PROBLEM NO. 5: "CAR"

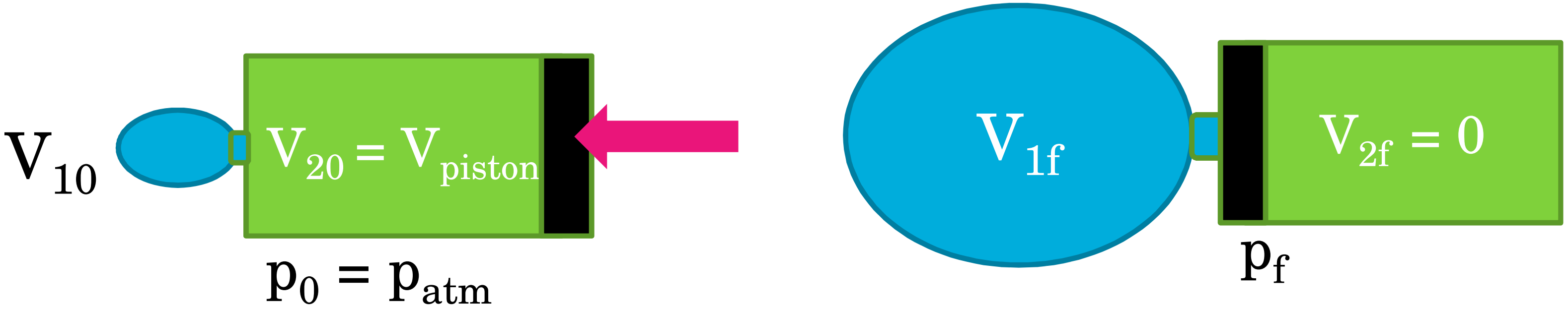
Build a model car powered by an engine using an elastic air-filled toy balloon as the energy source. Determine how the distance travelled by the car depends on relevant parameters and maximize the efficiency of the car.

OVERVIEW

- balloon observations
  - piston inflation/deflation model
    - energy input and output
    - rubber stretching losses
- construction
- observed motion
- parameters
  - initial conditions (volume, pressure)
  - jets, nozzles
  - maximised <u>travelled distance</u> and <u>efficiency</u>
- conclusion

PISTON MODEL ~ ENERGY

- piston contains all air later used for inflation/deflation
  - slowly pushed – balloon inflates
  - released piston – balloon deflates



$$p_0(V_{10} + V_{20}) = nRT = const$$

initial state

$$p_f V_{1f} = nRT$$

final state

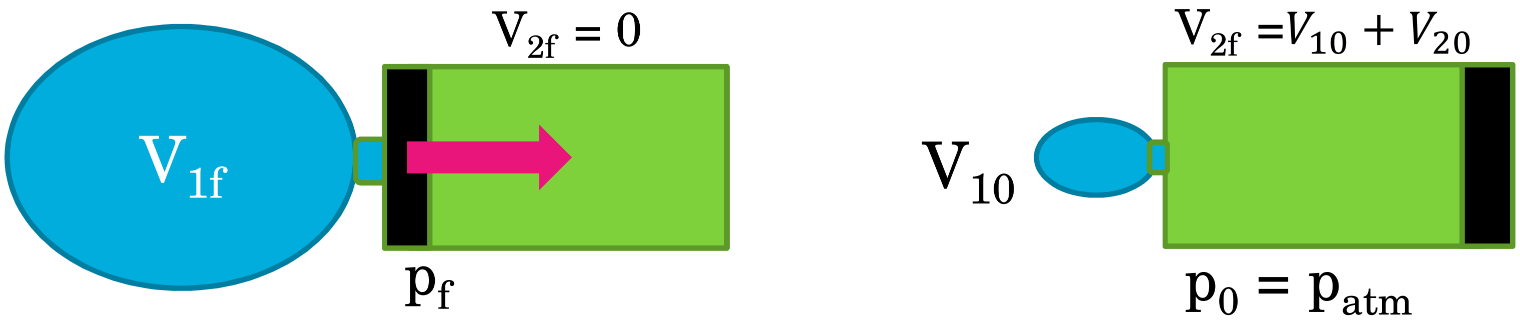
- at every moment ideal gas law equation states:

$$p(V_1 + V_2) = nRT \qquad R - gas\ constant\ = 8.314\frac{J}{molK}$$

$$dW = -(p - p_0)dV_2$$

integrating initial to final state

PISTON MODEL ~ ENERGY



$p_f V_{1f} = nRT$ $\quad$ $p_0(V_{10} + V_{20}) = nRT = const$

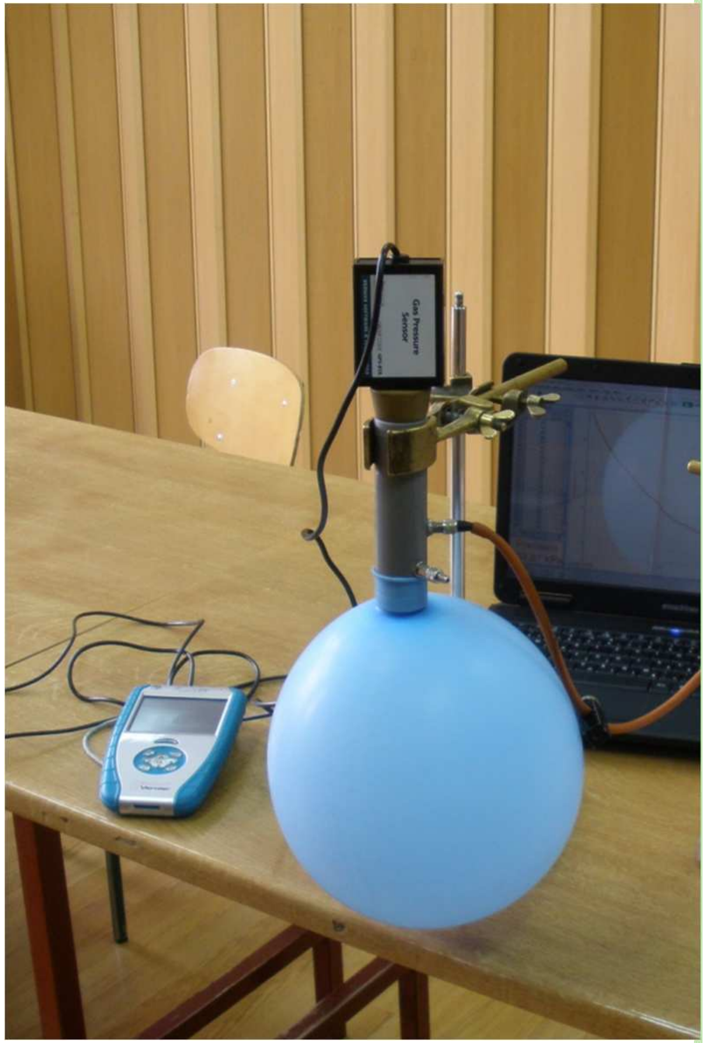
- maximal available energy
  - deflation work
- same equations valid

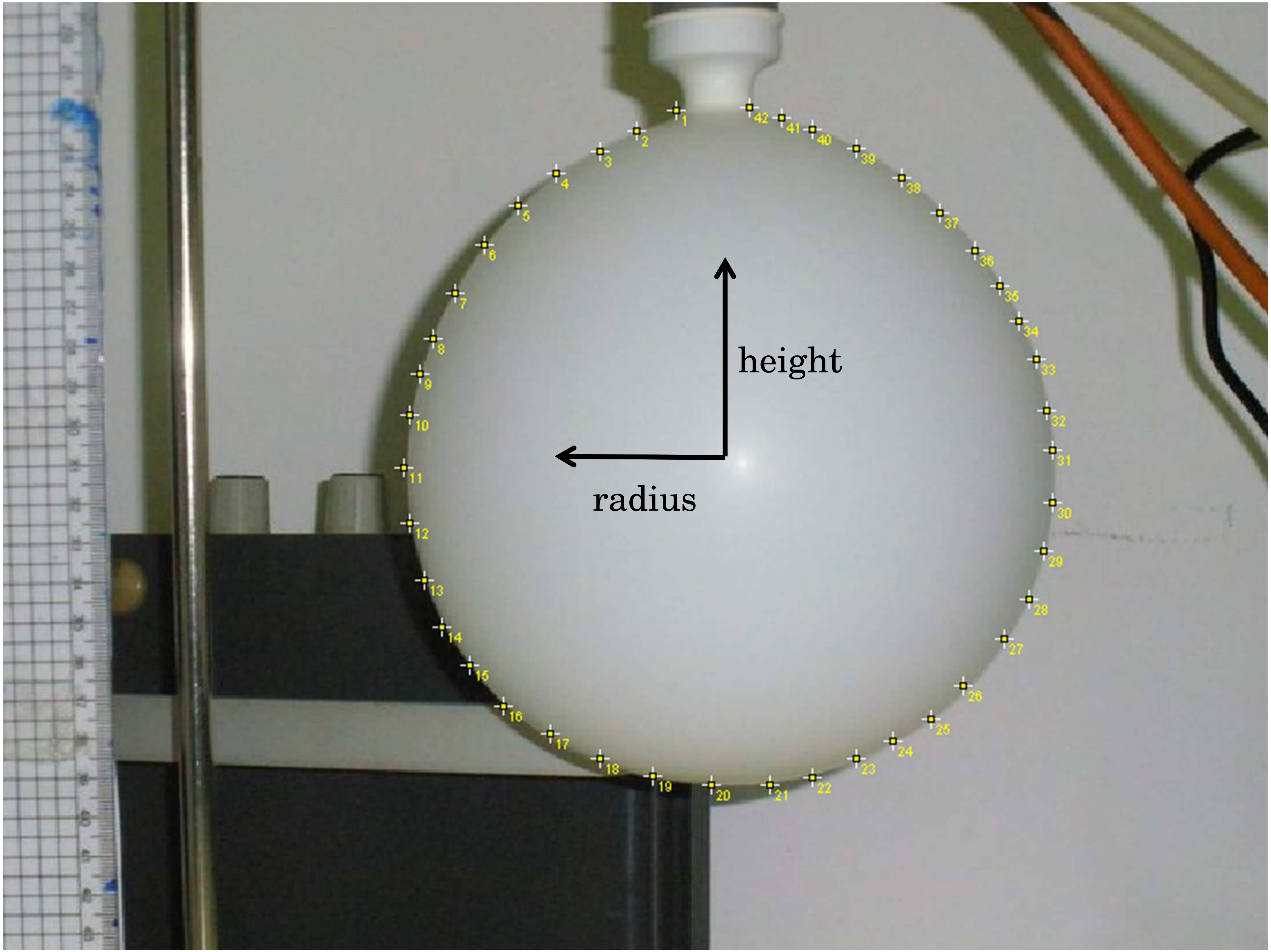
$$W = \int_{p_0}^{p_f} p\,dV_1 + p_f V_f \left(\ln\frac{p_f}{p_0} - 1\right) + p_0 V_{10}$$ **known conditions**

$\int_{p_0}^{p_f} p\,dV_1$ varies for inflation/deflation – ***determined experimentally***

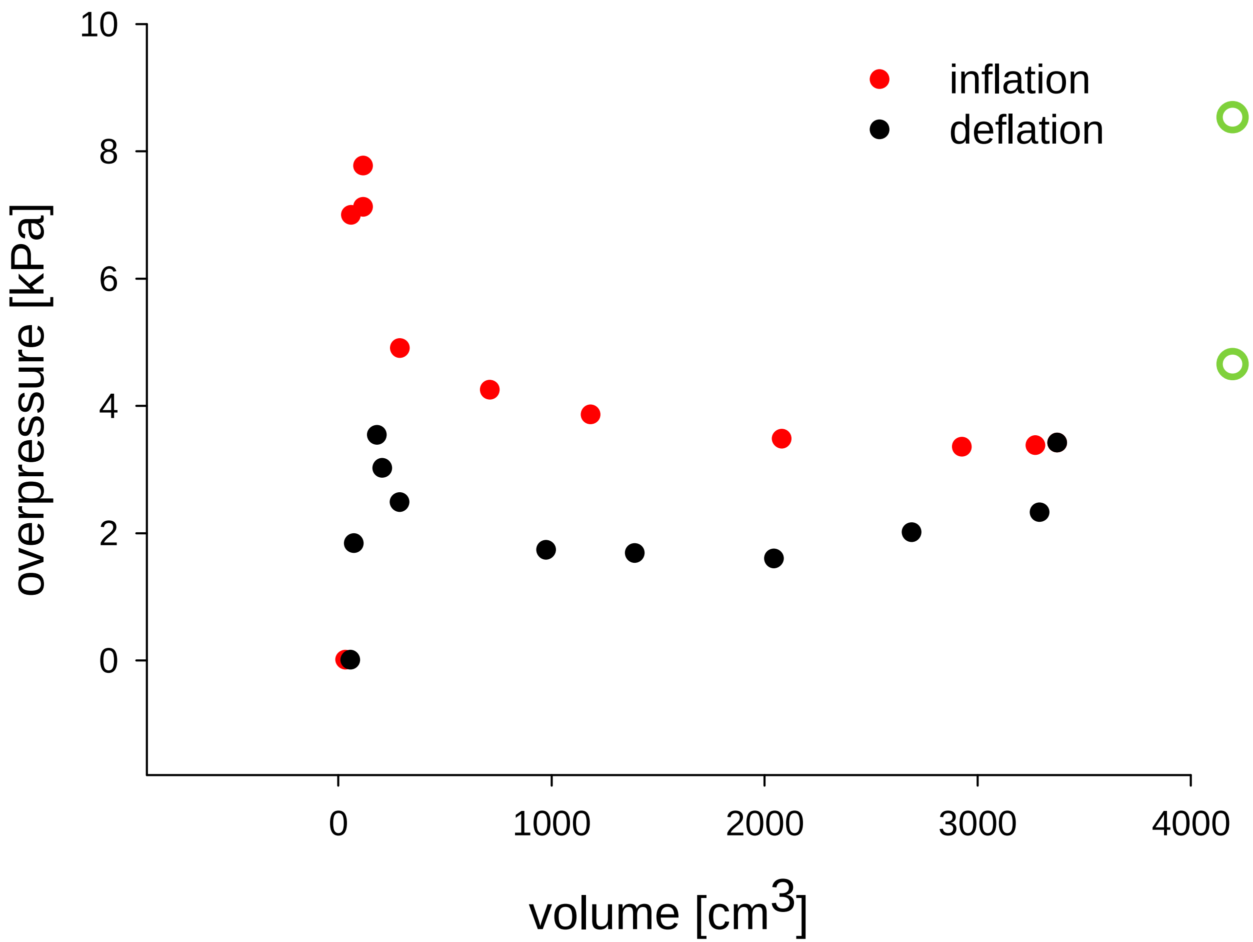
PRESSURE / VOLUME RELATION
INFLATION AND DEFLATION

- pressure
  - digital pressure sensor
- volume
    - photos taken at known pressure
  - balloon edge coordinates
    - balloon shape determined
  - balloon radius/height function
    - numerically integrated
  - shape rotation – body volume




height
radius

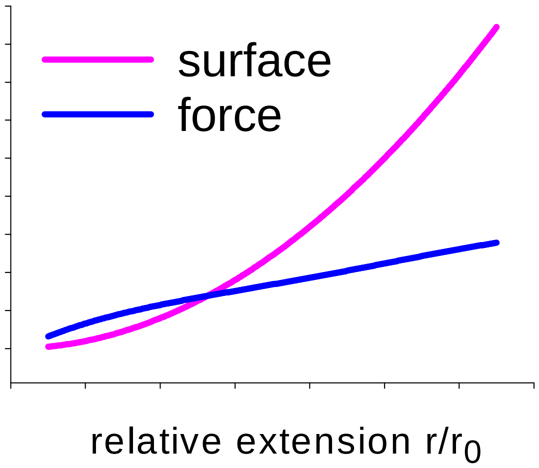
Typical pressure/volume relation



- *deflation curve is under the inflation curve*
- energy partly lost due to rubber deformations

BALOON STRETCHING – THEORETICAL CURVE

- pressure (force acting on a surfaca)
- balloon elastic energy
  - $U = 4\pi {r_0}^2 \kappa RT\left(2\lambda^2 + \frac{1}{\lambda^4} - 3\right)$*
    - $\lambda = \frac{r}{r_0}$ relative strain
    - $\kappa - rubber\ property$



- work needed to increase radius from r to r+*dr under pressure difference* ΔP (being overpressure)

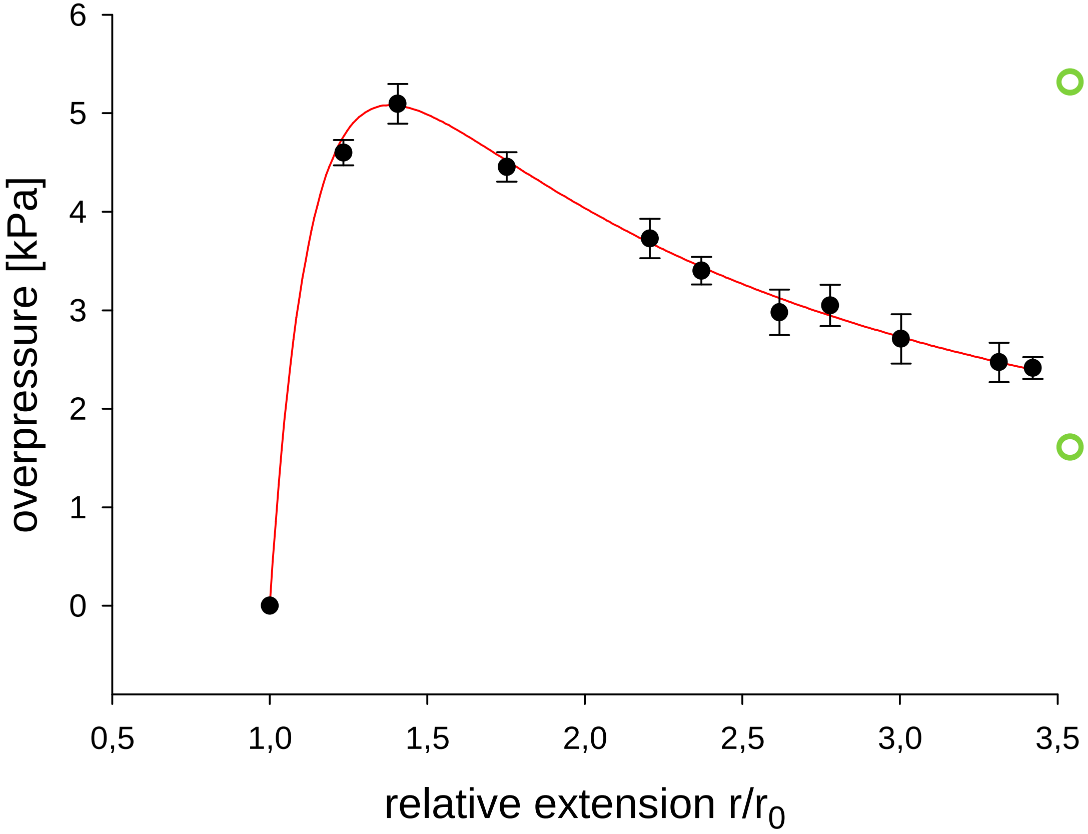
$$dW = \Delta P dV = \Delta P 4\pi r^2 dr = \left(\frac{dU}{dr}\right) dr$$

$$\Delta P = \frac{4\kappa RT}{r_0}\left(\frac{1}{\lambda} - \frac{1}{\lambda^7}\right)$$

*Y. Levin, F. L. da Silveira, Two rubber balloons: Phase diagram of air transfer, PHYS. REV. E (2004)

Balloon stretching

$$\Delta P = \alpha\left(\frac{1}{\lambda} - \frac{1}{\lambda^7}\right) - single\ parameter\ (\alpha\ rubber\ property) fit$$



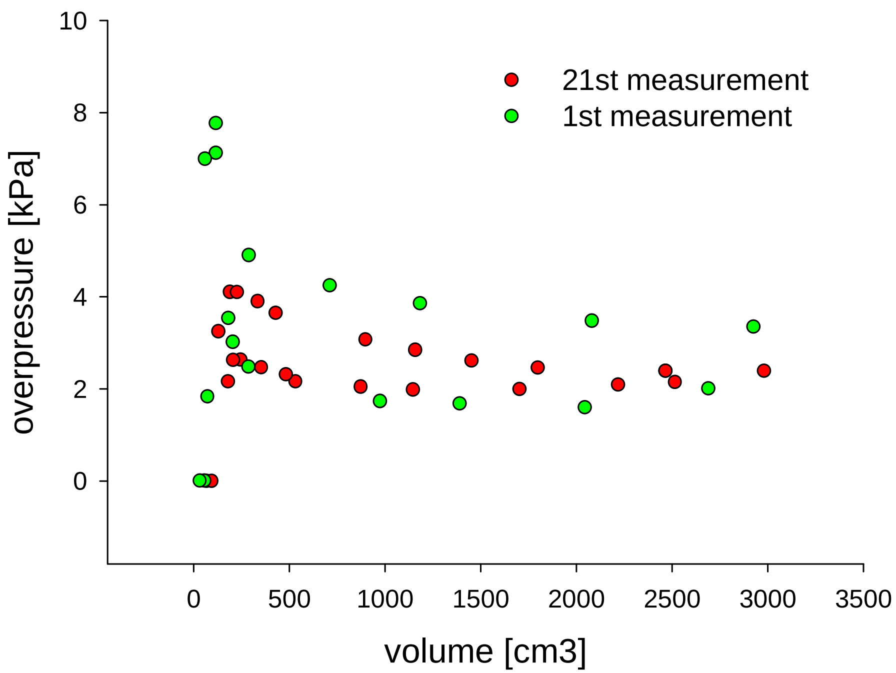
- *obtained curve has the expected maximum*
  - *due to surface/force relation*
- *explains and confirms the obtained shape of the overpressure/strain curve*

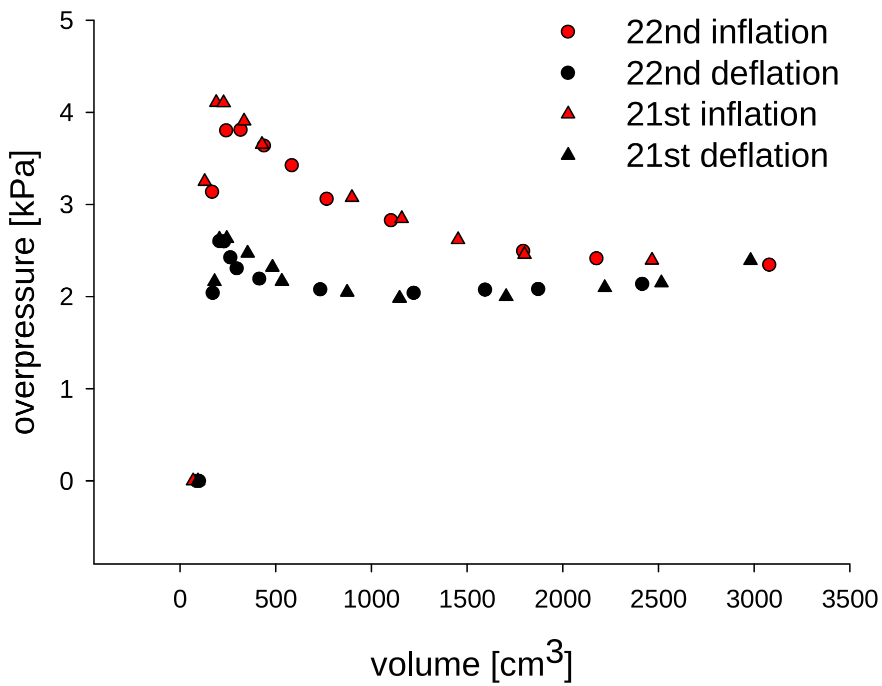
Consistency of measurements

- the stress-strain curve depends on the maximum loading previously encountered (Mullins effect)
  - later measurements – rubber already deformed
    - occurs when the load is increased beyond its prior value
    - less pressure difference needed to inflate to a certian volume
- pV relations
  - same balloon – multiple usage
  - initial volume (pressure)

  - OBSERVATION
    - pV graph for 1st and 10th measurement greatly differ

Consistency of measurements



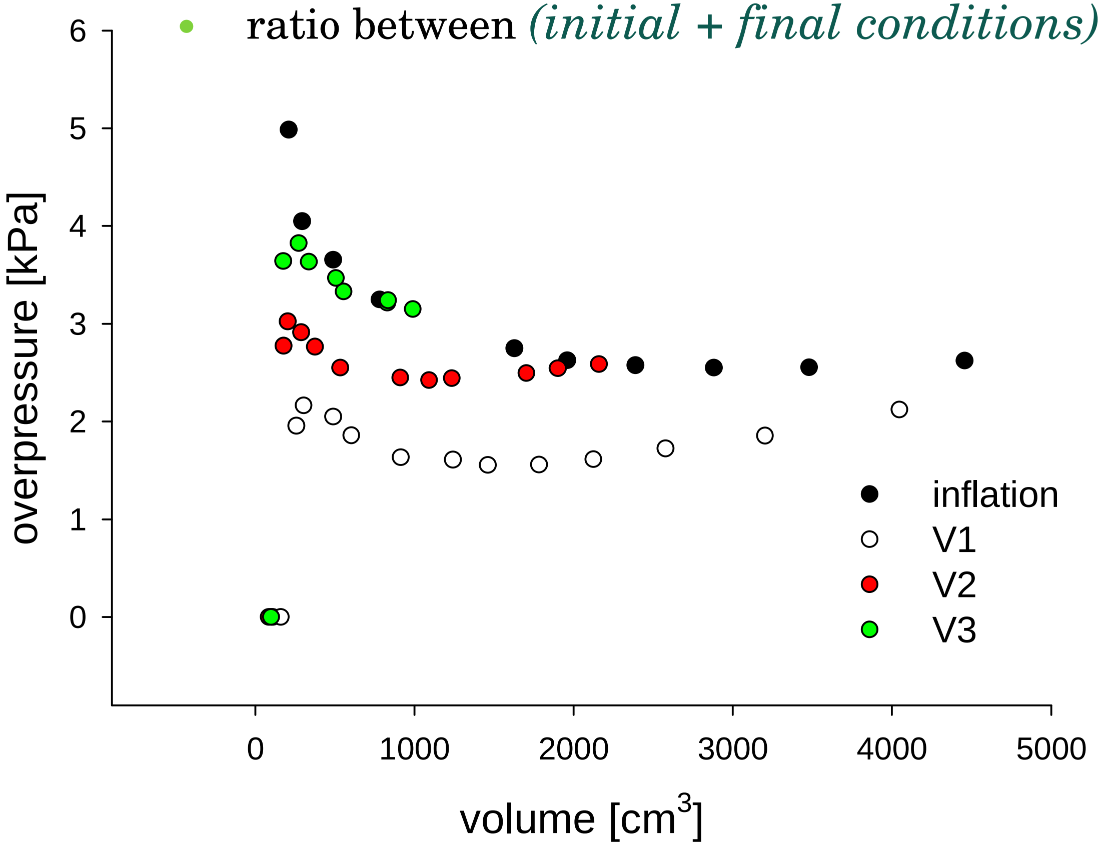


- 1st and 21st measurement vary greatly
- rubber deforms after few measurements

- 10th-25th measurement comparable (similar)
- afterwards balloon breaks

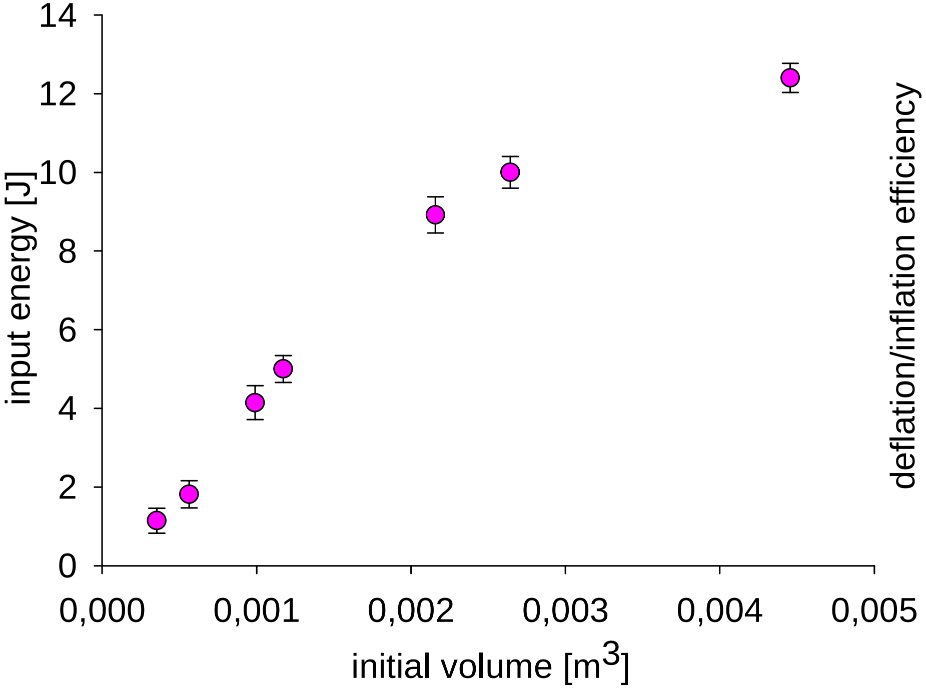
INITIAL VOLUME

- surface under the curve enlarges with initial volume
  - determining part of total input and output energy
- efficiency is determined by the deflation and inflation surfaces under the curve
  - ratio between *(initial + final conditions) + surface under the curve*

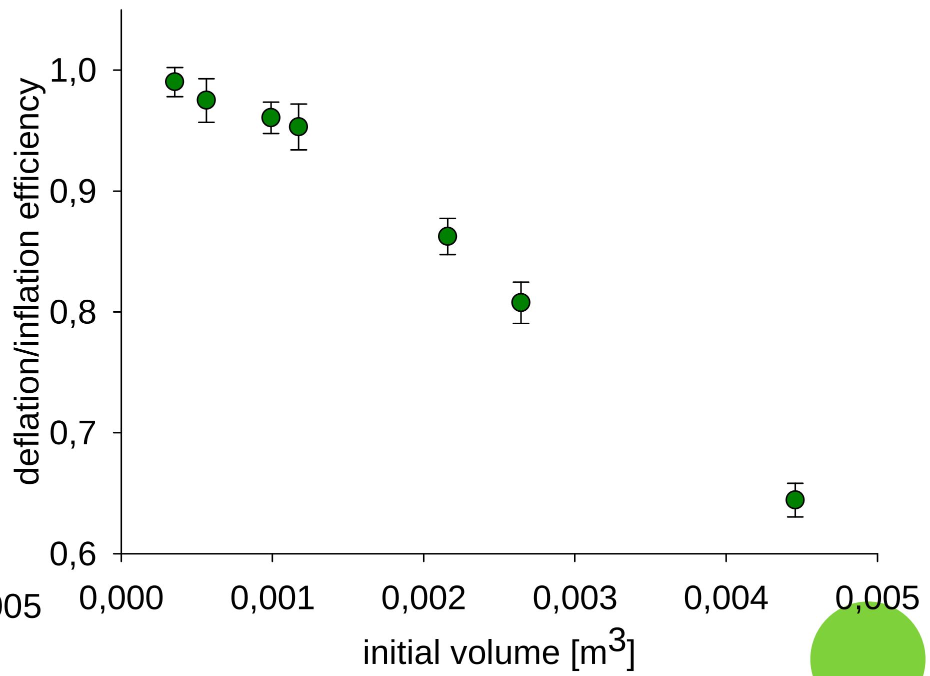


INITIAL VOLUME

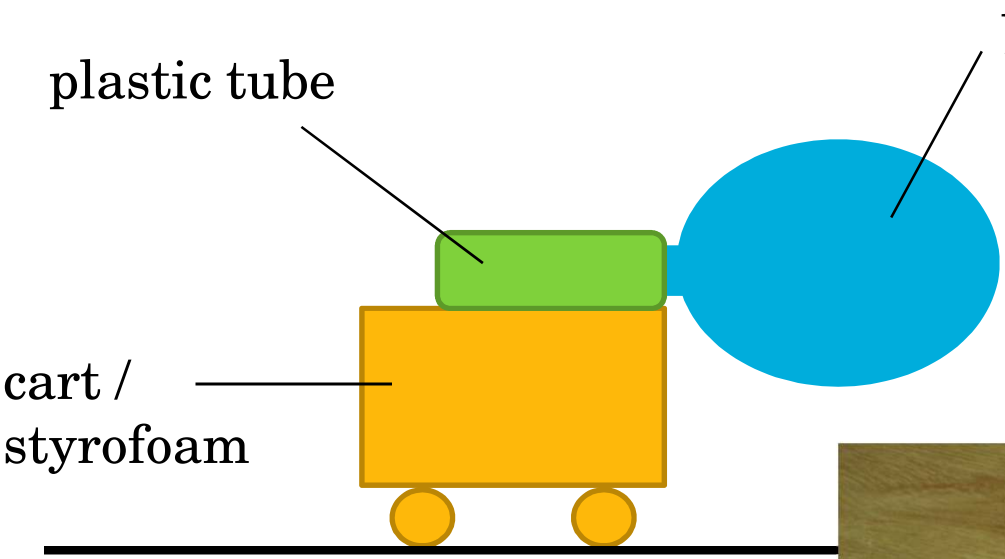
- input energy
  - inflation



- inflation/deflation varies
  - rubber stretching $\eta = \frac{E_d}{E_i}$

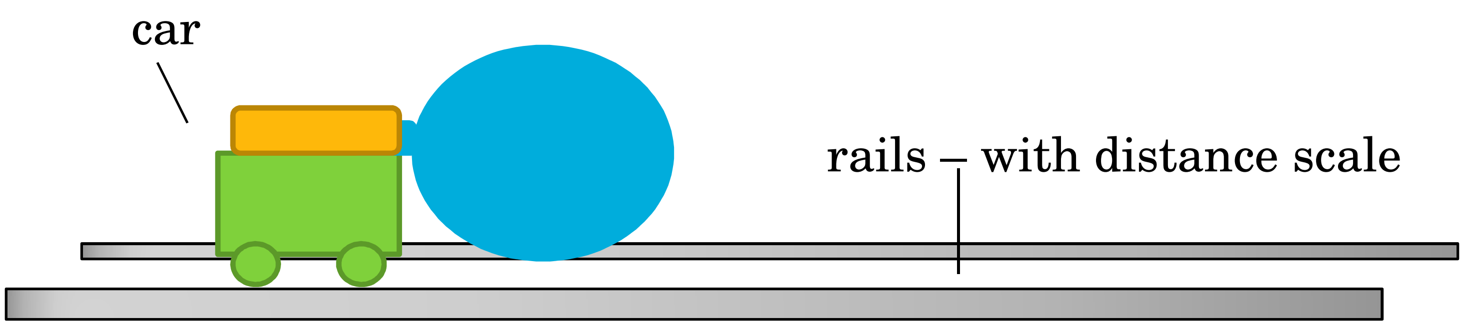


CONSTRUCTION

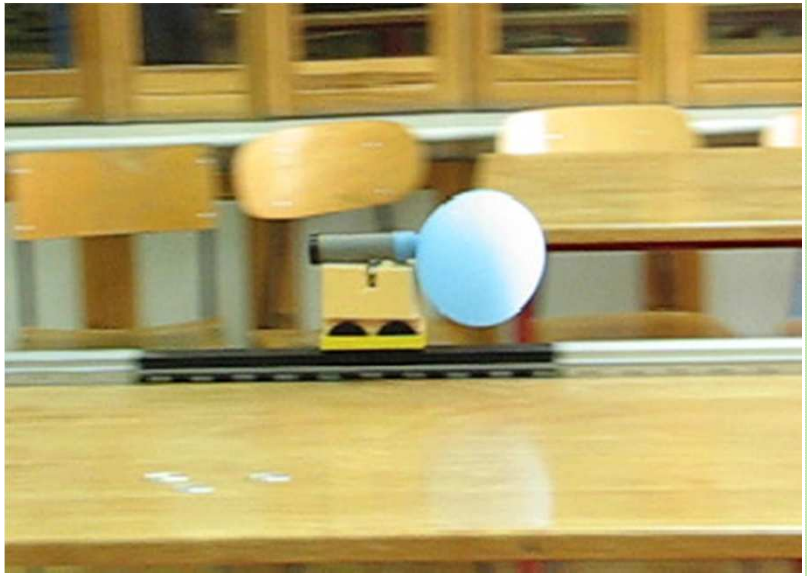


- plastic cart (lenght ~ 12cm)
- styrofoam
- plastic tube (d = 2,9 cm)
- metal jets / cardboard noozles
- car mass ~120g

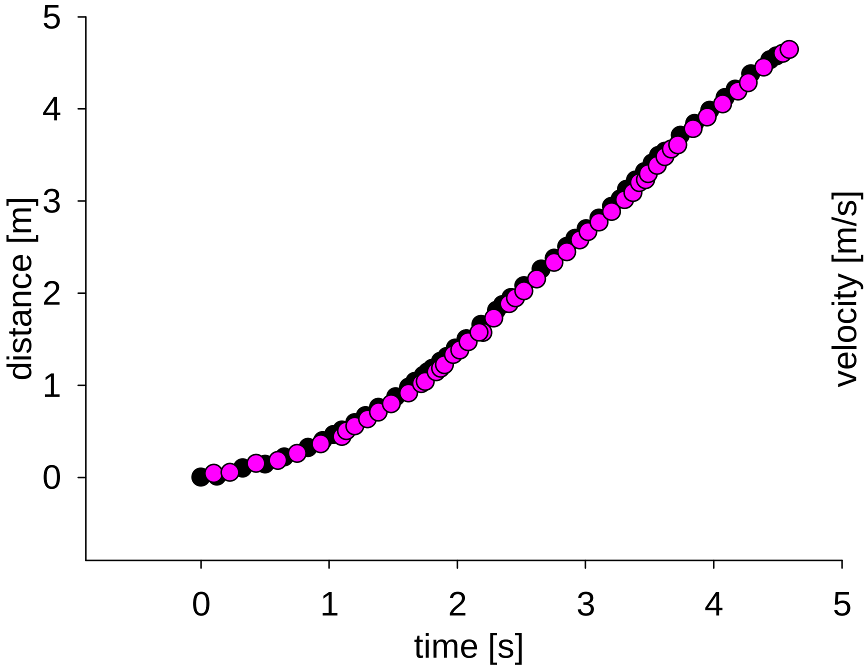
OBSERVED MOTION - SETUP

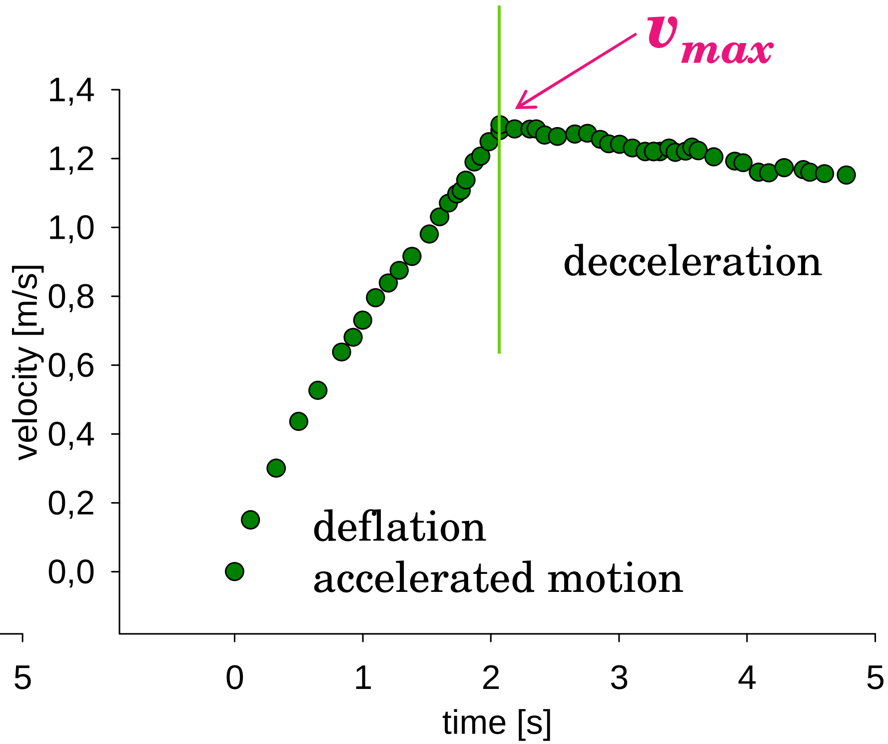


- rails
  - car moving straight
  - position determination
  - lenght 5 m
- video
  - camera 120 fps
  - placed 10 m from the rails
- analysis
- time / distance coordinates



OBSERVED MOTION



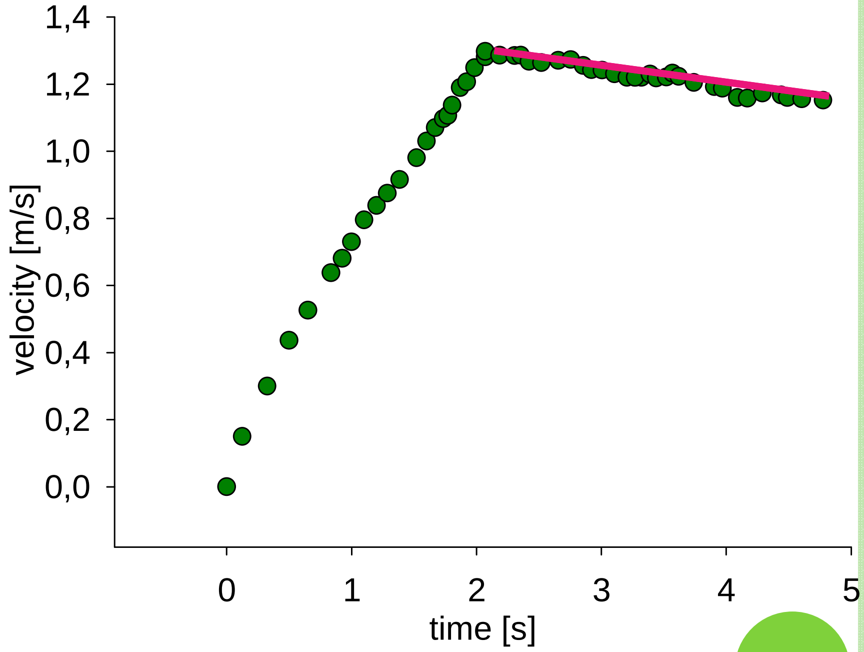


- distance/time graph obtained from video coordinates
- program time-distance graph:
  - 5 data points frame – cubic or linear curve fitted
  - derived: one point in the v-t graph
  - moves one point to the next frame

Travelled distance

- travelled distance: $s = s_0 + s_1$
  - acceleration path
    - v_{max} time
    - distance s_0
    - determined from the video
  - decceleration
    - a determined from the graph
    - linear fit coefficient
  - decceleration path

$$s_1 = \frac{v_{max}{}^2}{2a}$$



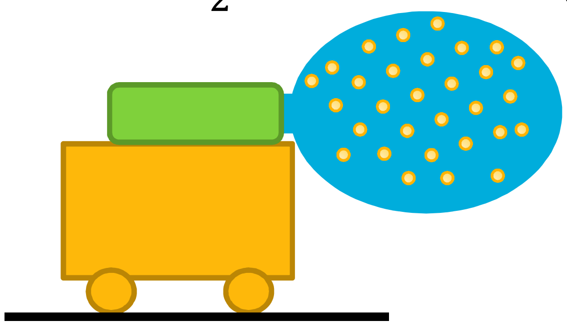
Basic working principle

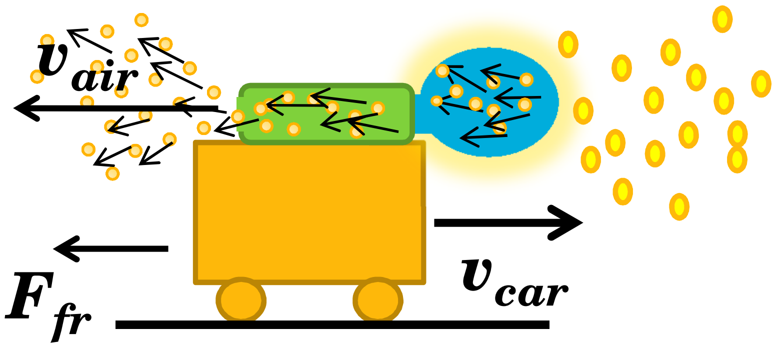
- input energy is used for inflation
- deflation transforms it into *kinetic energy of the air molecules*
    - losses due to resistance to movement (mutual collisions)
- momentum conservation

$$d(m_a v_a) - F_{fr} dt = m_c dv_c$$

- simplified model: air behavior
    - mass inside the balloon $\bar{p}V = \frac{m_a}{M} RT \rightarrow m_a = \bar{p}V \frac{M}{RT}$
    - velocity according to Bernoulli's principle (valid for turbulent flows)

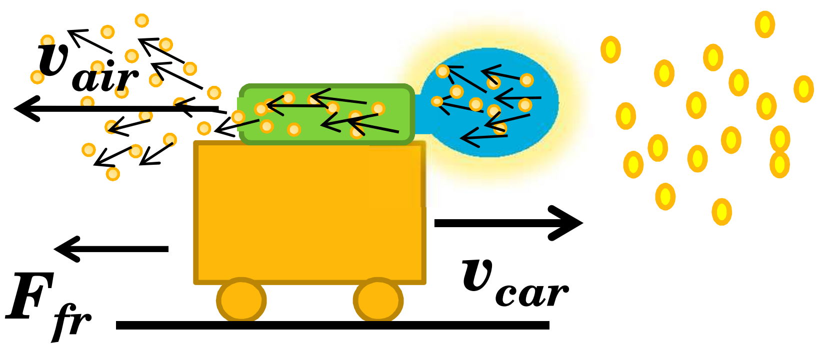
$$\frac{\rho v_a{}^2}{2} = \bar{p} \rightarrow va = \sqrt{\frac{2\bar{p}}{\rho}}$$

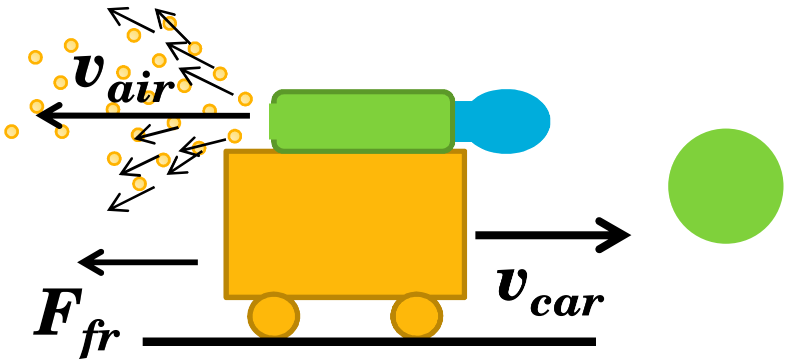




BASIC WORKING PRINCIPLE

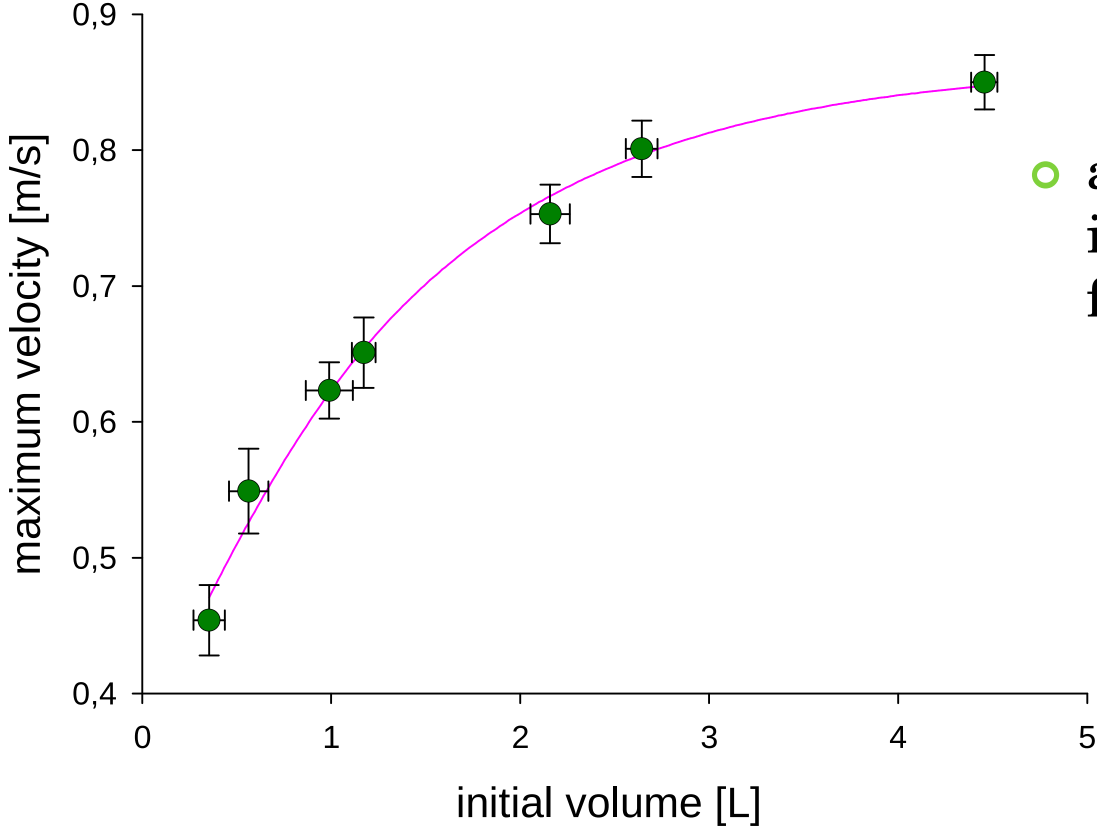
- simplified model: $m_a v_a - F_{fr} t = m_c \boldsymbol{v_c}$
  - friction force
    - main friction force source is the air drag $F_{fr} = \frac{1}{2} C\rho S \boldsymbol{v_c}^2 \sim V^{\frac{2}{3}} \boldsymbol{v_c}^2$
    - friction force duration - flow rate $Q = \frac{V}{t} = Sva \rightarrow t \sim \frac{V}{v_a}$
- $\bar{p}$ *is evaluated from pV graphs as* $\bar{p} = \frac{\int_{p_0}^{p_f} p dV_1}{V_f - \boldsymbol{V_0}}$
- maximum car velocity $\boldsymbol{v_c}$ initial $\boldsymbol{V_0}$ volume relation obtained





INITIAL VOLUME

- simplified model basic working principle fit
  - main friction force source is air drag



- as the initial volume increases the air drag force gains significance
  - reason of non-linear progression
    - simplification

$$m_a v_a - F_{fr} t = m_c v_c$$

MAXIMISATION – V_{MAX} IMPORTANCE

travelled distance

- motion consists of acceleration and decceleration
- *acceleration* to v_{max} is determined by:
  - initial conditions
    - volume/pressure
  - jets, noozles
  - travelled distance found from the video
- *decceleration* – same for all parameters
  - friction
    - air
    - wheels/rails/shaft
  - $v_{max} = \sqrt{2as} \rightarrow s = \frac{{v_{max}}^2}{2a}$

efficiency

- energy input and ouput ratio
  - $\eta = \frac{energy\ input}{energy\ output}$
- energy input
  - *work needed to inflate the balloon*
    - *piston model*
- energy output
  - $E_k = \frac{m_{car} v^2{}_{max}}{2}$
    - v_{max} car motion observation

PARAMETERS

- initial volume
  - total input energy
  - rubber stretching losses

- jet diameter
  - volumetric flow of air Q
    - Reynolds number 10^5
      - tubulent flow
  - exit velocity
    - deflation time

- nozzle
  - directs the flow

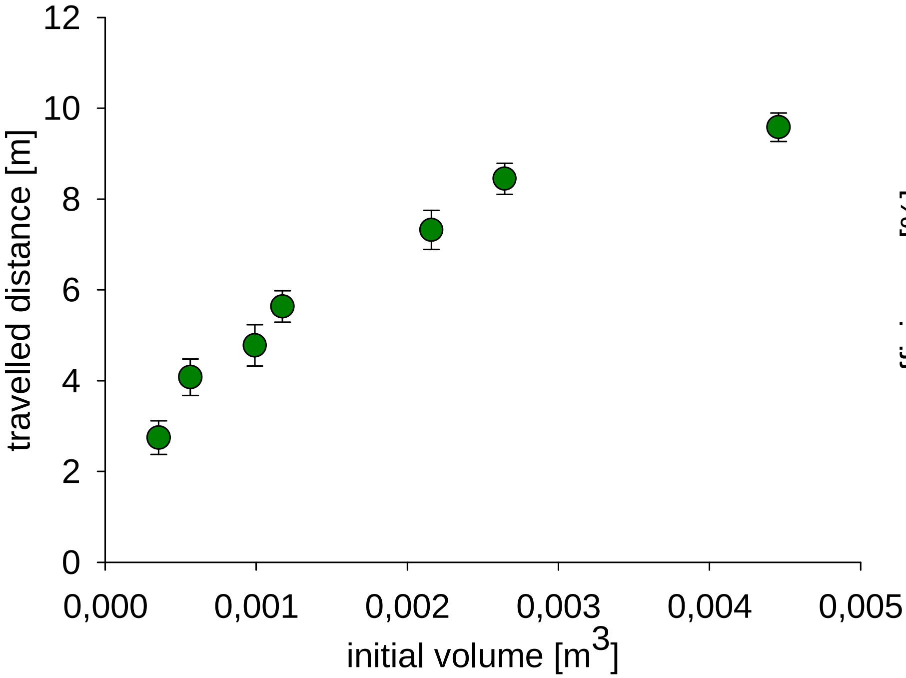
Initial volume

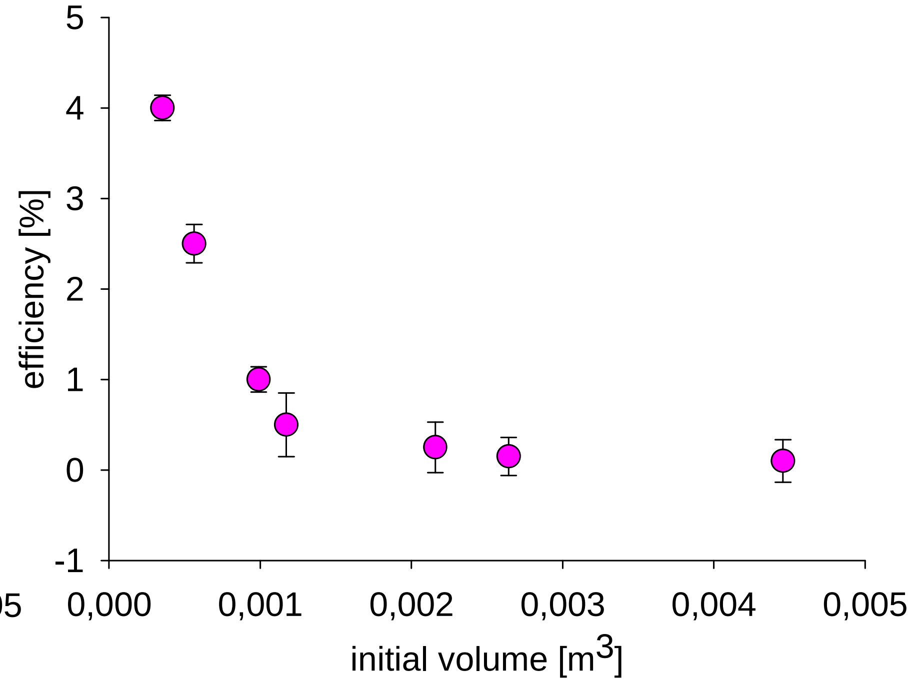
- initial volume! – pressure
- total input energy and deflation energy
  - critical sizes
    - min
      - needed to overcome static friction - start car movement
    - max
      - radius ~ car height
      - the balloon must not touch the floor
    - range ~ 0.15 dm^3 - 4.5 dm^3
      - deflated balloon ~ 0.075 dm^3
- air drag friction force

Initial volume
Results

- travelled distance:
  - $s = s_0 + s_1$
- maximisation largest balloon

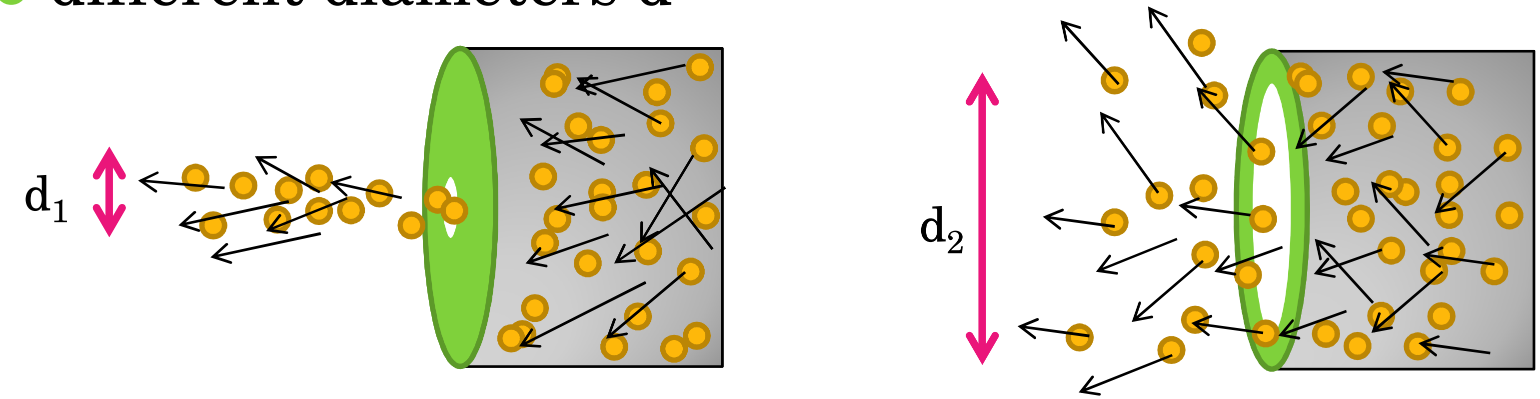
- efficiency:
  - output/input energy $\eta = \frac{m{v_{max}}^2}{2Ei}$
  - small
    - losses on the air kinetic energy
- maximisation smallest balloon





JETS - CIRCULAR TUBE OPENINGS

- different diameters d



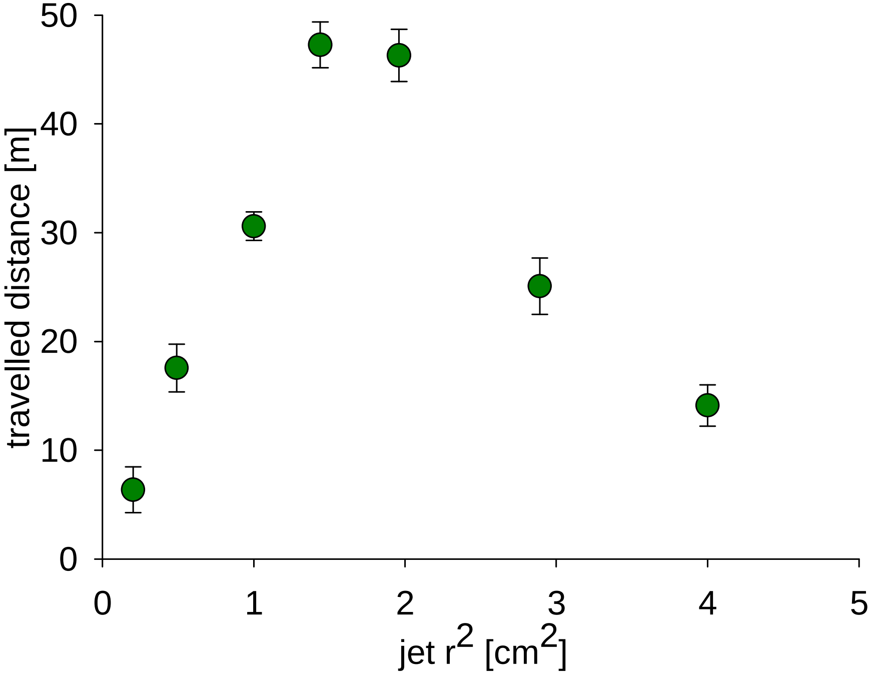
deflation time $t(d) >> t(D)$

- jet diameter changes
  - how well the air is directed
  - time needed for deflation
    - air drag duration
  - amount of resistance the air endures in mutual collisions
  - Reynolds number $Re = \frac{vd}{\nu}$, ν kynematic viscosity
    - turbulent flow Re>4000, 10_5

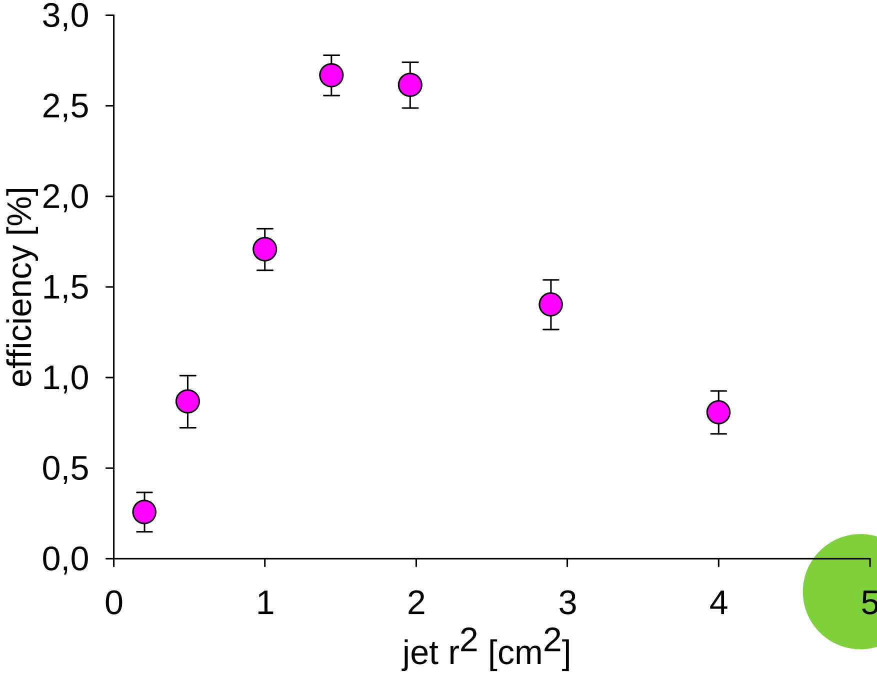
RESULTS (V~4DM3)

- different v_{max} for a certain diameter
  - both travelled distance and efficiency depend on v_{max}^2

max distance 47.3 m

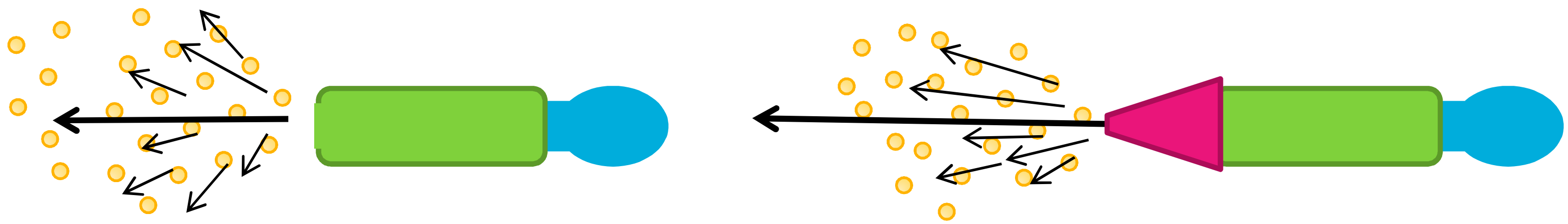


max efficiency 2.7%

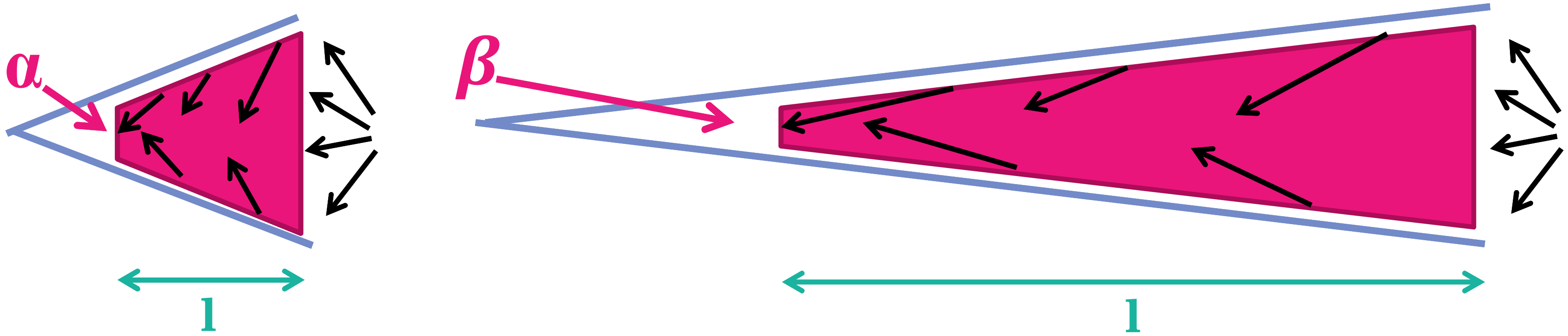


best jet diameter 1.2cm-1.4cm

NOZZLES – CONE SHAPED



effective velocity – horizontal component of rapid molecule movements

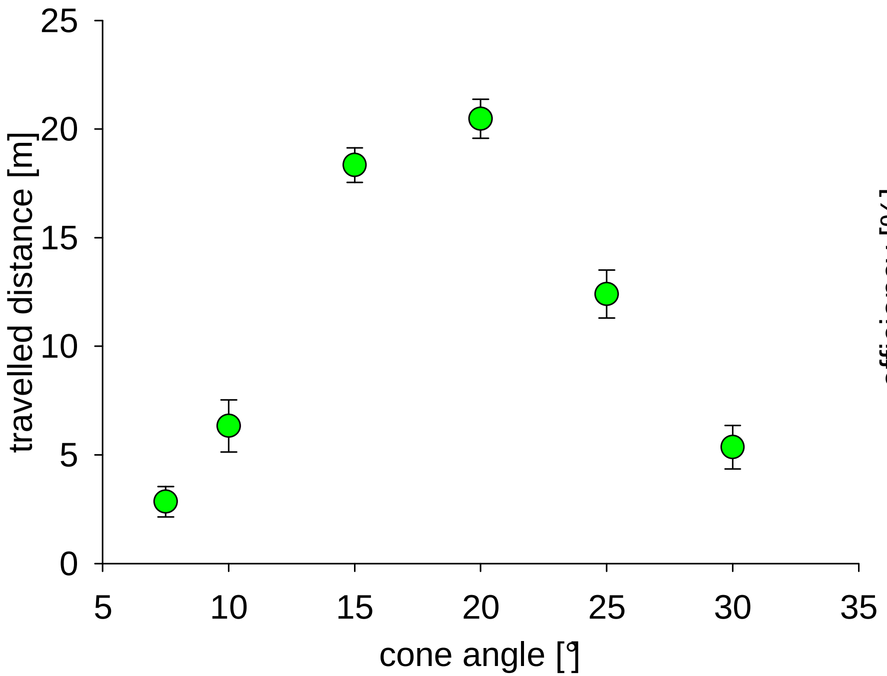


- different nozzles change
  - angle – how well the air is directed
  - tube lenght – amount of losses due to friction
- 6 cones – different angles α and thus lenghts (7.5°- 30°)

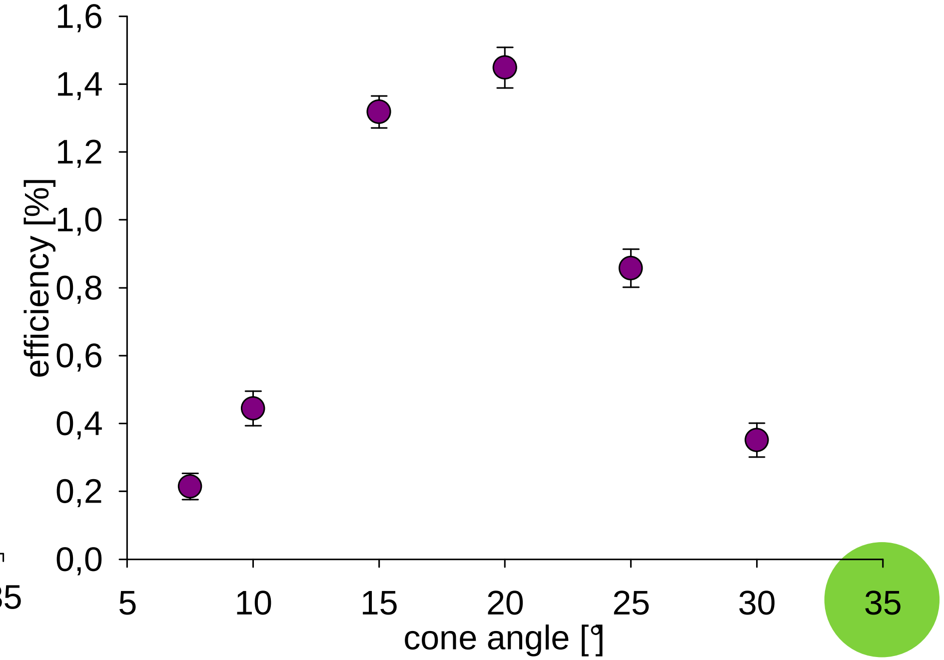
RESULTS (V~3DM^3) D=1CM

- different v_{max} for a certain diameter
  - both travelled distance and efficiency depend on v_{max}^2

max distance 20.5 m



max efficiency 1.5%



best cone angle 15°-20°

CONCLUSION

- piston inflation/deflation model – energy evaluation
- stretching losses – theoretical and experimental curve
- basic working priciple explained
- found maximum conditions for jets and noozles

travelled distance

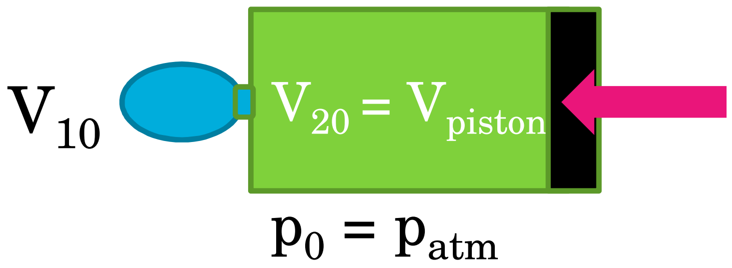
- initial V =4.5dm^3 *max*
- *optimal:*
  - *jet diameter1.2cm*
  - *nozzle angle 20°*
- *DISTANCE 70m*

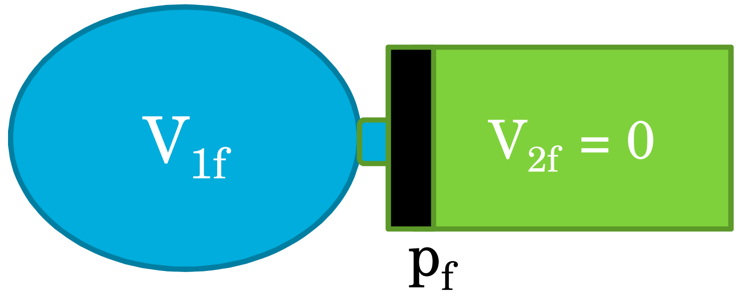
efficiency

- initial V=1.5dm^3 *min*
- *optimal:*
  - *jet diameter 1.2cm*
  - *nozzle angle 20°*
- *EFFICIENCY 6,4%*

Thank You!

PISTON MODEL ~ ENERGY





$$p_0(V_{10} + V_{20}) = nRT = const$$

$$p_f V_{1f} = nRT$$

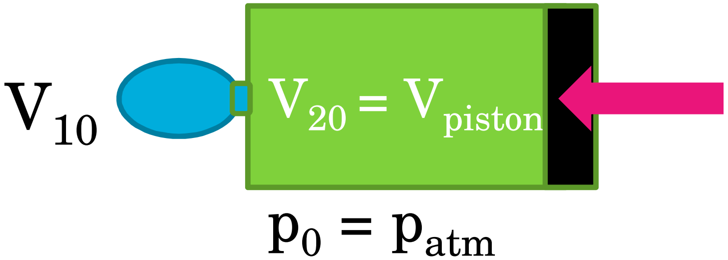
$$p(V_1 + V_2) = nRT$$

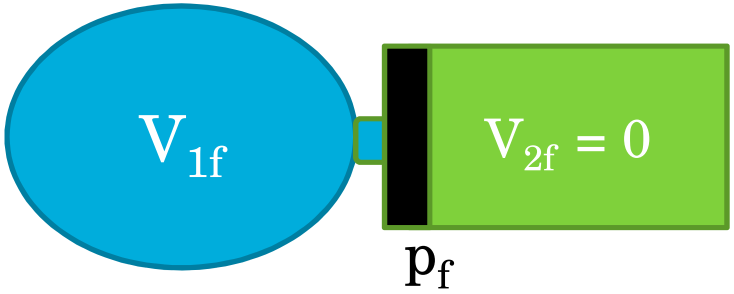
$$-dW = (p - p_0)dV_2$$

$$W = \int_{p_0}^{p_f} p dV_1 + p_f V_f (\ln \frac{p_f}{p_0} - 1) + p_0 V_{10}$$

$\int_{p_0}^{p_f} p dV_1$ varies for inflation/deflation – *determined from the graph*

PISTON MODEL ~ ENERGY





$$p_0(V_{10} + V_{20}) = nRT = const$$

$$p_f V_{1f} = nRT$$

$$p(V_1 + V_2) = p_f V_{1f}$$

$$dV_1 + dV_2 = -\frac{p_f V_{1f}}{p^2} dp \qquad dV_2 = -dV_1 - \frac{p_f V_{1f}}{p^2} dp$$

$$-dW = (p - p_0) dV_2$$

$$-dW = p\left(-dV_1 - \frac{p_f V_{1f}}{p^2} dp\right) - p_0 dV_2$$

$$\int_{p_0}^{p_f}$$

PISTON MODEL ~ ENERGY

$$-dW = p\left(-dV_1 - \frac{p_f V_{1f}}{p^2}dp\right) - p_0 dV_2$$

$$-dW = -pdV_1 - \frac{p_f V_{1f}}{p}dp - p_0 dV_2 \qquad \int_{p_0}^{p_f}$$

$$W = \int_{p_0}^{p_f} pdV_1 + p_f V_f \ln\frac{p_f}{p_0} + p_0(0 - V_{20})$$

$$p_0(V_{10} + V_{20}) = p_f V_f$$

$$V_{20} = \frac{p_f V_f}{p_0} - V_{10}$$

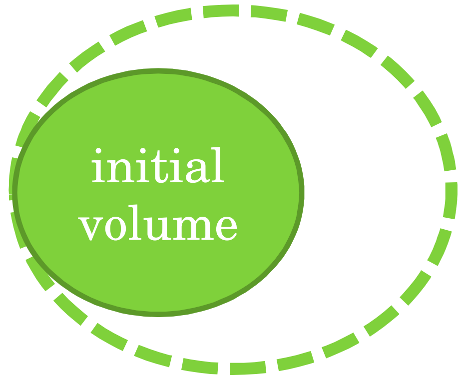
$$W = \int_{p_0}^{p_f} pdV_1 + p_f V_f \ln\frac{p_f}{p_0} - p_0\left(\frac{p_f V_f}{p_0} - V_{10}\right)$$

$$W = \int_{p_0}^{p_f} pdV_1 + p_f V_f\left(\ln\frac{p_f}{p_0} - 1\right) + p_0 V_{10}$$

$\int_{p_0}^{p_f} pdV_1$ varies for inflation/deflation – *determined from the graph*

Baloon stretching – theoretical curve

- elastic energy
  - $U = 4\pi r_0{}^2 \kappa RT \left(2\lambda^2 + \frac{1}{\lambda^4} - 3\right)$
    - $\lambda = \frac{r}{r_0}$ relative strain
    - $\kappa - rubber\ property$



- work needed to increase radius from r to $r+dr$ *under pressure difference* ΔP *(being overpressure)*

$$dW = \Delta P dV = \Delta P 4\pi r^2 dr = \left(\frac{dU}{dr}\right) dr$$

$$dV = \frac{d(\frac{4}{3}r^3\pi)}{dr} = 4\pi r^2 dr$$

Baloon stretching – theoretical curve

$$elastic\ energy\ U = 4\pi {r_0}^2 \kappa RT\left(2\lambda^2 + \frac{1}{\lambda^4} - 3\right)$$

$$dW = \Delta P 4\pi r^2 dr = \left(\frac{dU}{dr}\right) dr = 16\pi\kappa RT\left(r - \frac{{r_0}^6}{r^5}\right) dr$$

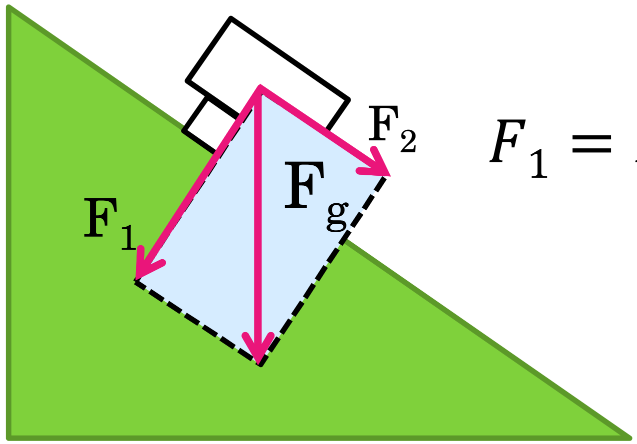
$$\Delta P = 4\kappa RT\left(\frac{1}{r} - \frac{{r_0}^6}{r^7}\right) = \frac{4\kappa RT}{r_0}\left(\frac{r_0}{r} - \frac{{r_0}^7}{r^7}\right)$$

$$\Delta P = \frac{4\kappa RT}{r_0}\left(\frac{1}{\lambda} - \frac{1}{\lambda^7}\right)$$

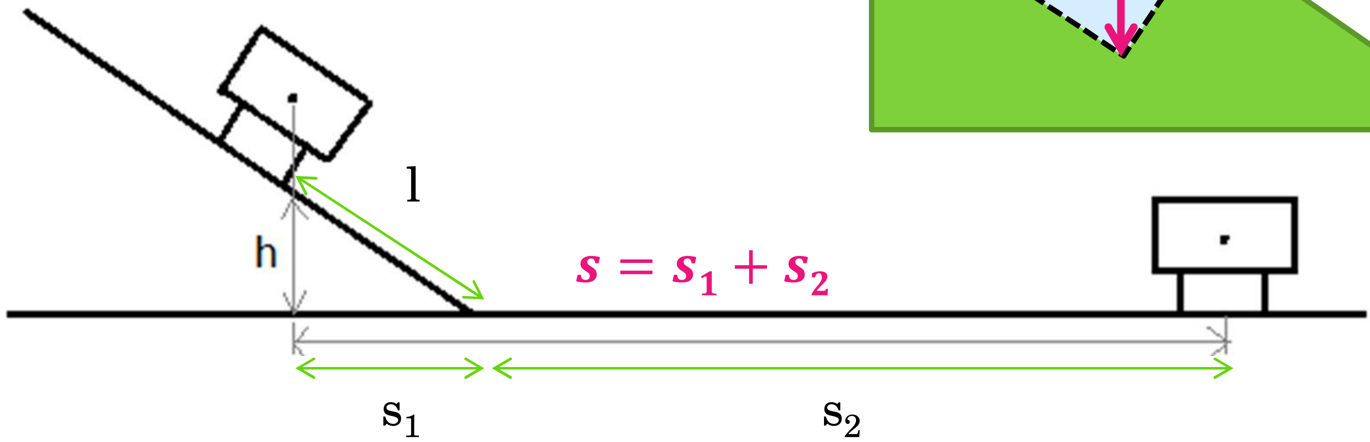
final expression

$$\Delta P = \alpha\left(\frac{1}{\lambda} - \frac{1}{\lambda^7}\right)$$

FRICTION



$$F_1 = Fg\frac{s_1}{l}$$



$$mgh = F_1\mu l + Fg\mu s_2$$

$$mgh = mg\mu\frac{s_1}{l}l + mg\mu s_2$$

$$h = \mu(\boldsymbol{s_1} + \boldsymbol{s_2})$$

h - car-surface distance
s - horizontal travelled distance

$$\mu = \frac{h}{s} = 4.5 \cdot 10^{-3}$$