Now we will substitute (1) into (2)

$$d_n = \frac{1}{2}\frac{(v_n - v_{n-1})^2}{a_n} + \frac{(v_n - v_{n-1})v_{n-1}}{a_n}$$

$$a_n = \frac{v_n - v_{n-1}}{d_n}\frac{v_n + v_{n-1}}{2} \qquad (3)$$

substituted back to (1): $t_n = \dfrac{2d_n}{v_n + v_{n-1}}$

In the second phase of calculation we use the following fact:

$F_a + \Theta v - F_g = F$, where

- F_a .... aerodynamic frictional force
- Θv .... magnetical friction force
- F_g .... gravitational force
- F .... resolving force acting on the magnet, we can calculate it for each interval in each experiment using formula (3).

There is a magnetic frictional force equal to zero in the glass tubes. Therefore we can use this case to investigate the dependence of aerodynamic friction on the speed of the magnet and we can consequently find the size of Θ. This is shown in graph 2.

SUMMARY

We have described the situation theoretically. We found out, on what and how the magnetical frictional forces acting on the falling magnet depend. We have also experimentally verified these theoretical results.

As we assured ourselves, the magnet's velocity in the tube will be stabilized after approx. 0.5 ÷ 1 s (depending which tube is used) at a certain value. We can see, that the magnetic friction is much higher than the aerodynamic one from the graph 1.

The difference between the theoretical results and practical measurments are caused mainly by the fact that the magnet does not fall precisely down the tube's axis, which means that the magnet has a tendency to turn, which increases the mechanical friction. It also has to include the aerodynamic friction.

Nevertheless, We can conclude that our experiments prooved that we can use our theoretical model in the simulation with reasonable accuracy.

Problem No.4 – Upper Boundary

We solved this problem both experimentally, using the device described below, and theoretically, using methods from mathematical statistics and a computer simulation.

As regards our technical possibilities, we conducted the experiments using the following set-up: As the "flexible plate", we used a steel plate glued to a loudspeaker. The signal from a tone generator was fed into the loudspeaker, amplified to sufficient power by an amplifier (15W). Also, we attached a glass tube to the plate and put a steel ball of 3mm diameter into it.

The amplitude of the plates's oscillation at low frequencies was 4mm, as measured. The frequency of the signal could be changed to any value, but only small range was of a practical meaning for us, because the oscillations' amplitude decreased abruptly at higher frequencies (already at approximatelly 100 Hz) and the ball didn't even jump off from the plate. This limitation was caused mainly by mechanical properties of the loudspeaker and couldn't be passed because there is actually no better equipment for such a purpose. After all, the work wasn't completely ideal even at lower frequencies (oscillations weren't perfectly harmonic and the results were also affected by the dependence of the amplitude on the frequency, which can't be expressed generally). The ball jumped to a maximum height of its own resonance frequency of the loudspeaker body (25 Hz), at higher frequencies it jumped lower and soon stopped jumping at all. Because of this the experiment could serve only for finding the qualitative basis, while other ways had to be used for its numerical expression.

An analytic solution of the problem appeared to be impossible, because it needs to solve a set of complicated differential equations, even with random quantities. Therefore we solved the problem numerically, with a computer simulation. Our program simulates the behaviour of a ball jumping on the flexible plate, numerically determines a moment of the collision, calculates impact velocity and maximum height of the jump and can even show the distribution of particular quantities' values. All the information obtained is of good quality and agrees with the experimental data within the limits of accuracy.

Let's consider a ball falling with the velocity v. If the coefficient of restitution is r and instantaneous velocity of the plate is u (sign positive, if the plate is going up, and negative, if it goes down). Then the velocity of the ball, related to the plate, is $u + v$ and the ball rebounds with velocity $r(u + v)$, pointing indeed upward. Because the coordinate system connected with the plate moves with velocity u related to the ground, the rebounding ball's velocity is $rv + (1 + r)u$ in the system of ground. As defined, v must always be positive, while u can differ from $-U$ to U, where U is the maximum possible velocity of the plate ($U = Y_m \sin \Omega$, Where Y_m is the amplitude of the oscillation and Ω its angular frequency). The new velocity may also be negative, meaning the ball falling onto quickly descending plate is "taken" by its motion and although it rebounds up, in relation to the plate, its further movement is downwards in our system.

Let's introduce a quantity $E' = 2E_m / m$. We can suppose that the potential energy of the ball impinging on the plate is zero (actually, it differs in different positions, but few milimeters of difference are negligible in comparison to heights of tens of centimeters, which the ball reaches, jumping). Then the "reduced energy" of the ball falling onto the plate is v^2 and that of the ball rebounding is $v^2r^2 + 2(1 + r)uv + (1 + r)^2u^2$. In a long-term scale, energy losses at rebound of the plate, which is not perfectly flexible, are equal to the energy increases, caused by the plate's movement and following "tennis-rocket effect", as we could call it. How much is that increase? Experiments and both simulations show that the phase of the plate's oscillation is quite random. As we can easily see, one jump of the ball takes considerably more time than one period of the oscillation, so any resonance is impossible (also because of many random influences). Therefore it's as probable that the ball falls onto the plate when its velocity u_1 as that it falls onto the plate at a velocity $-u_1$, mean value is then 0 (as we can also easily find with integrating, which is quite trivial in this case). However, the mean value of u^2 is (as we find again by integrating) $U^2/2$. If we record the mean impact velocity of the ball as v_1, it's possible to write

$$v_1^2 = r^2 v_1^2 + (1+r)^2 U^2/2, \text{ and then } v_1 = U\sqrt{[\tfrac{1}{2}(1+r)/(1-r)]}\,.$$

Because the maximum height of vertical throw up is $v^2/(2g)$, mean height of the ball's jumps will be $h_1 = U^2(1+r)/[4g(1-r)]$, i.e. proportional to the square of the plate oscillation frequency. Experiments and simulation show a very good agreement with this theoretical forecast.

Actually, the ball of course jumps to many different heights, both lower and higher than h. As measurements and experiments show, the maximum heights really reached are approximately $h_{max} = U^2(1+r)/[g(1-r)]$. Also in this case the resultant value is directly proportional to the square of maximum velocity of the plate.

SUMMARY

The conclusion of both theoretical calculations and the practical experiment is, that maximum height the ball can reach is directly proportional to the square of the plate's oscillation frequency (supposing the amplitude to be constant) and depends very significantly on the coefficient of restitution.

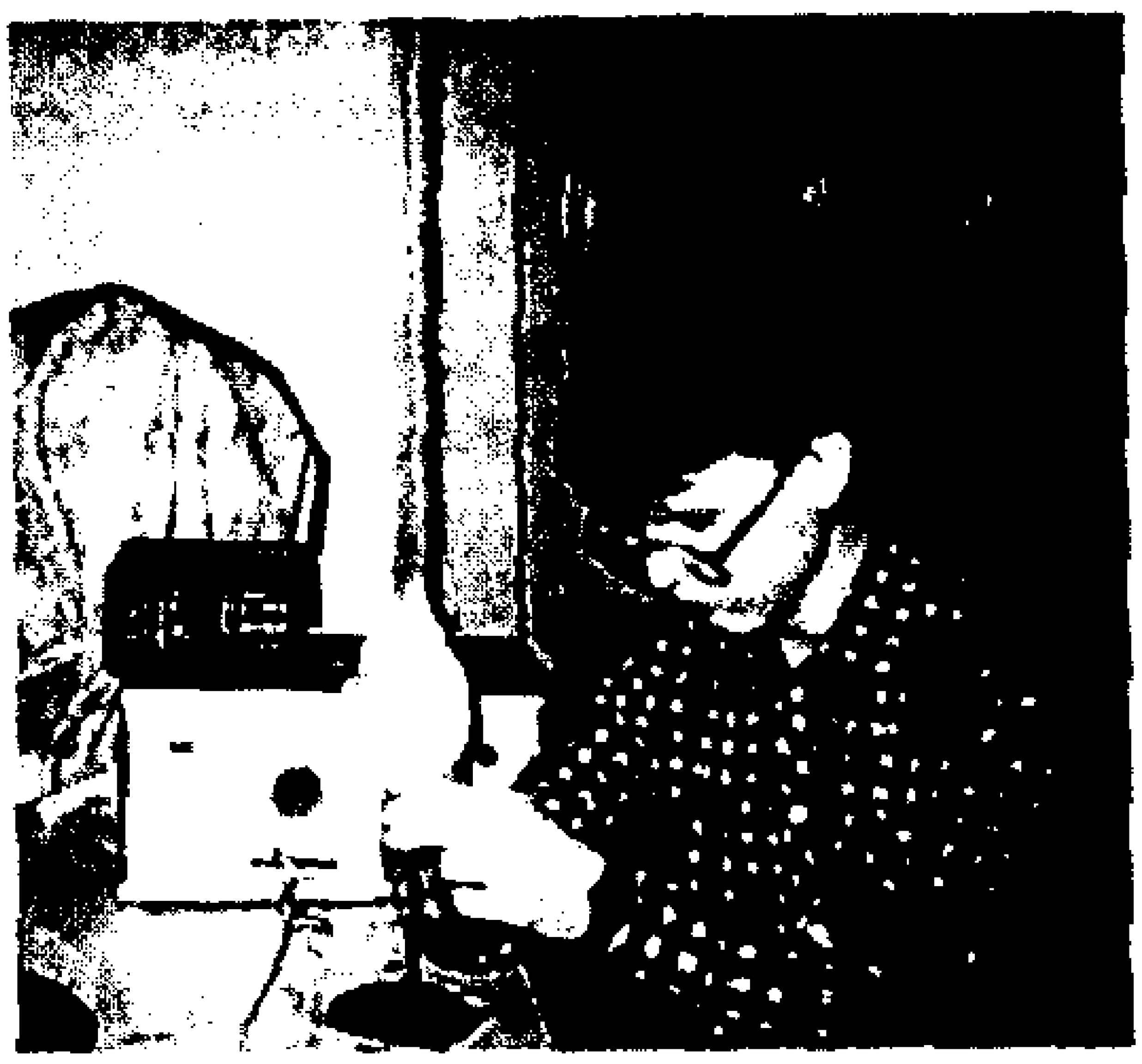

Fig.12 Experimental verification of the theoretical model.
Photo M. Prouza

Problem No.5 - Distribution Function

INTRODUCTION

This problem isn't set out in a way which is easily understanable. It is not clear, what the sense of words "what part of the time" is and what quantities H and dH should mean. After a thorough analysis of the whole problem we concluded that we are to find out the probability (here identified with long-term relative frequency) of the ball's occurence at certain place. Regarding the effective impossibility of determining any values experimentally due to very short intervals between successive falls of the ball to the platform the basis of our work consists of theoretical deduction and conducting experiments with a computer model of the given phenomenon.