prob2.tex **Problem 2**

Boiling Water

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1 Introduction

This problem could be solved either in theoretical or experimental way. Both of these possibilities may reveal satifactory results. However, since physics is one of natural sciences which try to join these two points of view, we made some experiments and consequent conclusions which we succeeded to support with a theoretical model.

2 Experimental part

The first experiment is determined already by the formulation of the problem. We measured time necessary for bringing water to boil in both cases (with and without a lid). Some of the experimental results are presented in Figs. 1 and 2. When we assume that the power input does not change in the measurement period, it means that the ratio of time saved is the same as the one of the energy. Thus we found out that while we are using a lid we spare approximatelly 20% of time. At this point we should at least scan through the problematics of the type of the heater. Within our experiments we used a gas heater and an electric heater. In the case of the gas heater we were able to save about 15% and using an electric one approximatelly 25% of time. Within the theoretical part we limited our solution just to the electric type since in such case it is easier to describe the heat transfer between the pot and the hot plate.

Another experiments were carried out as well. First we determined the total mass losses during the experiment (i.e. how much steam escapes out of the pot). This value helped us in confirming partial results of the theoretical solution. Just for completeness: when using a lid the losses were 2 grams per initial mass of 200 g, in the other case the losses were much higher — 11 g per 200 g.

Except of these basic experiments we were to determine plenty of material constants. One of the most important ones is the thermal conductivity of the pot. The experimental setup was very simple: known amount of water of a certain temperature was warmed up by, say, 10 kelvins in a pot of a given size. Then, according to relation

$$\lambda = \frac{c_m ml}{tS\ \Delta T} \tag{1}$$

($c_m = 4.2 kJkg^{-1}K^{-1}$ is the specific heat of water, m is the mass of water, l the thickness of the wall of the pot and S the area of its bottom, t is time taken to warm up the water by ΔT) we got the sought value of the thermal conductivity $\lambda = 0.4 Wm^{-1}K^{-1}$.

3 Theoretical part

3.1 Model of heat transfer to the pot

It is obvious that the most important role in the warming up the pot (including water) will play the heater's hot plate. Nevertheless, ambient air is a body with thermal capacity, too. This means there can be some heat flow from hot air to the pot or vice versa (in case of colder air). However, the temperature of air (derivable somehow from the temperature of the hot plate) and its flow is very hard to be described and so we tried to neglect the heat transfer between ambient air and the pot. We felt competent to do it because in general heat transfer between a gaseous phase and solid non-evaporating phase is very slow.

Another thing is the real amount of heat transferred to the pot. So as to evaluate it we needed to know the efficiency of the heater in the particular case. Knowing the power input of the heater we performed an experiment similar to the last one mentioned — water in a pot was heated up by ΔT in time interval t using a heater with power input P. Then its efficiency was

$$\eta = \frac{c_m \Delta T}{Pt}. \tag{2}$$

However, in a further stage of our solution we found that it is also a simplification to consider the efficiency of the heater to be constant and so we rather considered the hot plate as having constant temperature. Using the temperature difference we derived the heat flow.

3.2 On the problem of water-vapour pressure over the free surface of water

The pressure of vapours over the free surface of liquid is a quantity which characterises the equilibrium between the liquid and the gaseous phase of a substance. It depends mainly (i) on the kind of liquid and (ii) on the temperature. The Augustus and Clausius-Clapeyron equations are the ones which describe the dependence. From the last equation we get:

$$\ln \frac{p_1}{p_2} = -\frac{H_{\mathrm{evap}}}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right), \tag{3}$$

$$p = p_{\mathrm{atm}} \exp\left[-\frac{H_{\mathrm{evap}}}{R}\left(\frac{1}{T_1} - \frac{1}{T_{\mathrm{e}}}\right)\right], \tag{4}$$

where p_1, p_2, p are vapour pressures at temperatures T_1, T_2, T respectively, T_{e} is the boiling point under atmospheric pressure p_{atm}, R is the universal gas constant $(8.314 JK^{-1} mole^{-1})$ and H_{evap} is the molar evaporation enthalpy, equal to $42 kJ mole^{-1}$ for water. Having found out the partial pressure p of water vapour it is not very hard to determine the maximum absolute air humidity Φ_{max} according to the state equation:

$$\Phi_{\mathrm{max}} = \frac{p M_v}{RT} \tag{5}$$

(M_v is the molecular mass of water — $0.018 kgmol^{-1}$).

Note: the absolute air humidity Φ and relative air humidity ϕ fulfil the relation:

$$\Phi = \Phi_{\max}\phi \ . \tag{6}$$

3.3 Models of steam leakage

3.3.1 Pot without a lid

In this case steam escapes from the pot freely, the energy losses due to steam leakage are determined just by the rate of steam flow and the absolute humidity of it. So as to make a mathematically useful model we supposed the pot to be full, i.e. the surface of water is just at the top of the pot. We assume, that a quasistable state is set, in other words, the rate of steam flow, as well as its humidity after some not very long time, are constant. The latter is to be determined first. From the law of mass conservation follows that the amount of molecules evaporated in a given time interval must be equal to the sum of the amounts of recondensed and leaked molecules[1]. The first two quantities can be evaluated from the formula[2]:

$$\frac{\pi r^2 p_{\text{part}}}{\sqrt{2\pi M_v RT}} = \frac{1}{4}\pi r^2 v_a \Phi + n_{\text{leak}}, \tag{7}$$

where p_{part} is the temperature-dependent partial pressure of water vapour and v_a is the mean arithmetic velocity of molecules in gaseous phase (also temperature-dependent). The n_{leak} is the sought amount of molecules which leak per unit of time. The only other unknown variable is the absolute humidity of air within this equation. Now let us determine it. First, we will find out a Φ-dependent formula for n_{leak}. Let us consider an element of steam. The total force acting upon it is the sum of the gravitional force and the upper-lifting one:

$$\mathrm{d}\mathbf{F} = -\nabla p \cdot \mathrm{d}V - \rho \mathbf{g} \mathrm{d}V \ , \tag{8}$$

[1] Assuming that the air humidity does not change.

[2] see: L. Pátý, *Fizika niských tlaku*, *(Physics of low pressures)*, Academia, Praha 1968, pp. 86–89.

where ρ is the density of air, $\mathbf{g}$ the gravitional acceleration and p the atmospheric pressure in a given point. From this equation we come to a resulting relation

$$\mathrm{d}\mathbf{F} = -\frac{p_0\mathbf{g}}{R}\left(\frac{M_d}{T_0} - \frac{M_r}{T}\right)\mathrm{d}V \ , \tag{9}$$

where T_0 is the temperature of ambient air and T is the temperature of the air element[3], p_0 is the atmospheric pressure and

$$M_r = \frac{M_{\mathrm{O}_2}w(\mathrm{O}_2) + M_{\mathrm{N}_2}w(\mathrm{N}_2) + M_{\mathrm{H}_2\mathrm{O}}w(\mathrm{H}_2\mathrm{O})}{w(\mathrm{O}_2) + w(\mathrm{N}_2) + w(\mathrm{H}_2\mathrm{O})} \tag{10}$$

($w(x)$ stands for the molecular ratio of the particular component in air). M_d can be computed in a similar way — dry air is however supposed to have no water in itself, and thus $w(\mathrm{H}_2\mathrm{O}) = 0$.

Now, the force acting upon an element of air is integrated over its volume (the element is treated as a hollow cylinder of infinitesimal thickness and real height). At this point let us discuss the model of leaking steam. The steam column is supposed to be a cylinder. The cylinder is composed of a system of coaxial hollow cylinders of infinitesimal thickness. The rate of flow in a single cylinder of the last mentioned ones is the same all over its body. If v is the cylinder-radius dependent rate of flow, then according to Newton's postulate we can write:

$$\tau = \frac{\partial v}{\partial r}\eta, \tag{11}$$

where τ is the tension in tangential direction (parallel to the axis of the cylinder), η the air dynamic viscosity ($1.71 \times 10^{-5} Pas$). The tension caused by different flow rates in different layers multiplied by the area where this tension acts (surface of the cylinder) reveals a force which, however, must be compensated by a lifting force in a quasistable state. This equation can be used for determination of the v function and the total volume flow per unit of time, consequently.

[3] Obtained from the weighted mean $(\Phi T + \rho T_0)/(\Phi + \rho)$.

The final equation is:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \phi \frac{\pi r^4}{8\eta} \frac{p_0|\mathbf{g}|}{R} \left(\frac{M_d}{T_0} - \frac{M_r}{T} \right) . \tag{12}$$

The left side of this equation is equal to n_{leak} (see (**??**)). Now, the equation (**??**) is complete. It is a differential equation which is too difficult for analytical solution. Therefore a numerical solution appeared to be the best one for us. We found that the value of ϕ is all the time almost equal to 1 ($\phi > 0.99$), i.e. the steam is almost saturated. This idea is also supported by the fact that, in reality, the steam is white, i.e. it condensates.

Now we are able to determine mass losses rate. Let us take into account, that every molecule that leaks out of the pot has to evaporate[4]. This means, the specific evaporation heat multiplied by mass losses per unit of time gives the energy losses per time unit.

3.3.2 Pot with a lid

A model is in this case much simpler than in the previous one. It is supposed that all steam losses are caused by the thermal expansion of air and by increase of the maximum absolute air humidity. The steam within the pot is considered to be saturated (if in the previous case the vapour was saturated too, then now, when the air is in a partially closed space, the relative humidity should be even higher). Thus we easily come to a formula for energy losses per change of temperature (vapour pressure, respectively)[5]:

$$\mathrm{d}Q = l_{\mathrm{e}} V_{\mathrm{hr}} \left(\frac{\mathrm{d}p\, M}{RT} + \beta \Phi \,\mathrm{d}T \right) , \tag{13}$$

where l_{e} is the specific evaporation heat of water, V_{hr} is the volume of air in the pot, β the coefficient of thermal volume-expansion of air.

[4] Its warming up by approximatelly $80K$ needed negligibly small energy compared to the evaporation.

[5] We use the same relation between mass and energy losses as at the end of the previous paragraph.

3.4 Conclusion of total energetic losses

The last type of energy losses which has not been mentioned herein is the heat radiation. If the pot radiates into a half space, the sum of the energy it radiates at temperature T and the energy it receives from the environment (air) of temperature T_0 (everything per unit of time) is

$$P = (1 - a)S\ \sigma(T^4 - T_0^4), \tag{14}$$

where a is the albedo of the pot, S is area of its surface, σ is equal to $5.670 W m^{-2} K^{-4}$.

Now we come into the very last stage of the theoretical solution. The conservation law of energy means that all energy acquired by the pot from the heater must turn either to change of temperature or to energy losses. We are already able to substitute all these three types of energies with time- and temperature-dependent expressions. If we would integrate the resulting equation with respect to time, we would get a time-dependent temperature function, which would easily show when water reaches the given temperature. Actually, this equation is again analytically insolvable. It means, we had to solve it numerically using a computer. The results obtained for the cases without and with a lid are presented in Figs. 3 and 4. They show that time necessary to bring water to boiling is by approximatelly 20% longer in the case of no lid.

4 Summary

First we did some experiments according to the formulation of the problem. They showed that water can be brought to boiling about 20% faster in a pot with a lid than in the pot without a lid. Then we created two theoretical models — one for a pot with a lid, another for a pot without a lid. The result of the considerations concerning these models were time-dependent temperature functions. They were in a good agreement with the experimentally obtained data and in global result showed the time saved to be approximatelly 20%.

Figure 1: Temperature of water versus time. Experimental results for $0.2kg$ of water heated by a gas heater in a pot without a lid.

Figure 2: Temperature of water versus time. Experimental results for $0.2kg$ of water heated by an electric heater in a pot without a lid.

Figure 3: Time dependence of temperature calculated on a basis of the theoretical model for a pot without a lid ($m = 0.2kg$)

Figure 4: Time dependence of temperature calculated on a basis of the theoretical model for a pot with a lid ($m = 0.2kg$)

prob3.tex **Problem 3**

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Drop

The task was to explain a phenomenon of arising a system of rings when drying up a drop of salted water. An experiment was carried out to investigate this problem. It consisted of a number of smaller experiments in which drops of salted water were dried up under different conditions. Following factors were taken in consideration when preparing experiments: