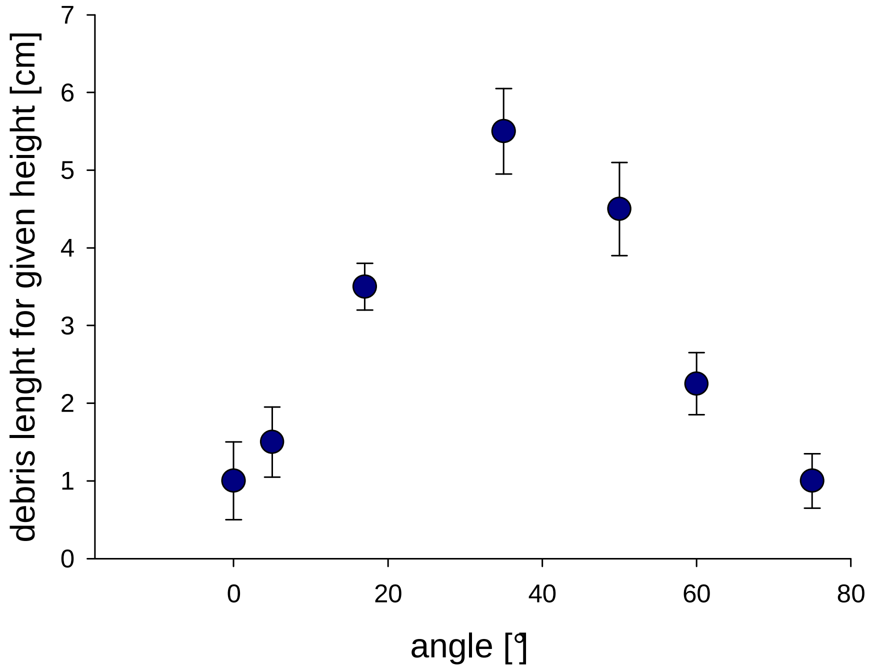




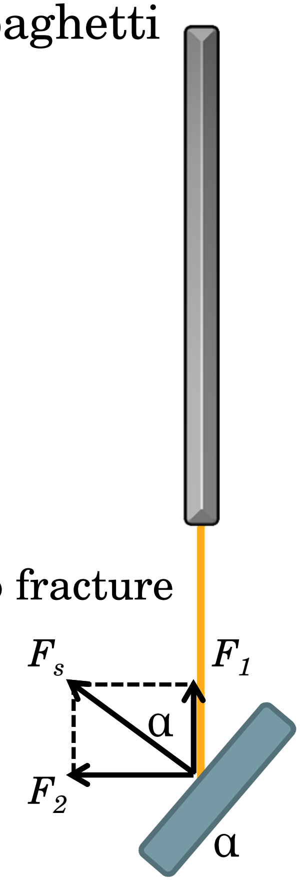


IMPACT ANGLE DEPENDENCE

- complex buckling/bending relation
  - as the angle increases, bending gains significance over buckling
    - strucututes break more easily under bending loads
  - angle ~30° friction force is not great enough to keep the spaghetti steady
    - slides – no fracture

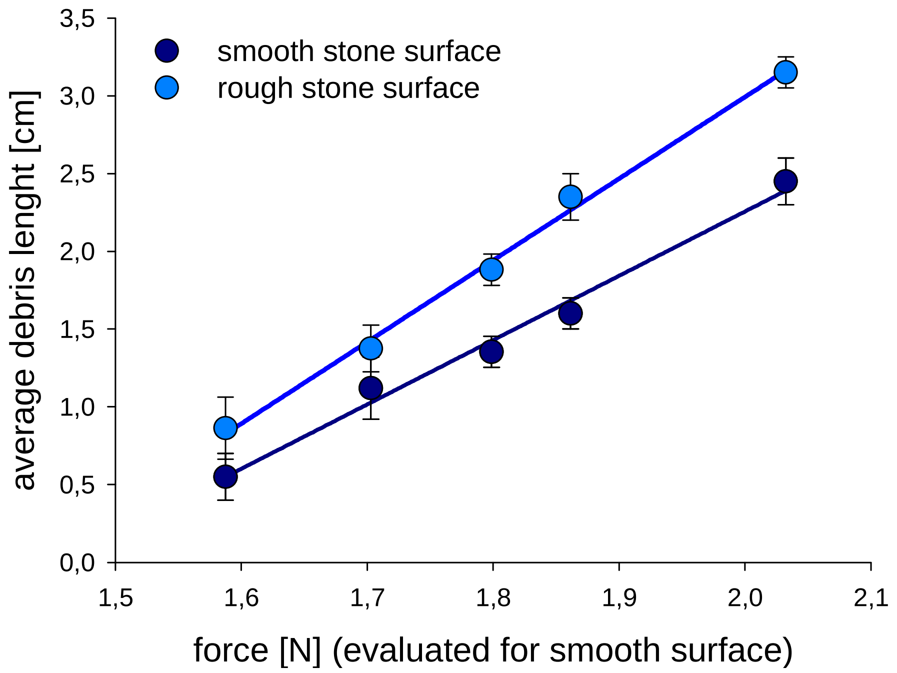


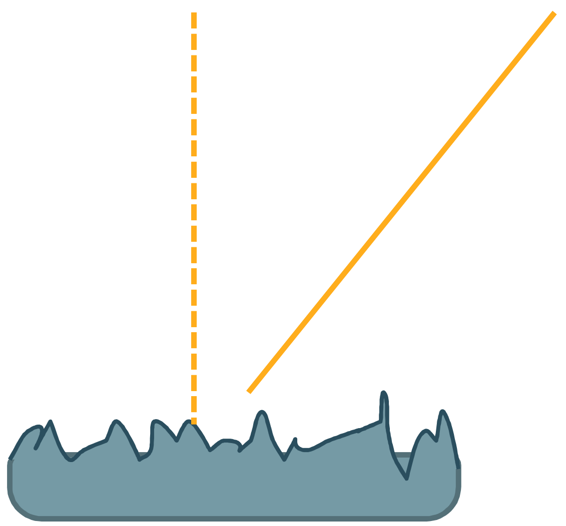
- tube height 3.25 m
- spaghetti °2
- at angles exceeding 80° no fracture



SURFACE DEPENDENCE ROUGHNESS

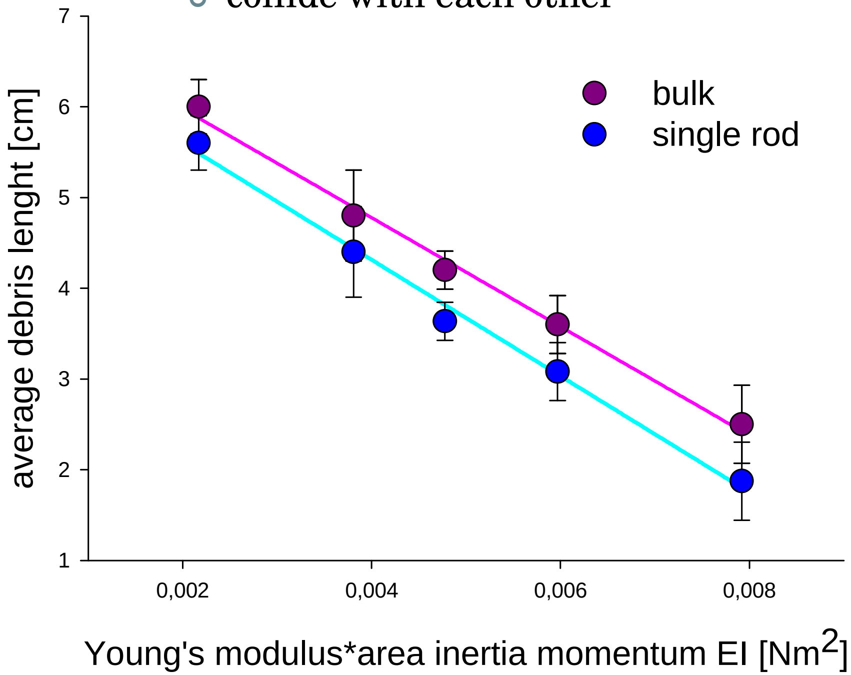
- spaghetti °4, same stone two sides – rough, smooth
- rough stone surface changes the spaghetti impact angle (surface imperfections)
  - greater angle results in more bending deformation – longer debris
- debris lenght zero for smooth surface (regression linear coefficient)
  - $F = 1.46 \pm 0.02N$ expected value (smooth) $F = 1.48 \pm 0.01N$

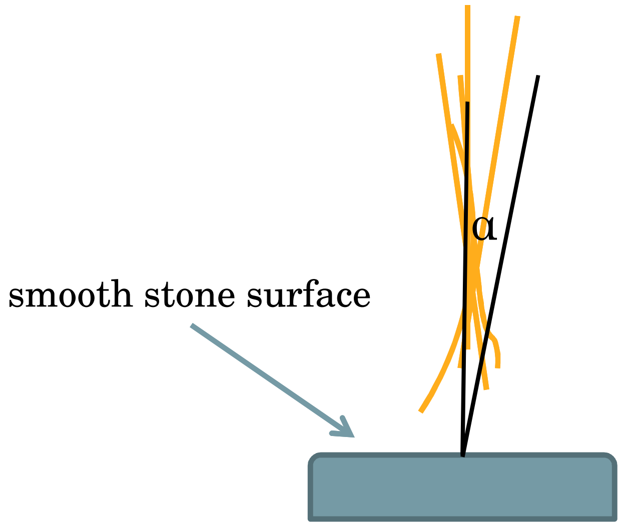




SINGLE ROD / BULK DEBRIS LENGHT COMPARISON

- too many movements for the force to be evaluated on camera
  - *force and debris lenght are proportional*
  - *on the same height≈ same force*
- spaghetti interact in a bulk
  - change direction, hit the surface under a small angle
    - greater angle results in more bending deformation – longer debris
    - collide with each other





CONCLUSION

- theoretical explanation buckling
- conducted experiment
    - conditions under which spaghetti does not break
  - <u>lowest fracture impact forces at vertical fall</u>
    - °1 $F = 0.59 \pm 0.03N$
    - °2 $F = 1.10 \pm 0.02N$
    - °3 $F = 1.29 \pm 0.02\ N$
    - °4 $F = 1.48 \pm 0.01N$
    - °5 $F = 2.26 \pm 0.03\ N$
- debris lenght at a force → *zero debris lenght*
  - predicted using simulation and measured – agreement
- surface hardness dependence
  - same minimum fracture forces
    - different impact duration and velocity change - confiration
- impact angle dependence
  - surface roughness dependence
  - number of spaghetti falling
    - changes the bending/buckling influence on dispacements

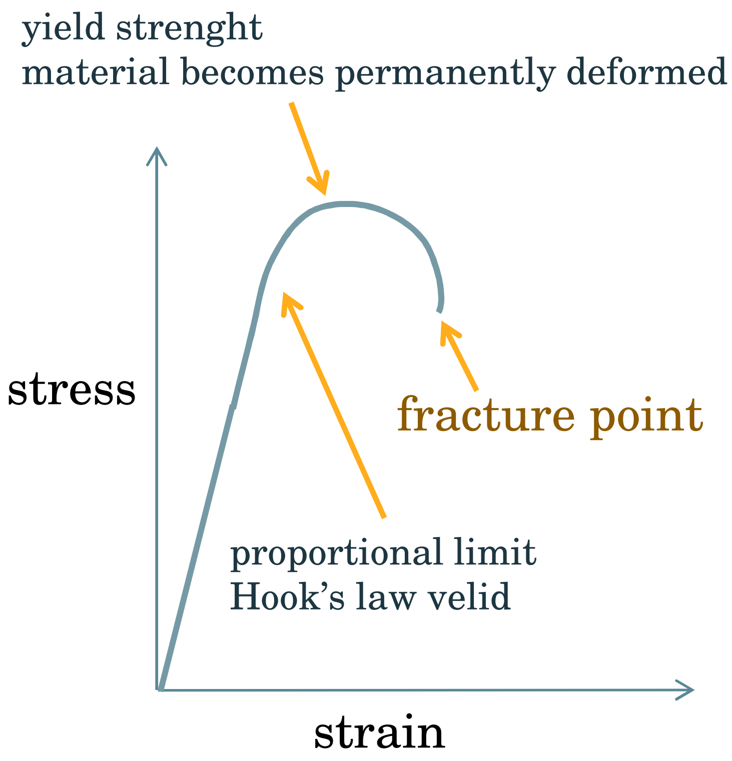
REFERENCES

- V.Šimić, Otpornost materijala 1, Školska knjiga, 1995.
- V.Šimić, Otpornost materijala 2, Školska knjiga, 1995.
- Halliday, Resnick, Walker, Fundamentals of physics, 2003.
- B. Audoly, S. Neukirch, http://www.lmm.jussieu.fr/spaghetti/

Thank you!

Impact

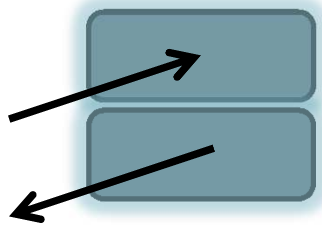
- typical stress strain curve for brittle materials
- Hook's diagram



$$stress\ \sigma = \frac{F}{A} = E\frac{\Delta x}{x_0}$$

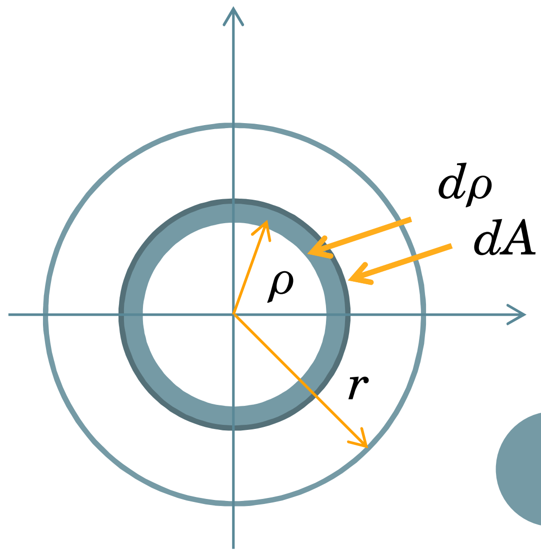
$$strain\frac{\Delta x}{x_0}$$

- fracture modes
  - $tearing\ mode = sliding\ mode$
    - for a long thin object



Area moment of inertia

- property of a cross section
  - geometrically: the strain in the beam
    - maximum at the top
    - decrease linearly to zero at the medial axis
    - continues to decrease linearly to the bottom
      - energy stored in a cross-sectional slice of the bent beam
        - proportional to the sum of the square of the distance to the medial axis
- circle
  - symmetrical (same on every axis)



- $dA = 2\pi\rho d\rho$

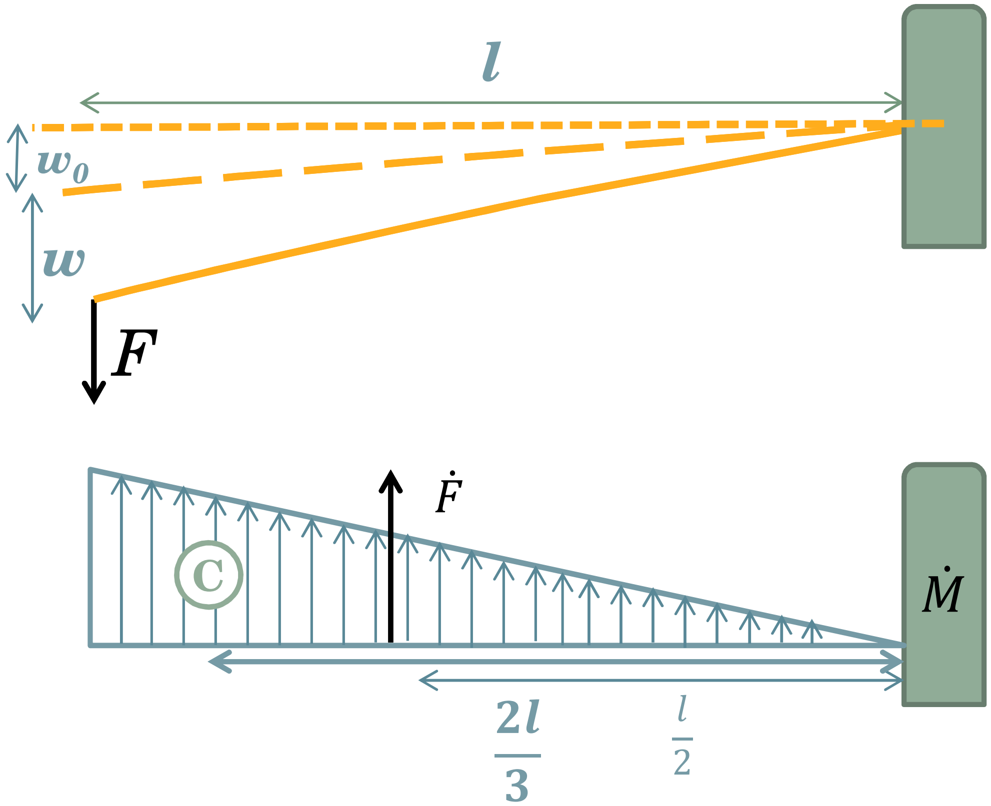
- $I = \int_A \rho^2 dA = \int_0^r 2\pi\rho^3 d\rho = \frac{r^4\pi}{2}$

BEAM DEFLECTION METHOD

- beams with complex loads, boundary deflections
- equation of the elastic line for a beam $M = M_F = \frac{d^2 w}{dx^2} EI$
  - load intensity and bending moment relation $\frac{d^2 M}{dx^2} = -q$

  - consider probe beam with load intensity of $\dot{q} = M$
    - same shaped stress diagram as the bending moment of our beam
  - *probe beam bending moment* $\frac{d^2 \dot{M}}{dx^2} = -\dot{q} = -M = -\frac{d^2 w}{dx^2} EI$
    - $EId^2w = d^2\dot{M}$ *integrating*
    - $EIw = M + Cx + D$ $-$ *boundary condition* $(fixed\ end, D = 0, C = 0)$
  - $w = \frac{\dot{M}}{EI}$

Beam deflection method



applied load-small weight

$$\dot{F} = Fl\frac{l}{2} \qquad \dot{M} = \dot{F}\frac{2}{3}l = \frac{Fl^3}{3}$$

- $\dot{F}$ *probe beam force*
- $\dot{M}$ *probe beam bending moment*

$$w = \frac{\dot{M}}{EI} = \frac{Fl^3}{3EI} \; deflection - load\ F$$

SIMULATION REGRESSION