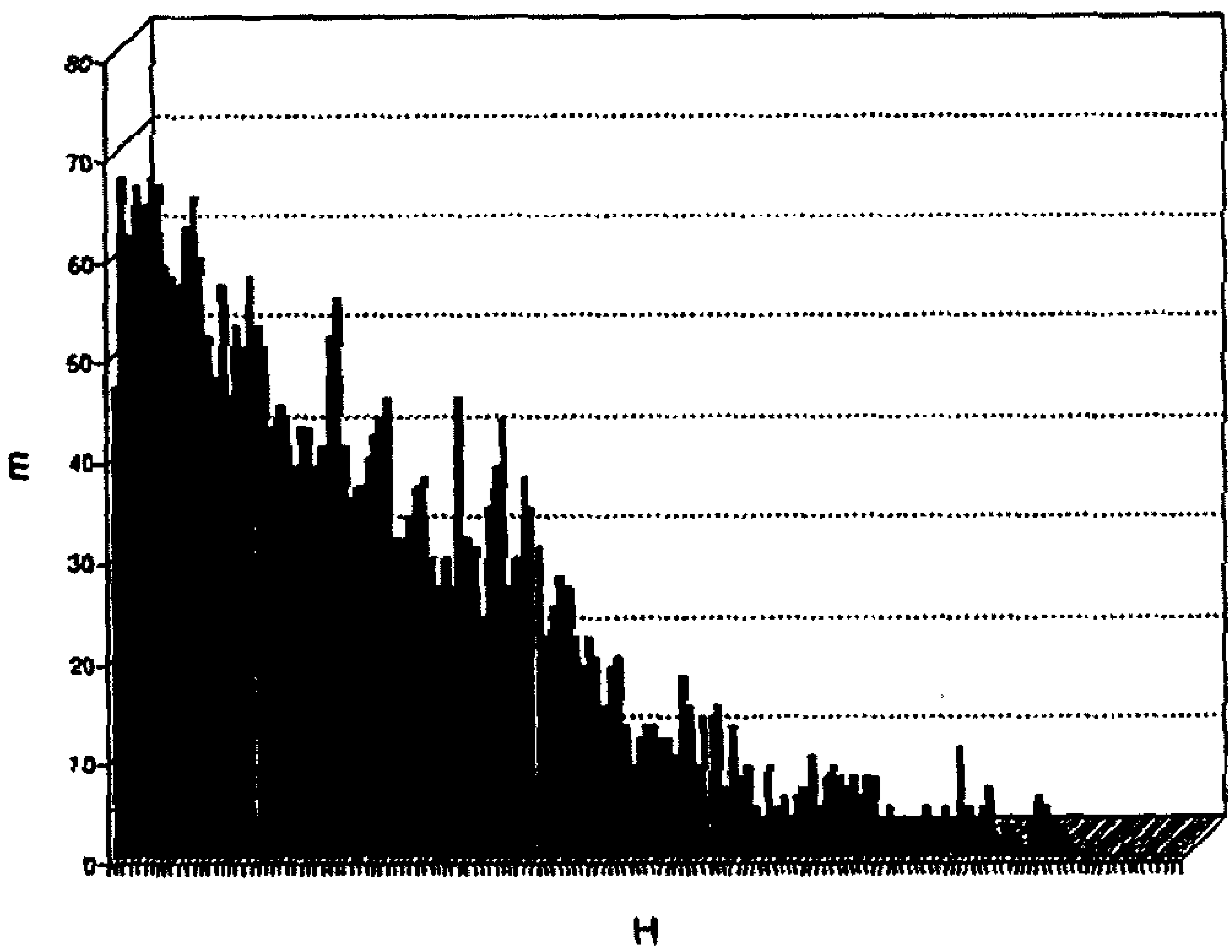

Fig. 13 The frequency of ball's position in particular heights

Problem No.6 – Acceleration

Let *s* consider one impact of the ball. According to the solution of problem # 4, a ball falling with velocity v has a velocity of magnitude $r \,.\, v + (1 + r) \,.\, u$ after the rebound, where u is momentary velocity of the plate. Further: because the ball reaches heights of approximatelly 10 cm, different positions of the rebound plate in the moment of the ball's impact can be neglected and potential energy of the ball in this point can be considered zero (error is at most 10%). Then overall mechanical energy of the ball during one jump (losses caused by air resistance are at low velocities in our case fully negligible) is equal to kinetic energy at the moment of impact and rebounding, respectively. Further, let's introduce quantity $E' = 2E_k / m$, where m is the mass of the ball. Then obviously E' before impact is equal to v^2 and after it

$$[rv + (1 + r)u]^2 = r^2v^2 + 2(1 + r)uv + (1 + r)^2u^2$$

Let's think on: As we've shown in problems # 4 and 5, the mean value of u is equal to zero and the one of u to one half of squared maximum value of u, i.e. $U^2/2$. Because the setting speaks of long-term average, we can without problems substitute variable quantities with their mean values and express energy after n-th rebound

$$E_{n+1}' = r^2v^2 + (1 + r)^2U^2/2$$

which in comparison with $E_n' = v^2$ results in relation for energy increment after n-th rebound

$$dE_n' = (1 + r)^2U^2/2 - (1 - r^2)v^2$$

After substituting $C = (1 + r)^2U^2/2$ and with knowledge of $v^2 = E'$ we get

$$dE_n' = C - (1 - r^2)E_n'$$

Let's look for dependence of E_n' on time and n, respectively, with initial value $E_0' = 0$. We get gradually

$$dE_1' = C,\ E_1' = C,\ dE_2' = C - (1 - r^2)C = Cr^2,\ E_2' = C(1 + r^2),$$

$$dE_3' = C - (1 - r^2)C(1 + r^2) = Cr^4,\ E_3' = C(1 + r^2 + r^4),$$

$$dE_4' = C - (1 - r^2)C(1 + r^2 + r^4) = Cr^6,\ E_4' = C(1 + r^2 + r^4 + r^6) \text{ etc.}$$

Some regularity is visible here and really, with mathematical induction it's easy to prove.

$$dE_n' = Cr^{2n-2},\ E_n' = C(1 + r^2 + \ldots + r^{2n-2}) = C(1 - r^{2n})/(1 - r^2).$$

So we can see that energy of the ball grows on a reverse exponential, asymptotically approaching limit $E_m' = C/(1 - r^2)$, calculated already in #4 as a long-term stable average. Of course, this is just mathematical approximation of the real, rather erratic behaviour of the ball, because it is influenced by many things that are quantitatively unexpressible or we neglected them (irregularities of ball and plate surfaces, air resistance, the rebounding takes some time). That causes the energy of the ball to decline from calculated values and sometimes even exceed E_m', but generally, as an average of many experiments, the results agree, with the theoretical model very well.

Problem No.7 – Aspen Leaf

Possible Air Flow

Firstly, we have to find the exact meaning of the term "windless". Since it is not mentioned in the Beaufort's scale of wind's strength (we used the scale from Encyclopedia Britannica), we supposed that "windless" is equal to the term "calm" which means that the velocity of wind is lower that 1 ms^{-1}. All these terms are, however, related to the horizontal speed of wind. Even if we suppose that this speed is zero (although this case is quite improbable), we cannot neglect the vertical air flow. This one is caused because of unequal air heating in different layers - this phenomenon occurs everywhere in nature and has much greater influence upon contingent leaf movement (because the direction of this flow is perpendicular to the plane of the leaf blade).

Leaf movement

Despite of all these theoretical considerations we decided to make sure that in windless weather (in our case it means in a closed vessel of negligible dimensions and therefore with negligible differences in air heating and air flow) the aspen leaves really do not move. For the experimental purposes two types of leaves were used: firstly ones grown when no natural aspen leaves were available - they were grown in a closed glass vessel and nourished using standard soil under conditions suitable for fast growth. The leaves were compared with the ones from a collection of the National Botanical Garden in Prague. They were equal. The second type of leaves was taken from living trees. Experiments with both types showed that there is no movement of leaves in conditions that do not allow any air flow. Using this experiments we were authorised to exclude all types of vital movements (e.g. caused by flow of nutritions in the leaf), that means, this is a physical one.