PROBLEM NO. 5: "CAR"

Build a model car powered by an engine using an elastic air-filled toy balloon as the energy source. Determine how the distance travelled by the car depends on relevant parameters and maximize the efficiency of the car.
OVERVIEW

- balloon observations
  - piston inflation/deflation model
    - energy input and output
    - rubber stretching losses

- construction

- observed motion

- parameters
  - initial conditions (volume, pressure)
  - jets, nozzles
  - maximised travelled distance and efficiency

- conclusion
PISTON MODEL ~ ENERGY

- piston contains all air later used for inflation/deflation
  - slowly pushed – balloon inflates
  - released piston – balloon deflates

\[ V_{10} = V_{piston} \]
\[ p_0 = p_{atm} \]
\[ p_0(V_{10} + V_{20}) = nRT = \text{const} \]
initial state

\[ V_{1f} \]
\[ V_{2f} = 0 \]

\[ p_f V_{1f} = nRT \]
final state

- at every moment ideal gas law equation states:

\[ p(V_1 + V_2) = nRT \]
\[ R \quad \text{gas constant} = 8.314 \frac{J}{molK} \]

\[ dW = -(p - p_0)dV_2 \]
integrating initial to final state
PISTON MODEL ~ ENERGY

- maximal available energy
  - deflation work
- same equations valid

\[ W = \int_{p_0}^{p_f} p \, dV_1 + p_f V_f \left( \ln \frac{p_f}{p_0} - 1 \right) + p_0 V_{10} \] known conditions

\[ \int_{p_0}^{p_f} p \, dV_1 \] varies for inflation/deflation – determined experimentally
PRESSURE / VOLUME RELATION INFLATION AND DEFLATION

- pressure
  - digital pressure sensor
- volume
  - photos taken at known pressure
  - balloon edge coordinates
    - balloon shape determined
  - balloon radius/height function
    - numerically integrated
  - shape rotation – body volume
Typical pressure/volume relation

- Deflation curve is under the inflation curve.
- Energy partly lost due to rubber deformations.

Graph:
- Overpressure [kPa] on the y-axis.
- Volume [cm³] on the x-axis.
- Red dots represent inflation.
- Black dots represent deflation.

Table:

<table>
<thead>
<tr>
<th>Volume [cm³]</th>
<th>Overpressure [kPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1000</td>
<td>1</td>
</tr>
<tr>
<td>2000</td>
<td>4</td>
</tr>
<tr>
<td>3000</td>
<td>8</td>
</tr>
<tr>
<td>4000</td>
<td>10</td>
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</tbody>
</table>
Balloon stretching – theoretical curve

- pressure (force acting on a surface)
- balloon elastic energy
  - \( U = 4\pi r_0^2 \kappa RT \left(2\lambda^2 + \frac{1}{\lambda^4} - 3\right) \)
  - \( \lambda = \frac{r}{r_0} \) relative strain
  - \( \kappa \) – rubber property
- work needed to increase radius from \( r \) to \( r + dr \) under pressure difference \( \Delta P \) (being overpressure)
  
  \[ dW = \Delta P dV = \Delta P 4\pi r^2 dr = \left(\frac{dU}{dr}\right) dr \]

\[ \Delta P = \frac{4\kappa RT}{r_0} \left(\frac{1}{\lambda} - \frac{1}{\lambda^7}\right) \]

Balloon stretching

\[ \Delta P = \alpha \left( \frac{1}{\lambda} - \frac{1}{\lambda^7} \right) \] — single parameter (a rubber property) fit

- obtained curve has the expected maximum
- due to surface(force) relation
- explains and confirms the obtained shape of the overpressure/strain curve
CONSISTENCY OF MEASUREMENTS

- the stress-strain curve depends on the maximum loading previously encountered (Mullins effect)
  - later measurements – rubber already deformed
    - occurs when the load is increased beyond its prior value
    - less pressure difference needed to inflate to a certain volume

- pV relations
  - same balloon – multiple usage
  - initial volume (pressure)

- OBSERVATION
  - pV graph for 1st and 10th measurement greatly differ
CONSISTENCY OF MEASUREMENTS

- 1st and 21st measurement vary greatly
- rubber deforms after few measurements
- 10th-25th measurement comparable (similar)
- afterwards balloon breaks
**Initial Volume**

- Surface under the curve enlarges with initial volume
  - Determining part of total input and output energy
- Efficiency is determined by the deflation and inflation surfaces under the curve
  - Ratio between \((\text{initial} + \text{final conditions}) + \text{surface under the curve}\)
**INITIAL VOLUME**

- **input energy**
  - inflation

\[
\text{inflation/deflation varies} \quad \eta = \frac{E_d}{E_i}
\]

![Graph showing input energy and deflation/inflation efficiency against initial volume.](image-url)
CONSTRUCTION

- plastic cart (length ~ 12cm)
- styrofoam
- plastic tube (d = 2.9 cm)
- metal jets / cardboard nozzles
- car mass ~120g
**Observed Motion - Setup**

- **Rails**
  - Car moving straight
  - Position determination
  - Length 5 m

- **Video**
  - Camera 120 fps
  - Placed 10 m from the rails

- **Analysis**

- **Time / Distance Coordinates**
### Observed motion

- Distance/time graph obtained from video coordinates
- Program time-distance graph:
  - 5 data points frame – cubic or linear curve fitted
  - Derived: one point in the v-t graph
  - Moves one point to the next frame

![Graph showing distance and velocity over time](image-url)
**TRAVELLED DISTANCE**

- **travelled distance:** $s = s_0 + s_1$
  - acceleration path
    - $v_{\text{max}}$ time
    - distance $s_0$
    - determined from the video
  - decceleration
    - $a$ determined from the graph
    - linear fit coefficient
  - decceleration path
    $$s_1 = \frac{v_{\text{max}}^2}{2a}$$

![Graph showing velocity over time with data points and a fitted line.](image-url)
Basic working principle

- Input energy is used for inflation
- Deflation transforms it into kinetic energy of the air molecules
  - Losses due to resistance to movement (mutual collisions)
  - Momentum conservation
    \[ d(m_a v_a) - F_{fr} dt = m_c dv_c \]
- Simplified model: air behavior
  - Mass inside the balloon \( \bar{p}V = \frac{m_a}{M}RT \to m_a = \bar{p}V \frac{M}{RT} \)
  - Velocity according to Bernoulli’s principle (valid for turbulent flows)
    \[ \frac{\rho v_a^2}{2} = \bar{p} \to v_a = \sqrt{\frac{2\bar{p}}{\rho}} \]
**Basic Working Principle**

- **simplified model:** \( m_a v_a - F_{fr} t = m_c v_c \)
  - friction force
    - main friction force source is the air drag \( F_{fr} = \frac{1}{2} C \rho S v_c^2 \sim V_0^2 v_c^2 \)
    - friction force duration - flow rate \( Q = \frac{v}{t} = Sv a \rightarrow t \sim \frac{v}{v_a} \)

- \( \bar{\rho} \) is evaluated from \( pV \) graphs as \( \bar{\rho} = \frac{\int_{p_0}^{p_f} pdv_1}{V_f - V_0} \)
- maximum car velocity \( v_c \) initial \( V_0 \) volume relation obtained
**Initial Volume**

- simplified model basic working principle fit
  - main friction force source is air drag

\[ m_a v_a - F_r t = m_c v_c \]
MAXIMISATION – $V_{\text{MAX}}$ IMPORTANCE

travelled distance

- motion consists of acceleration and decceleration
- acceleration to $V_{\text{max}}$ is determined by:
  - initial conditions
    - volume/pressure
  - jets, noozles
  - travelled distance found from the video
- decceleration – same for all parameters
  - friction
    - air
    - wheels/rails/shaft
  - $V_{\text{max}} = \sqrt{2as} \rightarrow s = \frac{V_{\text{max}}^2}{2a}$

efficiency

- energy input and output ratio
  - $\eta = \frac{\text{energy input}}{\text{energy output}}$

- energy input
  - work needed to inflate the balloon
    - piston model

- energy output
  - $E_k = \frac{m_{\text{car}}V_{\text{max}}^2}{2}$
    - $V_{\text{max}}$ car motion observation
PARAMETERS

- initial volume
  - total input energy
  - rubber stretching losses

- jet diameter
  - volumetric flow of air $Q$
    - Reynolds number $10^5$
    - turbulent flow
  - exit velocity
    - deflation time

- nozzle
  - directs the flow
INITIAL VOLUME

- initial volume! – pressure
- total input energy and deflation energy
  - critical sizes
    - min
      - needed to overcome static friction - start car movement
    - max
      - radius ~ car height
      - the balloon must not touch the floor
      - range ~ 0.15 dm$^3$ - 4.5 dm$^3$
      - deflated balloon ~ 0.075 dm$^3$
- air drag friction force
INITIAL VOLUME

RESULTS

- travelled distance: \( s = s_0 + s_1 \)
- efficiency:
  - output/input energy \( \eta = \frac{mv_{max}^2}{2Ei} \)
  - small losses on the air kinetic energy

- maximisation largest balloon
- maximisation smallest balloon
JETS - CIRCULAR TUBE OPENINGS

- different diameters \( d \)
  - deflation time \( t(d) \gg t(D) \)

- jet diameter changes
  - how well the air is directed
  - time needed for deflation
    - air drag duration
  - amount of resistance the air endures in mutual collisions
  - Reynolds number \( Re = \frac{vd}{\nu} \), \( \nu \) kynematic viscosity
    - turbulent flow \( Re > 4000 \), \( 10^5 \)
RESULTS (V~4DM3)

- different $v_{\text{max}}$ for a certain diameter
  - both travelled distance and efficiency depend on $v_{\text{max}}^2$

max distance 47.3 m

max efficiency 2.7%

best jet diameter 1.2cm-1.4cm
NOZZLES – CONE SHAPED

different nozzles change
- angle – how well the air is directed
- tube length – amount of losses due to friction
- 6 cones – different angles $\alpha$ and thus lengths ($7.5^\circ$ - $30^\circ$)

effective velocity – horizontal component of rapid molecule movements
RESULTS ($V \sim 3Dm^3$) $D=1cm$

- different $v_{max}$ for a certain diameter
  - both travelled distance and efficiency depend on $v_{max}^2$

max distance 20.5 m  
max efficiency 1.5%

![Graph showing travelled distance and efficiency vs. cone angle]
CONCLUSION

- piston inflation/deflation model – energy evaluation
- stretching losses – theoretical and experimental curve
- basic working principle explained
- found maximum conditions for jets and nozzles

<table>
<thead>
<tr>
<th>travelled distance</th>
<th>efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>initial V =4.5dm³ max</td>
<td>initial V=1.5dm³ min</td>
</tr>
<tr>
<td>optimal:</td>
<td>optimal:</td>
</tr>
<tr>
<td>• jet diameter 1.2cm</td>
<td>• jet diameter 1.2cm</td>
</tr>
<tr>
<td>• nozzle angle 20°</td>
<td>• nozzle angle 20°</td>
</tr>
<tr>
<td>DISTANCE 70m</td>
<td>EFFICIENCY 6.4%</td>
</tr>
</tbody>
</table>
Thank You!
PISTON MODEL ~ ENERGY

\[ V_{20} = V_{\text{piston}} \]

\[ p_0 = p_{\text{atm}} \]

\[ p_0(V_{10} + V_{20}) = nRT = \text{const} \]

\[ (V_1 + V_2) = nRT \]

\[ -dW = (p - p_0)dV_2 \]

\[ W = \int_{p_0}^{p_f} pdV_1 + p_f V_f \left( \ln \frac{p_f}{p_0} - 1 \right) + p_0 V_{10} \]

\[ \int_{p_0}^{p_f} pdV_1 \text{ varies for inflation/deflation – determined from the graph} \]
PISTON MODEL ~ ENERGY

\[ V_{10} \quad \text{p}_0 = p_{\text{atm}} \]

\[ p_0 (V_{10} + V_{20}) = nRT = \text{const} \]

\[ p(V_1 + V_2) = p_f V_{1f} \]

\[ dV_1 + dV_2 = - \frac{p_f V_{1f}}{p^2} dp \]

\[ -dW = (p - p_0) dV_2 \]

\[ -dW = p \left( -dV_1 - \frac{p_f V_{1f}}{p^2} dp \right) - p_0 dV_2 \]

\[ \int_{p_0}^{p_f} \]
PISTON MODEL ~ ENERGY

\[-dW = p \left( -dV_1 - \frac{p_fV_1f}{p^2} dp \right) - p_0 dV_2\]

\[-dW = -pdV_1 - \frac{p_fV_1f}{p} dp - p_0 dV_2 \quad \int_{p_0}^{p_f} \]

\[W = \int_{p_0}^{p_f} pdV_1 + p_fV_f \ln \frac{p_f}{p_0} + p_0(0 - V_{20}) \quad p_0(V_{10} + V_{20}) = p_fV_f\]

\[W = \int_{p_0}^{p_f} pdV_1 + p_fV_f \ln \frac{p_f}{p_0} - p_0 \left( \frac{p_fV_f}{p_0} - V_{10} \right) \]

\[W = \int_{p_0}^{p_f} pdV_1 + p_fV_f \ln \frac{p_f}{p_0} - 1 + p_0 V_{10} \]

\[\int_{p_0}^{p_f} pdV_1 \text{ varies for inflation/deflation – determined from the graph}\]
Balloon Stretching – Theoretical Curve

- **elastic energy**
  - $U = 4\pi r_0^2 \kappa RT \left(2\lambda^2 + \frac{1}{\lambda^4} - 3\right)$
  - $\lambda = \frac{r}{r_0}$ relative strain
  - $\kappa$ – rubber property

- work needed to increase radius from $r$ to $r+dr$ under pressure difference $\Delta P$ (being overpressure)

$$dW = \Delta P dV = \Delta P 4\pi r^2 dr = \left(\frac{dU}{dr}\right) dr$$

$$dV = \frac{d\left(\frac{4}{3}r^3 \pi\right)}{dr} = 4\pi r^2 dr$$
Balloon stretching – theoretical curve

elastic energy $U = 4\pi r_0^2 \kappa RT \left( 2\lambda^2 + \frac{1}{\lambda^4} - 3 \right)$

$dW = \Delta P 4\pi r^2 dr = \left( \frac{dU}{dr} \right) dr = 16\pi \kappa RT \left( r - \frac{r_0^6}{r^5} \right) dr$

$\Delta P = 4\kappa RT \left( \frac{1}{r} - \frac{r_0^6}{r^7} \right) = \frac{4\kappa RT}{r_0} \left( \frac{r_0}{r} - \frac{r_0^7}{r^7} \right)$

$\Delta P = \frac{4\kappa RT}{r_0} \left( \frac{1}{\lambda} - \frac{1}{\lambda^7} \right)$

final expression

$\Delta P = \alpha \left( \frac{1}{\lambda} - \frac{1}{\lambda^7} \right)$
**Friction**

\[ F_1 = Fg \frac{s_1}{l} \]

\[ mgh = F_1 \mu l + Fg \mu s_2 \]
\[ mgh = mg \mu \frac{s_1}{l} l + mg \mu s_2 \]
\[ h = \mu (s_1 + s_2) \]

- \( h \) - car-surface distance
- \( s \) - horizontal travelled distance

\[ \mu = \frac{h}{s} = 4.5 \cdot 10^{-3} \]