(as we find again by integrating) $U^2/2$. If we record the mean impact velocity of the ball as v_1, it's possible to write

$$v_1^2 = r^2 v_1^2 + (1+r)^2 U^2/2, \text{ and then } v_1 = U\sqrt{[\tfrac{1}{2}(1+r)/(1-r)]}\,.$$

Because the maximum height of vertical throw up is $v^2/(2g)$, mean height of the ball's jumps will be $h_1 = U^2(1+r)/[4g(1-r)]$, i.e. proportional to the square of the plate oscillation frequency. Experiments and simulation show a very good agreement with this theoretical forecast.

Actually, the ball of course jumps to many different heights, both lower and higher than h. As measurements and experiments show, the maximum heights really reached are approximately $h_{max} = U^2(1+r)/[g(1-r)]$. Also in this case the resultant value is directly proportional to the square of maximum velocity of the plate.

SUMMARY

The conclusion of both theoretical calculations and the practical experiment is, that maximum height the ball can reach is directly proportional to the square of the plate's oscillation frequency (supposing the amplitude to be constant) and depends very significantly on the coefficient of restitution.

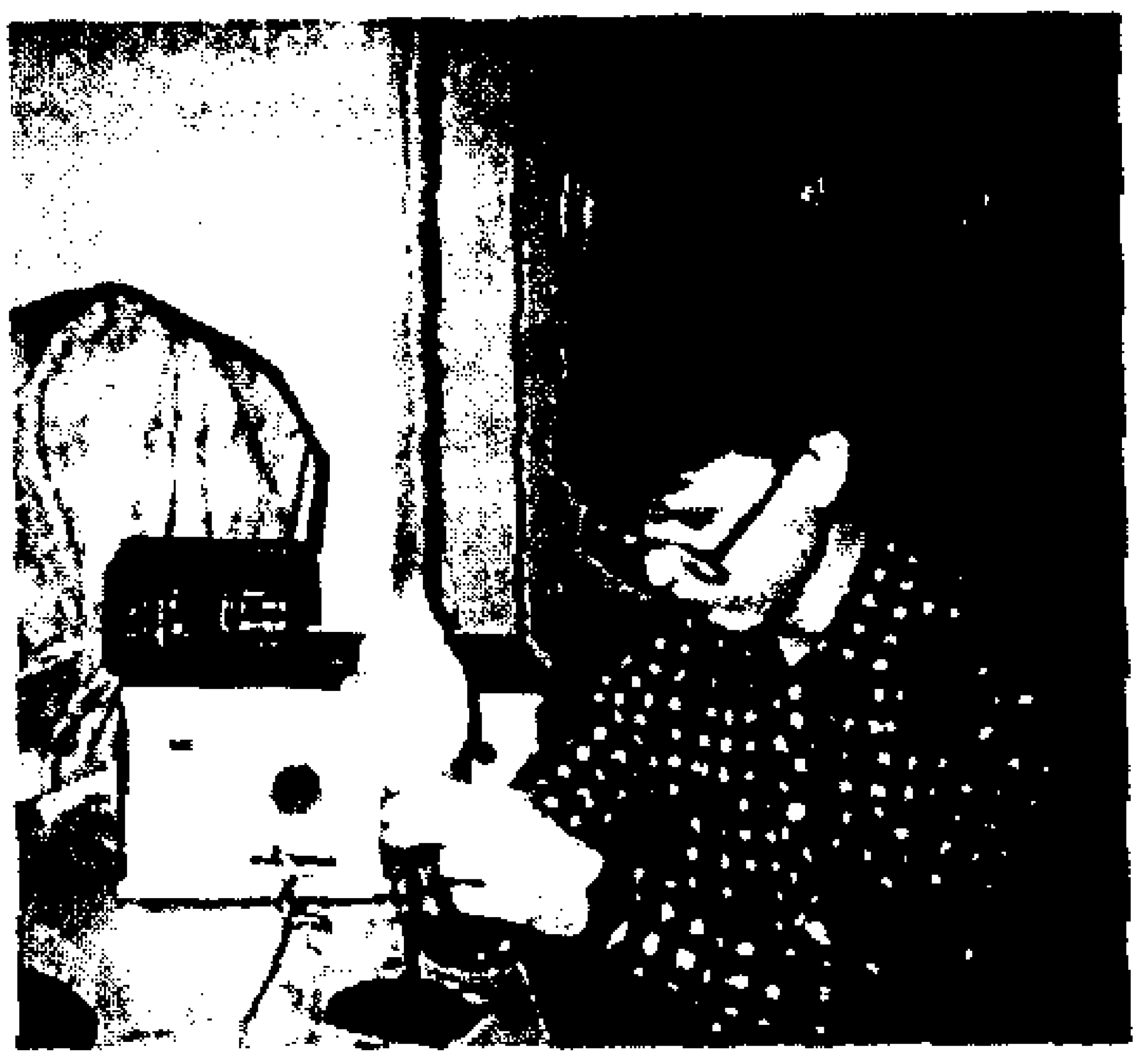

Fig.12 Experimental verification of the theoretical model.
Photo M. Prouza

Problem No.5 - Distribution Function

INTRODUCTION

This problem isn't set out in a way which is easily understanable. It is not clear, what the sense of words "what part of the time" is and what quantities H and dH should mean. After a thorough analysis of the whole problem we concluded that we are to find out the probability (here identified with long-term relative frequency) of the ball's occurence at certain place. Regarding the effective impossibility of determining any values experimentally due to very short intervals between successive falls of the ball to the platform the basis of our work consists of theoretical deduction and conducting experiments with a computer model of the given phenomenon.

SOLUTION

Let's consider the course of the function we are looking for during one particular jump. Let s assume that the ball reaches a maximum height H; then its overall mechanical energy during the jump is $E = mgh$. Searched function of distribution, i.e. $f(h) = dt/dh$, holds

$$f(h,H) = dt/dh = (dh/v)/dh = 1/v = 1/\sqrt{2(E-mgh)/m} =$$

$$= 1/\sqrt{2(mgH-mgh)/m} = 1/\sqrt{2g(H-h)} = \frac{1}{\sqrt{2g}} \cdot \frac{1}{\sqrt{(H-h)}}$$

A Graph of this function is given in the supplement.

Further, let the density of probability of H itself be $p(H) = dp/dH$. We can easily derive that the overall formula for $f(h)$ without dependence on H is found by integrating

$$f(h) = \int_0^\infty p(H)\, f(h,H)\, dH$$

where integrating is done along H and h is considered as a parameter. General analytic solution is out of our capabilities, though, and so we expressed $f(h)$ using computer simulation (see problem 4).

In such a case the solution could have been done in two ways. The first one was repeated many times, choosing t at random, finding of respective h and drawing a histogram for sufficiently small dh. The other possibility was numerically carrying out the above mentioned integration: with empirical knowledge of $p(H)$ or in fact $p(H)dH$ for small dH it would be possible to determine $f(h)$ for certain fixed h by summing along all dH; then repeat the procedure for h increasing dh. We chose the first method.

In order to fulfill this task, few small changes in the program were done. We computed the ball's jumping during quite a long time (300s), we randomly chose 6 000 moments in this interval and in the course of the program we were printing the ball's position in given moment. The results are processed into a graph, i.e. histogram with step $dh = h_{max}/500$.

CONCLUSION

The result (shown at Fig. 13) is finding frequency (in the graph marked m) of the ball's position in particular heights (in graph marked H, given in meters). For norming to the relative frequency divide m by 6 000. The distribution has the distinct shape of the Gauss curve, at which only random deviations show up. It's obvious that this function has a maximum of h near zero, its value decreases quickly with growing h and it is almost zero, for $h \to h_{max}$. The distribution dt/dh shows a remarkable similarity to the distribution dp/dh, i.e. density of probability of the ball's jumps' maximum height.

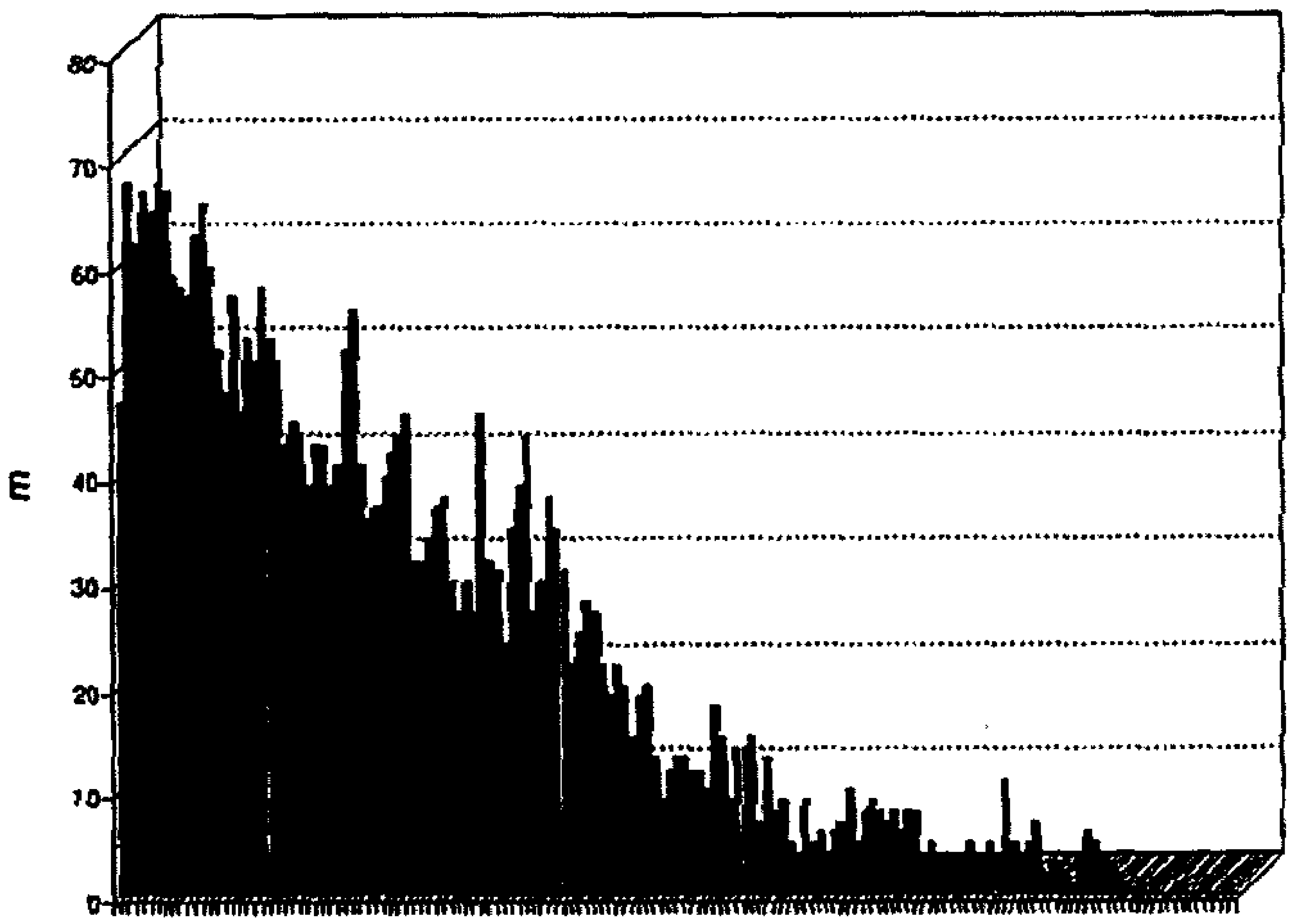

Fig. 13 The frequency of ball's position in particular heights

Problem No.6 – Acceleration

Let *s* consider one impact of the ball. According to the solution of problem # 4, a ball falling with velocity v has a velocity of magnitude $r \,.\, v + (1 + r) \,.\, u$ after the rebound, where u is momentary velocity of the plate. Further: because the ball reaches heights of approximatelly 10 cm, different positions of the rebound plate in the moment of the ball's impact can be neglected and potential energy of the ball in this point can be considered zero (error is at most 10%). Then overall mechanical energy of the ball during one jump (losses caused by air resistance are at low velocities in our case fully negligible) is equal to kinetic energy at the moment of impact and rebounding, respectively. Further, let's introduce quantity $E' = 2E_k / m$, where m is the mass of the ball. Then obviously E' before impact is equal to v^2 and after it

$$[rv + (1 + r)u]^2 = r^2v^2 + 2(1 + r)uv + (1 + r)^2u^2$$

Let's think on: As we've shown in problems # 4 and 5, the mean value of u is equal to zero and the one of u to one half of squared maximum value of u, i.e. $U^2/2$. Because the setting speaks of long-term average, we can without problems substitute variable quantities with their mean values and express energy after n-th rebound

$$E_{n+1}' = r^2v^2 + (1 + r)^2U^2/2$$

which in comparison with $E_n' = v^2$ results in relation for energy increment after n-th rebound

$$dE_n' = (1 + r)^2U^2/2 - (1 - r^2)v^2$$

After substituting $C = (1 + r)^2U^2/2$ and with knowledge of $v^2 = E'$ we get

$$dE_n' = C - (1 - r^2)E_n'$$