Problem No.3 – Magnetism

INTRODUCTION

In this task we are supposed to formulate a problem ourselves using the given theme – a cylindrical magnet falling through a vertical copper tube. Our problem runs as follows: *"On what and how does the movment of a cylindrical magnet which falls through a vertical copper tube* depend? *Compare the experimental results with the theory and explain the differences."*

THE THEORETICAL SOLUTION

This task was both theoretically and experimentally very difficult.

We must not approximate the magnet as a dipole, because this approximation is used for a field of larger distances greater than 10 dimensions of the magnet. The distances we are interested in are comparable with the magnet's dimension or less, therefore this approximation would be too rough to be useful.

We considered a cylindrical magnet falling in the axis of an infinitely long copper tube with a constant diameter. We are evaluating the magnetic friction between the magnet and the tube.

The experimental device was a copper tube (approximatelly 2 m long) equipped with an optical bar at each end connected to a computer and with several coils wrapped round the tube in it s lower part, connected to ISES.

h .... thickness of a tube's wall
r .... tube's radius
v .... velocity of the magnet
$B(r, z)$... magnetical induction in point [r,z]
$\alpha(r, z)$.. angle between **B** and plane XY
ρ .... resistance of the copper

Let us think about a thin elemental ring with a height dz and a diameter dr. There is a current $I_i = U_i/R$, where R is the resistance of the ring, U_i is the potential inducted in it. We can write:

$$d^2 I_i = \frac{d^2 U_i}{R} = \frac{U_i \, dr \, dz}{\rho 2\pi r}$$

$U_i = Blv = B\cos\alpha \; 2\pi r v$, B and α are functions of (r, z).

For the force acting on a conductor with a current, which is placed in a magnetic field, the following is valid: $F = BIl$, where l is the length of the conductor. So we can write:

$$d^2 F = B\cos^2\alpha \; d^2 I_i 2\pi r = \frac{B\cos\alpha \; 2\pi r \, dr \, dz}{\rho 2\pi r}, \text{subst.} U_i:$$

$$d^2 F = \frac{B^2 \cos^2\alpha \; 2\pi v \, dr \, dz}{\rho}$$

We will find out the force acting on a half of the tube, because the second half causes force of the same size and both of the are decreasing (from Lenz's law). $F' = 1/2\, F$

$$F' = \int_0^{\infty}\int_{r_1}^{r_2} \frac{B^2 \cos^2\alpha \; 2\pi r v}{\rho} \, dr \, dz = \int_0^{\infty}\int_{r_1}^{r_2} B^2 \cos^2\alpha \; r \, dr \, dz$$

$$F = \frac{4\pi v}{\rho} \underbrace{\int_0^{\infty}\int_{r_1}^{r_2} B^2 \cos^2\alpha \; r \, dr \, dz}_{} \quad (1)$$

The double integral (1) is uncountable for us, because B and α are functions of (r, z) which we cannot analytically formulate. Fortunately, the double integral (1) is a constant for given arrangement, so we can say, the retarding force F is directly proportional to the velocity of the magnet and inversely proportional to the resistance of the tube.

We can introduce a constant Θ:

$$\Theta = \frac{4\pi}{\rho} \int_0^{\infty}\int_{r_1}^{r_2} B^2 \cos^2\alpha \; r \, dr \, dz$$

For sufficiently thin tubes we can simplify even this expression. We can suppose, that B and α are not dependent on r. In this case we can eliminate $B^2 . \cos^2\alpha$ from the internal integral:

$$\Theta = \frac{4\pi}{\rho} \int_0^{\infty} B^2 \cos^2\alpha \left[\int_{r_1}^{r_2} r \, dr \right] dz = \frac{2\pi}{\rho} \int_0^{\infty} B^2 \cos^2\alpha \left(r_2^{\,2} - r_1^{\,2} \right) dz =$$

$$= \frac{2\pi}{\rho} \int_0^{\infty} B^2 \cos^2\alpha \; h(r_1 + r_2) \, dz = \frac{4\pi h r}{\rho} \int_0^{\infty} B^2 \cos^2\alpha \, dz$$

In this case we can say the force acting on the magnet is directly proportional to tube's thickness. This is true just for tubes with small thickness, of course.

In view of the fact that the intensity of the retardation depends on the magnet's velocity, this velocity will be stabilized (after some time) at a value, which we will name v_{MAX}.

Now let us think about the equation of motion, which is valid for the magnet:

$$F = ma$$

$$m \frac{d^2 z}{dt^2} = mg - \Theta \frac{dz}{dt}$$

from this formula we can obtain a differential equation of the second order:

$$\frac{d^2 z}{dt^2} + \frac{\Theta}{m} \frac{dz}{dt} = g$$

But we are interested in $\frac{dz}{dt}$ (it means v), not $\frac{d^2 z}{dt^2}$.

$\frac{dv}{dt} + \Theta v / m = g$, substituting $v = w + k$, we need $\Theta k / m = g$, therefore $k = mg / \Theta$

$\frac{dw}{dt} + \Theta w/m = \Theta k/m = g$

$w + \Theta w/m = 0$, solving by separation of variables:

$dw/w = -\Theta dt/m$, integrating:

$\ln|w| = -\Theta t/m + \ln|c|$

$|w/c| = \exp(-\Theta t/m), \; w \in R^-$:

$w = -|c| \exp(-\Theta t/m)$, substituting v for w:

$v = k - |c| \exp(-\Theta t/m) = mg/\Theta - |c| \exp(-\Theta t/m)$

The constant will be found from the initial terms:

$v|_{t=0} = 0 = mg/\Theta - |c| \exp(0) \Rightarrow |c| = mg/\Theta$

$$\boxed{v = mg/\Theta(1 - \exp(-\Theta t/m))}$$

We see, when $t \rightarrow \infty$, then $v \rightarrow v_{MAX} = mg/\Theta$

THE EXPERIMENTAL SOLUTION

The first part of our experiments was just observing the falling magnet. When we saw the falling magnet in the tube, we found out that the magnet has a tendency to turn around its horizontal axis. This effect occurs, when the magnet does not fall in the tube's axis, i.e. it is closer to one side of the tube than to the other one, which causes faster descent on one side than on the other one and so consequently the magnet tries to turn around. This effect was removed by sticking a suitable stripe on magnet, whose length was greater than the magnet's.

We have prepared following device for proper experiments:

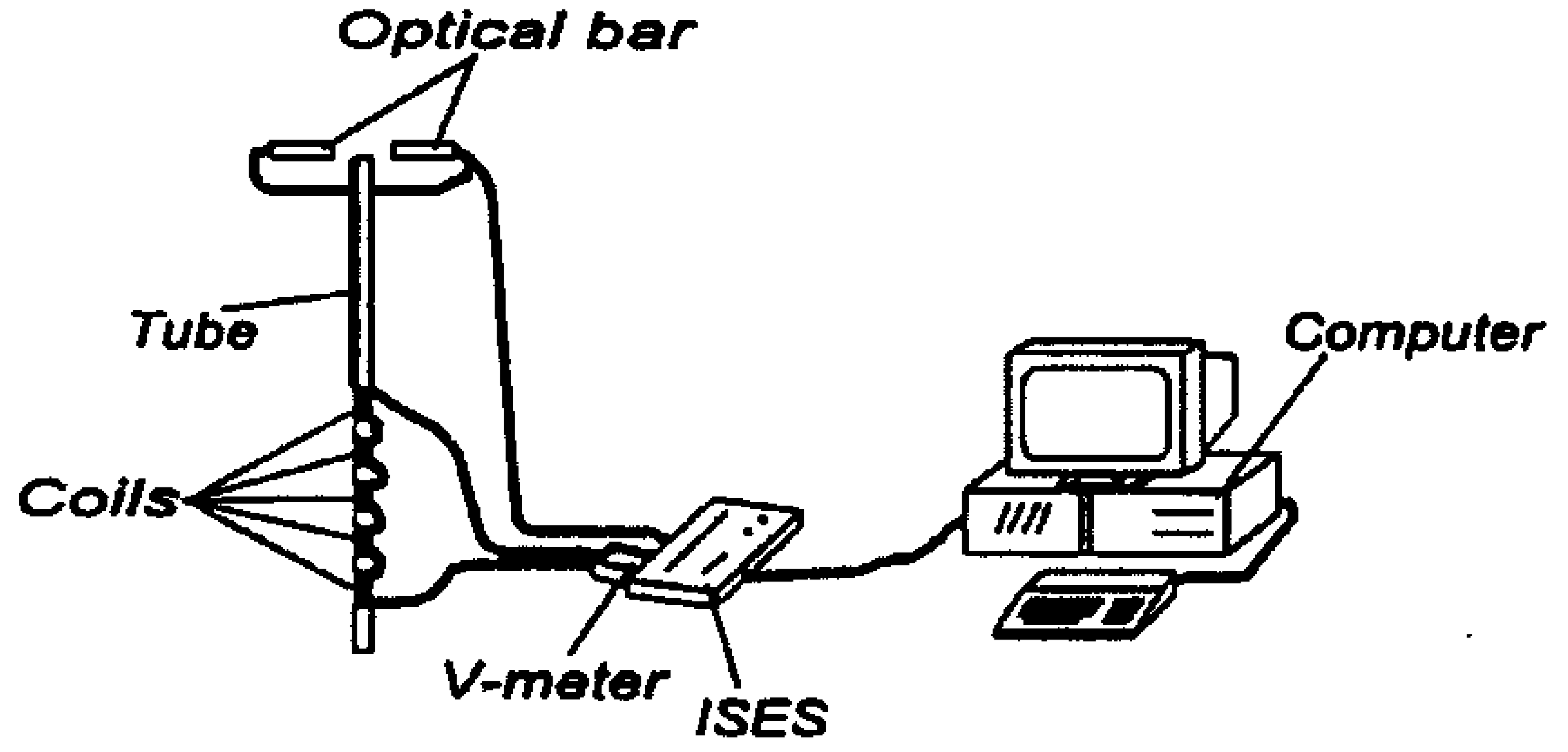

Fig.10

An optical bar is place close to the top of the tube, which starts measurement, when the magnet starts to fall through the tube. 80 cm from the tube's top is the first of five coils, which are at regular intervals of 10 cm. The ends of reels are connected with a voltmeter of ISES – Intelligent School Experimental System. Our computer records a course of the voltage on reels, relative to time. We can find out the times of the magnet's passages through the coils by means of ISES with great accuary. We can simply find out the corresponding velocities.

We will obtain a graph like this from ISES:

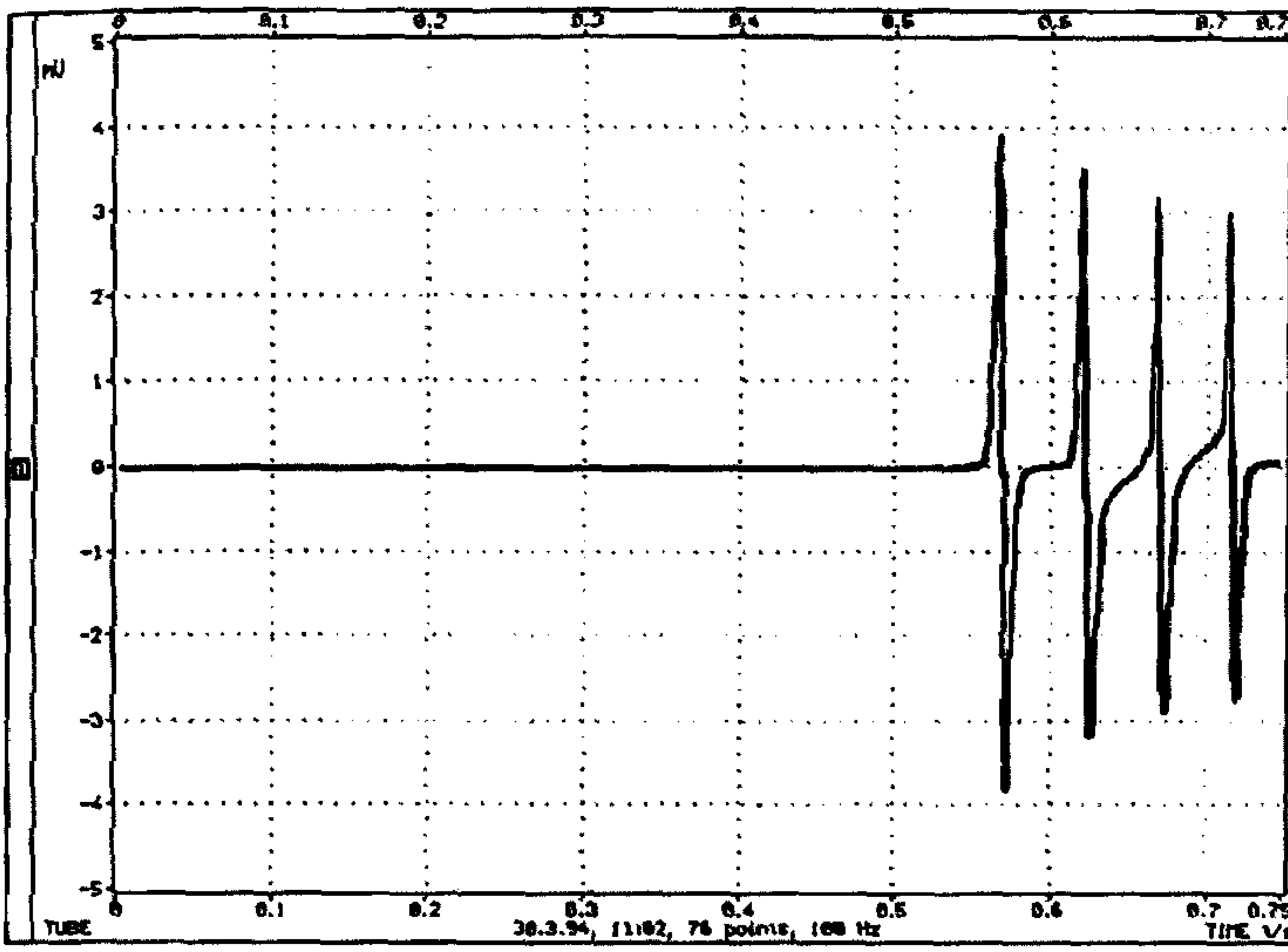

Fig.11

We carried out experiments with these tubes:

Num.	Material	Radius [mm]*	Thickness [mm]
1	Copper	10.0	1.0
2	Copper	7.0	1.5
3	Copper	6.0	1.0
4	Copper	6.0	0.5
5	Alumin.	6.0	1.0
G1	Glass	7.5	1.25
G2	Glass	6.0	1.01

* internal radius

We performed nine experiments with each tube to gain a reliable statistics.

Graph 1 shows the velocities of the magnet in different tubes (values approximated from those 9 experiments). Performing also experiments with glass tubes instead of just a copper ones was extremely useful for us, because in these tubes the speed of the magnet is decreased just by air friction, which we cannot analytically express. In this way, it could be measured simply.

In the following calculations we used the measured data. Using numerical methods we approximated the speed of the magnet at a certain interval with uniformly accelerated movement. The times and accelerations in each interval were enumerated using variables related to previous intervals:

time: $t_n = \frac{v_n - v_{n-1}}{a_n}$ (1)

distance $d_n = 1/2\, a_n t_n^2 + v_{n-1} t_n$ (2)

Now we will substitute (1) into (2)

$$d_n = \frac{1}{2}\frac{(v_n - v_{n-1})^2}{a_n} + \frac{(v_n - v_{n-1})v_{n-1}}{a_n}$$

$$a_n = \frac{v_n - v_{n-1}}{d_n}\frac{v_n + v_{n-1}}{2} \qquad (3)$$

substituted back to (1): $t_n = \dfrac{2d_n}{v_n + v_{n-1}}$

In the second phase of calculation we use the following fact:

$F_a + \Theta v - F_g = F$, where

F_a .... aerodynamic frictional force
Θv .... magnetical friction force
F_g .... gravitational force
F .... resolving force acting on the magnet, we can calculate it for each interval in each experiment using formula (3).

There is a magnetic frictional force equal to zero in the glass tubes. Therefore we can use this case to investigate the dependence of aerodynamic friction on the speed of the magnet and we can consequently find the size of Θ. This is shown in graph 2.

SUMMARY

We have described the situation theoretically. We found out, on what and how the magnetical frictional forces acting on the falling magnet depend. We have also experimentally verified these theoretical results.

As we assured ourselves, the magnet's velocity in the tube will be stabilized after approx. 0.5 ÷ 1 s (depending which tube is used) at a certain value. We can see, that the magnetic friction is much higher than the aerodynamic one from the graph 1.

The difference between the theoretical results and practical measurments are caused mainly by the fact that the magnet does not fall precisely down the tube's axis, which means that the magnet has a tendency to turn, which increases the mechanical friction. It also has to include the aerodynamic friction.

Nevertheless, We can conclude that our experiments prooved that we can use our theoretical model in the simulation with reasonable accuracy.

Problem No.4 – Upper Boundary

We solved this problem both experimentally, using the device described below, and theoretically, using methods from mathematical statistics and a computer simulation.

As regards our technical possibilities, we conducted the experiments using the following set-up: As the "flexible plate", we used a steel plate glued to a loudspeaker. The signal from a tone generator was fed into the loudspeaker, amplified to sufficient power by an amplifier (15W). Also, we attached a glass tube to the plate and put a steel ball of 3mm diameter into it.