6 SOLUTIONS OF THE VIITH IPYT PROBLEMS

Problem No.1 – Optics

In this task we decided to solve the problem of the maximum distinguishability of a lens telescope (known as a refractor). We used the principles of wave optics and compared theoretical results with practical observations.

Introduction

Firstly, we should explain how this type of telescope works: let's consider a stream of parallel beams of the same phase (i.e. they form a plane wave front). First they pass through a round entry slot where so called Fraunhoffer's difraction occurs, next they pass through the objective which is a converging lens with a large focal distance concentrating the beams on the focal point. There is a second converging lens behind the focal point (an eyepiece) from which the beams, which were divergent up to this point, emerge parallel. These can be observed visually. Using geometrical optics we can assume that while using an ideal objective an object at an infinite distance will be projected on to the focal plane at a single point. In reality, however, thanks to wave base of light a divergency of parallel beams in the entry slot occurs. That means that the object is appears as a disc. We will now attempt to describe it (its size and the intensity of its parts).

Fraunhoffer's diffraction on the slot

According to Huygens' principle the beam that has come through the slot propagates in all directions behind the screen. This is true in the case of the telescope entry slot as well. The beams deflected away from the optical axis will be concetrated into a single point in the focal plane, due to the objective. We are interested in its intensity in relation to the angle of their deflection from the optical axis. The evaluation will be done using the complex amplitude (it's second power is equal to the intensity of light at a certain point) – it can be expressed by planar integration of the elements of the area of the section perpendicular to the deflected optical axis. The result for a round entry slot is a so called Bessel's function. This can be approximated using goniometrical functions (e.g. $((\sin x) : x)^2$ – see Fig.9). We can see that the intensity of the points of light oscilates with increasing distance from the middle of the disc. The first minimum occurs for the angular distance $\alpha = 0.61 \,.\, 1 : D$ (1 is the wave length, D is the diameter of the objective). The other maxima follow – altough they are much weaker then the first one (Fig.9) and so they can be neglected. (The parameter of the disc is assumed to be the distance of the first minimum from the middle of the disc.)

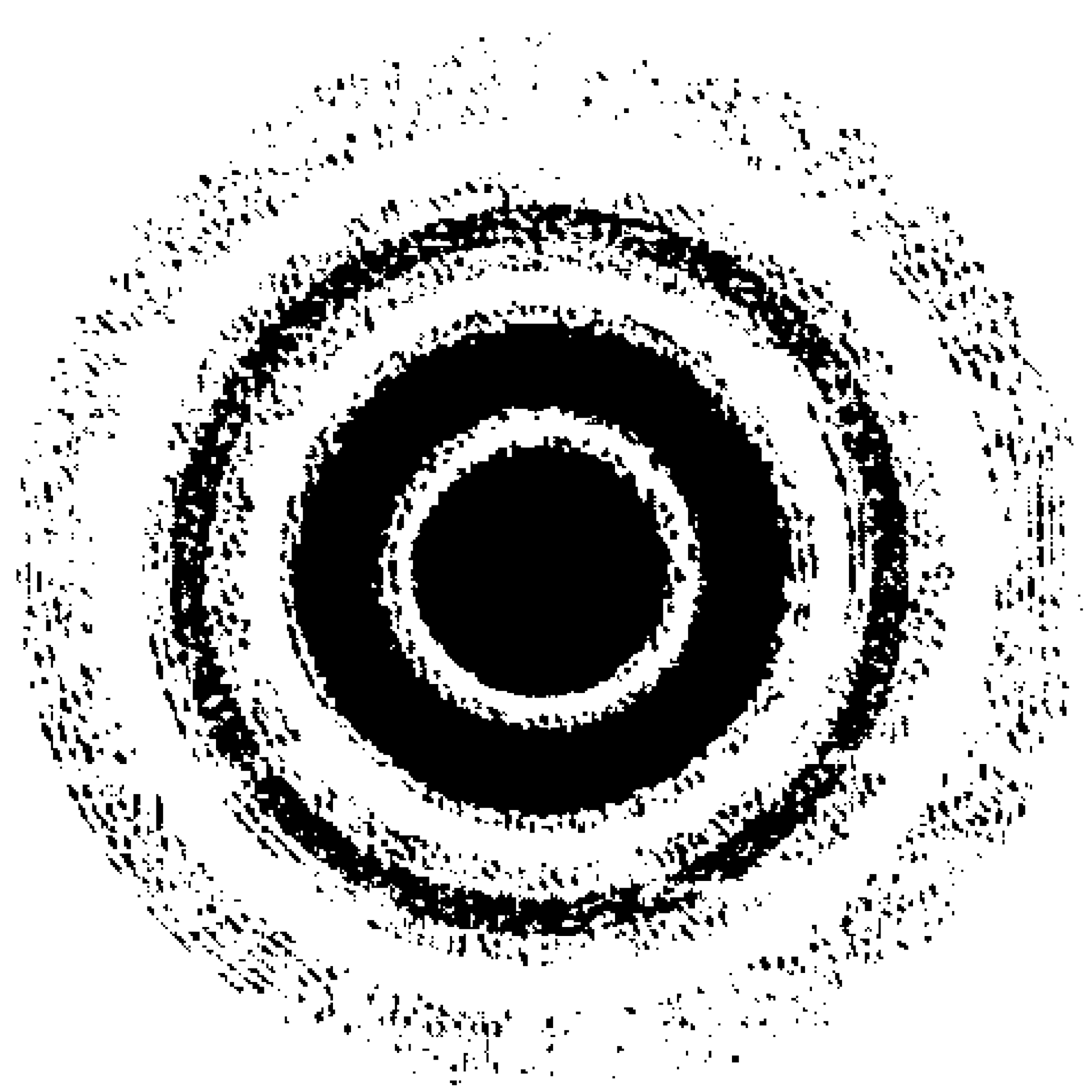

Fig.9 Mathematical model of the disc projected on the screen

Evaluation

First we have to establish the contribution to the complex amplitude caused by wave motion that is caused by waves emerging from the element from the area $d\sigma$ of the plane of the entry slot. If we assume that their amplitude is directly proportional to this area $d\sigma$, we can say that:

$$dS = c\,d\sigma \exp(i(ut + 2)\pi d : l) \qquad (1)$$

where dS is the addition to the complex amplitude we are looking for, u is the frequency of light, l is the wave length of light and d is the difference in distance of the particular element (signed $d\sigma$) of the wave front that passes through the center of the lens from the plane of the slot (this distance is of course measured in the direction of the wave), so we can say that $d = x \sin\alpha$.

Now it is suitable to set in the polar coordinates β and δ (β - distance from the middle of the slot, δ - angular coordinate, r - perimeter of the slot), that means:

$$x = r + \delta\cos\beta \quad (2),\quad d\sigma = d\beta d\delta\beta \qquad (3)$$

from (2) follows $d = r\sin\alpha + \delta\cos\beta\sin\alpha$ (4). Substituting (3) and (4) into (1) we get:

$$dS = c\exp(i(ut + 2\pi(r\sin\alpha + \delta\cos\beta\sin\alpha) : l))\,\beta d\delta d\beta \qquad (5)$$

after arrangement:

$$dS = c\exp(i(ut + (2\pi r\sin\alpha) : l))\exp((2\pi i\delta\cos\beta\sin\alpha) : l)\,\beta d\delta d\beta$$

after integration over the whole area of the entry slot (that means β from 0 to r and δ from 0 to 2π) we get a form which gives every wave of a certain frequency a complex amplitude:

$$k = c\int_{\beta=0}^{2\pi}\int_{\delta=0}^{r}\beta\exp(2\pi i\cos\delta\sin\alpha)\,d\beta d\delta \qquad (6)$$

if we multiple k by a number that is complexly conjugated to it, we get the value of the square of the real amplitude, i.e. the intensity of light at a certain point in the focal plain of the lens (the integral (6) can be converted to the so called Bessel s function):

$$I = I_0\pi^2 r^4\left(1 - \tfrac{1}{2}m + (m^2 : 2!)^2 : 3 - (m^3 : 3!)^3 : 4 + (m^4 : 4!)^2 - \ldots\right)^2 \qquad (7)$$

where $m = \pi r(\sin\alpha) : l$. The last equation (7) consequently shows the relation of intensity of a certain point of the focal plain with the angle of deflection α. As we have said, we are interested in the first minimum of I, which according to the equation (7) occurs for $\alpha = 0{,}61 r : l$.

The problem of maximal distinguishability

We have just found the size of the disc projected on the screen, placed in the focal plane. Now we want to find how far from each other the discs of two different objects must be in order to be able to distinguish them from each other. Let's examine a section which crosses the centre of each of the discs. There are three possible types of graph which show the course of the intensity – they are marked *a*, *b* and *c*. We can see that in case *c* we are no longer able to distinguish the two discs from each other. Case a shows the two discs as two independent objects and finally, case *b* shows the discs from the nearest point from where they are still distinguishable. It is obvious that the lowest point between the two maxima in the intensity course must be sufficiently deep for the two maxima to be distinguishable. For visual observation its depth must be at least 20% of the maximum intensity (Rayleigh's criterion). When using modern equipment we can distinguish 10% or even 5% as well. We stayed at the level of 20% because the equipment in question is rather difficult to come by. This level reflects the distance of the middles of the discs equal to the perimeter of the discs. We will make it equal to the maximum distinguishability – in our case the smallest distinguishable anuglar distance equals 0,61 . r : l.

Experimental comparison of the results

In view of the fact that theoretical results are rarely identical to real ones (e.g. because of the fact that we assumed the lens was ideal), we decided to observe several binary stars. All the photographs come from the observatory at Petřín in Prague. We used telescopes made by The Carl Zeiss Jena company with objective diameters of 180 and 200 mm.

The following table shows all of the observed objects (photos taken on 28th and 29th March 1994):

No.	object	angular distance
1	pi Boo	5,6"
2	gamma CrB	0,7"
3	gamma Leo	4,4"

Summary

The photographical materials confirm our evaluations fairly well. Especially gamma Coronae Borealis with an angular distance 0,7" confirmed that the theoretical and practical results are in sufficient conformity. So we can see that the distinguishability of the telescope doesn t depend on overlighting (very considerable esp. in Prague) and by a calm atmosphere very good results can be reached.

At the same time we can see the binary stars with very small angular distance (close to the maximal distinguishability) are really very hard to distinguish (e. g. mentioned gamma CrB) and a good photo of such an object must come only from the hand of a skilled photographer.

Problem No. 2 - Compass

The problem, as formulated by our team, is based on a detailed analysis the one described by Cherry-Garrar. We decided to state it in the following way:

Describe the magnetic field of the Earth which affects the magnetic compass needle and explain how to eliminate the sudden changes in the direction of the needle, in order to the determine which direction north is?

If we want to obtain a precise picture of Earth's geomagnetic field, we would have to measure the intensity and direction of the magnetic field of the entire globe. Unfortunately, the Earth's magnetic field varies in time and it is also very