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prob13.tex The Hungarian team's solution for problem no. 13. at the

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13. Air lens

Lenses are usually made of solids and sometimes made of liquids. Construct an optical lens made of air in such a way that light can travel through the lens without crossing any material but air. Determine on which factors the focal length of an air lens depends.

ABSTRACT

An iron lens was constructed to investigate the problem of air lenses. It was heated up to $200-300°C$ so a focus line appeared at a distance of approximately $1m$. Two models were made to explain which are the main effects causing the lens focus. The basic idea of the first model was that the temperature inside the lens is constant, thus it behaves like an ordinary lens made from glass. However, experiments showed, that the lens works in different way, which can not be explained with the help of this model. According to the second model the temperature at the axis of the lens is definitely lower than at the walls. The experimental data was found to be in agreement with the effects predicted by the model.

# 12 INTRODUCTION — PROPERTIES OF AIR

If one wants to focus light, the light beam must be bent. This means, that the refractive index of the air must be changed. We found that heating up the air is the best way to do that.

The refractive index of a gas changes as its density changes. Since the refractive index depends on interaction between gas particles and light, it does not change when the density of gas is kept constant (Actually, the absolute refractive index of a gas is not influenced by any other of its global properties). At low density and constant pressure the absolute refractive index ($n_0$) depends on the temperature in the following way:

$$n_0 = 1 + \frac{C}{T} \ ,$$

where $C$ is a constant, its value is approximately $6.8 \times 10^{-2} K^{-1}$ .

It follows that the relative refractive index between two masses of air with their temperatures being $T_0$ and $T$ is approximately ($T_0$ is the outside temperature):

$$n_\mathrm{r} = \frac{1 + C/T}{1 + C/T_0} \simeq 1 + \frac{C}{T} - \frac{C}{T_0} \ .$$

For example the value of $n_\mathrm{r}$ at $300°C$ is $1 - 1.1 \times 10^{-4}$.

Figure 5: The structure of the lens.

Figure 6: Isothermal lines inside the lens.

# 13 THE LENS

In Fig. 1 the structure of the lens we made can be seen. The lens was made of iron and was kept at a constant temperature during the experiment. The temperature inside the lens was approximately $500 - 600K$. Through the narrow vertical hole of the lens the beam of laser light passes and, according to the experiments, it is bent inside the lens. The air flows in the lens at the sides and leaves it at the top hole. The supposed streamlines of the air are indicated in the figure. Fig. 2 shows the supposed isotherm lines inside the lens.

We carried out the following experiments with this lens: we measured the focal length of the lens and it was found to be approximately $1m$. A vertical line appeared as focus line on the screen. When air was blown into the lens through the top hole, the focus line did not change.

Figure 7: Ideal lens for the first model.

Figure 8: Ideal lens for the second model.

# 14 FIRST MODEL

The first model is built on an analogy with thin lenses. The main idea is shown in Fig. 3. There is a region inside the lens at a constant temperature with curved boundary surfaces at the sides. Since this is an approximation, we can suppose that the curvature radius of the boundary of this region is $R$, half the diameter of the lens.

From the equation for thin lenses:

$$\frac{1}{f} = (n_\mathrm{r} - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

an estimate for the focal length ($f$) can be calculated: $f \simeq 13m$ ($R_1 = R_2 = R = 2.5mm$). This result is not in agreement with the experimental data. And this is not the only shortcoming of the first model. Here, the curvatures of the boundary surfaces and thus the radius of the equivalent geometrical lens always depends on the direction of the stream. Changing the direction of the air stream would change the convergent lens into a divergent lens, so the focus line would surely disappear.

But later experiments showed that blowing air into the lens via the top hole does not change the focus at all (if the blowing is not so strong that it cools down the lens). It follows that we must find a theoretical model based on a different idea.

# 15 SECOND MODEL

This model is based on the assumption that the isothermal lines are parallel to the axis of the lens. In Fig. 4 the top view of this model can be seen.

The main difference between this and the previous model is the following: in the first model the direction of light beams changes only when they enter and leave the lens, and the temperature of air inside the lens is constant.

On the contrary, the second model claims that the heat conductivity of air is much lower. The temperature at the horizontal axis of the lens will not differ from the outside temperature because the air masses will not have time to be heated up.

To understand the idea behind this model, it must be clearly seen that light behaves like propagating waves and not like geometrical lines. A commonly used model for describing the propagation of waves is the Huygens-principle. In Fig. 5 a light beam can be seen in a region where the refractive index of the substance changes in the direction perpendicular to that of the beam (it increases in the direction of the big arrow). This phenomenon can be explained with the help of Fig. 6. The elemental waves are represented by small semicircles; these semicircles are larger where the velocity of propagation is higher (or the refractive index is smaller). The distance

Figure 9: Bending wave in a region where $n_{\mathrm{r}}$ increases in the direction of the arrow.

Figure 10: Explanation for bending.

traveled by a small elemental wave is $c\Delta t$ (velocity of propagation at that part of the substance times the interval of time it travels). It means that the wave front is bent. Consequently, the direction of propagation is altered.

Fig. 7 shows a plot of $n_{\mathrm{r}}$ as a function of $y$, the distance from the axis of the lens. The relative refractive index $n_{\mathrm{r}}$ must have a maximum at the horizontal axis of the lens (the air is the coldest there). For simplifying the calculations, we can suppose that it is a parabolic curve, thus

$$n_{\mathrm{r}} = 1 - Ky^2 \ .$$

The temperature of air changes from $T_0$ (outside temperature) to $T$ $(500 - 600K)$ as we move from the horizontal axis of the lens towards its wall. According to this, the difference of the relative refractive indexes of air at the two temperatures is $Ky^2$, and we obtain

$$K = \frac{4C}{d^2}\left(\frac{1}{T_0} - \frac{1}{T}\right) \ .$$

This gives $K \simeq 15m^{-2}$.

From these approximations the focal length of the lens can be calculated. The calculation is based on the Fermat-principle which

Figure 11: Refractive index vs. the distance from the axis of the lens.

Figure 12: Focusing: all light from the indicated angle reach the screen.

claims that any path of the light-beam (or any other wave) has a local minimum of time of traveling from the source to any point of the space. (The principle can be proved for the cases of reflection and refraction). In Fig. 8 light is collected (the paths of light are indicated); it means that the time needed for reaching the focus is the same for all beams. If it is not so, light will propagate only at that special path where the time is minimal (this way of thinking is correct only in the case when all light beams from a solid angle are collected, because there might be more local minimums of time to get from the source to the screen).

Figure 13: The parallel beams are collected into the focus point by the air lens.

Fig. 8 shows two beams of light reaching the region of the lens.

The incoming beams are parallel and they meet in the focus. The time of traveling of light is the same for the two beams, so considering that the velocity reduces by the factor of $n_{\mathrm{r}}$, we obtain:

$$l + f = l\left(1 - K\frac{D^2}{4}\right) + \sqrt{f^2 + \frac{D^2}{4}}\ .$$

For $l = 5.5cm$ we obtain the following result (the approximate solution of the previous equation):

$$f \simeq \frac{1}{2Kl} \simeq 0.6m\ .$$

The measured value of the focal length of the lens is in good agreement with the result we get from the calculations based on the second model. These results are independent from the direction of the air stream's velocity. The difference between the theory and measurement is caused by the approximations (parabolic profile of temperature, temperature inside the lens), but we found the above mentioned to be main effects.

# 16 APPENDIX

This result can be applied for improving glass fibers. The problem of the ordinary fibers is that a short impulse of light spreads out; some part of the beam is reflected many times so it takes more time to reach the end of the fiber. With this idea the problem can be solved since it takes the same time for every beam to get from one focus to other (this fiber must be constructed in a way to have no reflections; reflection would mean other local minimum).

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# 18 BIBLIOGRAPHY

*R. P. Feynman, R. B. Leighton, M. Sands*, The Feynman Lectures on Physics, vol. I, chapter 31, Reading, 1969.