prob5.tex **Problem 5**

SOUND

Thomas Bark, Marcus Meisinger, Wolfgang Kastaun, Carsten Geckeler

The task in the problem "Sound" was to transform the energy of a charged capacitor into sound with the highest possible efficiency. The $100\mu F$ capacitor is charged up to $30V$ and certainly no external source of power is allowed. According to formulas

$$Q = UC \;, \qquad E = \frac{1}{2}U^2C \;,$$

(C — capacitance of a capacitor, U — voltage on a capacitor, Q — charge in a capacitor, E — energy stored in a capacitor) the capacitor contains a charge of $Q = 3.0 \times 10^{-3}C$ and the stored energy is $E = 0.045J$. In order to illustrate: with this energy one may warm up one cubic centimeter of water by about $0.01K$.

We developed various types of electronic circuits, that were fed by the capacitor. The most important part of such circuits is the electro-acoustic transducer, i.e. the component that transforms the electric energy into sound. Soon we found out, that ordinary, so called dynamic loudspeakers are not appropriate for this task. They consist of a coil that is mechanically connected to the membrane. The coil is placed in a pot-like magnet. The problem is, that they need a relatively high current to produce an audible sound. But the capacitor cannot deliver high current for a long time, because it contains only little charge.

So we decided to use a piezo-loudspeaker (called simply "piezo"). They are well-known from electronic watches, their particular features are low "consumption" and high efficiency. The piezo is a capacitor with a special crystal as dielectric. If it is charged in one direction (one polarity), the crystall contracts, charged in the other way (reverse polarity), the crystall expands. By this deformation, electronic oscillations may be transformed into sound waves.

Crystals that show piezoelectric effect must possess so called polar axis. Around such an axis one finds rotational symmetry, both directions of this axis are not equivalent. One of well-known piezoelectrics is baryum titanate. At temperatures higher than the ferroelectric Curie-temperature it exists in the cubic form. The crystal has symmetry axes but no polar one and therefore this form is not piezoelectric. At the Curie-temperature the crystal undergoes a transformation. Below this temperature two crystal sublattices (anions and cations) are displaced a bit in one of the six possible directions (tetragonal form). This displacement produces an electric field in the crystal. If this field is increased or decreased by an outer electric field, the crystal expands or contracts. It is important to note, that the relative change in a crystal size is proportional to the external electric field $\mathcal{E}$ (some other crystals reveal much weaker electrostriction efect which is proportional to $\mathcal{E}^2$.)

As said before, the piezo is a special type of capacitor. When one charges it, it makes sound. But after this, it already contains the greater part of the energy, that was put into it, stored in its electric field. We have developed two ways to "recycle" this energy:

1. The simplest way is to discharge the piezo by a short-circuit after it has been charged. Nearly the same sonic energy as during charging is set free.

2. The most effective way is to build an LC resonant-circuit with the piezo as a capacitor and a coil. If one could exclude ohmic resistance and other losses, the whole energy in the piezo would finally be transformed into sound.

Fig. 1

Figure 1 shows the circuitry with such an LC-circuit. It is a Meissner oscillator, which works on a base of the feedback system. The energy swaps from inductance $L1$ to the piezo and back. Losses of energy in this circuit are compensated by the help of the transistor. It allows some small quantities of energy from the main capacitor to flow into the LC-circuit. The transistor is controlled by the inductance $L2$. The frequency of such an oscillator is

$$f = \frac{1}{2\pi\sqrt{L_1 C}} \ .$$

Unfortunately, the losses in the inductance $L1$ are relatively high. Since the capacity of the piezo is small ($110nF$), the inductance must be relatively big. This means that the wire in the coil must be long and that an iron-core is needed. Both are sources of losses, that result in a high consumption of the circuit. After 2-3 seconds the energy of the main capacitor is used up.

The next circuitry, presented in Fig. 2, works according to the first method of "energy recycling". It consists of two parts: the left part is an astable multivibrator, which creates the alternating current. The two outputs oscillate in paraphase (push-pull system) and control the right part.

Fig. 2

The clock-signal generator (left part) is the most common and simple type of multivibrator. If output #1 is at high level, the piezo is charged over transistor T3. Half a period later output #2 is at high level. Now, by transistor T4, the base of the PNP transistor T5 is connected to the ground. It is switched on and the piezo discharges over T5 (short circuit). This circuitry is relatively loud, it is also content with little power. It works for 9 seconds with decreasing sound volume. This circuitry was more effective than the other one and we used it for the measurements and calculations of efficiency.

If one has enough data about a piezo, one can calculate the amount of produced sound energy and, consequently, the efficency of the device.

As mentioned before, the contraction y of the piezo is proportional to the electric field $\mathcal{E}$ and to the thickness d of the crystal (α

is called piezoelectric coefficient):

$$y = \alpha d\mathcal{E} = \alpha U \tag{15}$$

If the piezo is connected to the source of sinus-shaped alternating-current, the elongation of the crystal changes according to harmonic oscillations and sound waves are emitted .

Fig. 3

Let us consider a small volume element $\mathrm{d}V = A\mathrm{d}x$ (A is the crystal surface perpendicular to the x-axis, as seen in Fig. 3). For the coordinate $x = x_1$ the pressure is p_1. For $x_2 = x_1 + \mathrm{d}x$ it is $p_2 = p_1 + \mathrm{d}p$. The force acting onto the left side is $F_1 = p_1 A$, on the right side — $F_2 = p_2 A$. The resulting force is

$$F = F_1 - F_2 = A\mathrm{d}p\ .$$

This force causes acceleration of the volume element according to Newton's law. If y denotes the displacement, the acceleration is $a = \mathrm{d}^2/\mathrm{d}t^2 = y''$. Mass of the element is $\mathrm{d}m = \rho\mathrm{d}V$, where ρ — material density. From Newton's law results

$$\begin{aligned} -A\mathrm{d}p &= \mathrm{d}my'' = \rho\mathrm{d}Vy'' = \rho A\mathrm{d}xy'', \\ -\mathrm{d}p &= \rho\mathrm{d}xy''\ . \end{aligned}$$

As we mentioned, y conforms to harmonic oscillations:

$$\begin{aligned} y &= y_0 \sin 2\pi(t/T - x/\lambda)\ , \\ y'' &= -4(\pi/T)^2 y_0 \sin 2\pi(t/T - x/\lambda)\ . \end{aligned}$$

T denotes here the oscillation period, t — time, λ — the wavelength, y_0 — the amplitude. So we get

$$\mathrm{d}p = 4(\pi/T)^2 \rho y_0 (\sin 2\pi(t/T - x/\lambda))\mathrm{d}x\ .$$

Integration gives:

$$p = p_{\rm A} + (2\pi/T^2)\rho y_0 \lambda \cos 2\pi(t/T - x/\lambda) \ ,$$

where $p_{\rm A}$ is an integration constant. Reasonably, it is set equal to the atmospheric pressure. The sound pressure is superimposed on the atmospheric pressure.

With $2\pi/T = \omega$ (circular frequency) and $\lambda/T = c$ (sound velocity) we get for the sound pressure:

$$p = \omega \rho c y_0 \cos 2\pi(t/T - x/\lambda) \ .$$

If we substitute y_0 from equation (1), we get as the amplitude of the sound pressure

$$p_0 = \omega \rho c \alpha U_0 \ ,$$

where U_0 is the amplitude of the voltage.

The intensity I of the sound is

$$I = \frac{1}{2}\frac{p_0^2}{\rho c} = \frac{1}{2}\omega^2 U_0^2 \rho c \alpha^2 \ .$$

The sound power emitted is $P = IA$.

Unfortunately, we did not have the data (A, α) about our piezo, thus we made mesurements in order to determine the sound power. We found out that the sound pressure is proportional to the voltage. But this is naturally true only for voltages not to high. That is why we first determined the "electroacoustic transducing factor" B of the loudspeaker. This is the ratio of sound pressure p to voltage U:

$$B = p/U \ .$$

The sound pressure is measured in $1m$ distance with $1000Hz$ frequency. For ideal loudspeakers factor B should be constant for all voltages. As it is difficult to measure the sound pressure directly, we measured the sound level (sound-pressure level) with a phonometer. The relationship between sound level and sound pressure:

$$PW = 10 \lg \frac{p_{\rm eff}^2}{p_h^2} = 20 \lg \frac{p_{\rm eff}}{p_h} \ ,$$

where PW is the sound level in dB, p_{eff} — sound pressure and p_h — the threshold of audibility, $p_h = 2 \times 10^{-5} Pa$.

The results of the measurements are presented in Fig. 4. The graph shows the transducing factor measured for different voltages.

Fig. 4

It is obvious that one ought to work with voltages lower than $6V$ on the piezo. If our circuitry is fed with $30V$, the capacitor gets effectively $3.3V$. In one-meter distance we measured a sound level of $PW = 79.0dB$, which corresponds to sound pressure of $p_{\text{eff}} = 0.159Pa$. According to formula

$$I = \frac{1}{2}\frac{{p_0}^2}{\rho c} = \frac{p_{\text{eff}}^2}{\rho c}$$

we get sound intensity of $I = 5.9 \times 10^{-5} Wm^{-2}$. But the loudspeaker does not emit evenly into space. It has an exponential horn and most of the energy is emitted in an angle of 20° around the middle axis (Fig. 5).

Fig. 5

The surface of an irradiated part of the sphere (of a radius r) is

$$A = 2\pi r^2 (1 - \cos\alpha) \ .$$

For $r = 1m$ and $\alpha = 20°$, we get $A = 0.379m^2$. If we multiply A by I, we get as result the sound power of our device $P = 2.24 \times 10^{-5} W$. This is the sound power if the circuitry is fed with $30V$. But the voltage on our storage capacitor decreases and that is why the sound volume also decreases. As we looked at the current our device needs for different voltages, we discovered that the impedance

Z, the apparent resistance of our circuitry keeps nearly constant ($Z = 2.94 \times 10^4\,\Omega$). This means, that the voltage U_C on the storage capacitor decreases exponentially with time t:

$$U_C(t) = 30V \cdot \exp\left(\frac{-t}{ZC}\right) ,$$

where C is capacity of the storage capacitor, like in the followig formula.

The voltage on the piezo is proportional to the supply voltage, it decreases according to the same exponential law. Similarly the sound pressure, which is proportional to the piezo-voltage. But the intensity, and so the sound power P, is proportional to the square of the sound pressure. Therefore it decreases with double speed:

$$P(t) = P_0 \exp\left(\frac{-2t}{ZC}\right) .$$

The circuit works for 9 seconds and if we want to calculate the entire sound energy, we just have to integrate $P(t)$ from 0 to $9s$:

$$\int_0^{9s} P(t)\mathrm{d}t = \int_0^{9s} P_0 \exp\left(\frac{-2t}{ZC}\right)\mathrm{d}t = \frac{-P_0 ZC}{2}\left[\exp\left(\frac{-2t}{ZC}\right)\right]_0^{9s} = 3.24\times10^{-5}J$$

The energy stored in the capacitor was $0.045J$. We get an efficiency of $\eta = 3.24 \times 10^{-5}/4.5 \times 10^{-2} = 0.072\%$.

Problems and objections raised by the opponent and answers of the reporter

1. "The loudspeaker also emitts out of the 20° angle !"

— This is true. So the efficiency is probably a bit higher. But the result is anyway not too precise, due to the inaccuracy of the phonometer.

2. "The device does not produce sinus shaped oscillations whereas the calculations require harmonic oscillation !"

— In fact it is a triangle-like oscillation (linearly increasing, perpendicular release, flat, linearly increasing ...). But any periodic function may be decomposed into sinus and cosinus functions (Fourier or harmonic analysis). They are emitted by the piezo without problems (superposition of waves).

"But this does not work for every function!"

— Some "unnatural" functions, e.g. rectangular, may not be decomposed properly, especially the edges. But the effect is small and does not change the efficiency.

3. "May one increase the efficiency by decreasing some resistances in the circuit?"

— No. If you want to charge a capacitor (piezo) to a certain voltage, always the same energy is lost in the ohmic resistance, irrespective of its value. For smaller resistances, only the time is shorter.