Let's look for dependence of E_n' on time and n, respectively, with initial value $E_0' = 0$. We get gradually

$dE_1' = C$, $E_1' = C$, $dE_2' = C - (1 - r^2)C = Cr^2$, $E_2' = C(1 + r^2)$,

$dE_3' = C - (1 - r^2)C(1 + r^2) = Cr^4$, $E_3' = C(1 + r^2 + r^4)$,

$dE_4' = C - (1 - r^2)C(1 + r^2 + r^4) = Cr^6$, $E_4' = C(1 + r^2 + r^4 + r^6)$ etc.

Some regularity is visible here and really, with mathematical induction it's easy to prove.

$dE_n' = Cr^{2n-2}$, $E_n' = C(1 + r^2 + ... + r^{2n-2}) = C(1 - r^{2n})/(1 - r^2)$.

So we can see that energy of the ball grows on a reverse exponential, asymptotically approaching limit $E_m' = C/(1 - r^2)$, calculated already in #4 as a long-term stable average. Of course, this is just mathematical approximation of the real, rather erratic behaviour of the ball, because it is influenced by many things that are quantitatively unexpressible or we neglected them (irregularities of ball and plate surfaces, air resistance, the rebounding takes some time). That causes the energy of the ball to decline from calculated values and sometimes even exceed E_m', but generally, as an average of many experiments, the results agree, with the theoretical model very well.

Problem No.7 – Aspen Leaf

Possible Air Flow

Firstly, we have to find the exact meaning of the term "windless". Since it is not mentioned in the Beaufort's scale of wind's strength (we used the scale from Encyclopedia Britannica), we supposed that "windless" is equal to the term "calm" which means that the velocity of wind is lower that 1 ms^{-1}. All these terms are, however, related to the horizontal speed of wind. Even if we suppose that this speed is zero (although this case is quite improbable), we cannot neglect the vertical air flow. This one is caused because of unequal air heating in different layers - this phenomenon occurs everywhere in nature and has much greater influence upon contingent leaf movement (because the direction of this flow is perpendicular to the plane of the leaf blade).

Leaf movement

Despite of all these theoretical considerations we decided to make sure that in windless weather (in our case it means in a closed vessel of negligible dimensions and therefore with negligible differences in air heating and air flow) the aspen leaves really do not move. For the experimental purposes two types of leaves were used: firstly ones grown when no natural aspen leaves were available - they were grown in a closed glass vessel and nourished using standard soil under conditions suitable for fast growth. The leaves were compared with the ones from a collection of the National Botanical Garden in Prague. They were equal. The second type of leaves was taken from living trees. Experiments with both types showed that there is no movement of leaves in conditions that do not allow any air flow. Using this experiments we were authorised to exclude all types of vital movements (e.g. caused by flow of nutritions in the leaf), that means, this is a physical one.

This opinion was confirmed by the scientists from the Botanical Garden in Prague as well.

Requirements of the Movement

Of course, we must deal with the question of why this movement occurs only in aspen leaves and not for example with oak ones. These conditions can be characterized in the whole as morphological. The best way to explain this is a comparison.

Firstly - the aspen leaf-stalk is quite flat, that means it is very easy to be bend (the section of the oak one is round). Next fact is that the joint between the stalk and the blade has only very small area (the plane of the blade is perpendicular to the plane of the stalk) and therefore although it is very firm, it is very flexibile as well. Besides that the aspen stalk is quite long compared to the oak and so less air flow is necessary to make the leaf move. We must consider as well, that the axis of the stalk and the plane of the blade contain an angle of 20 degrees. All mentioned gifts of the aspen leaf are shown on the picture.

There is yet another experiment that was done. We were finding the centre of gravity of the leaf with and without the stalk (see photo). The result showed that in both cases it is placed quite near to the point of joint between the stalk and the blade of the leaf, which makes the leaf even more easy to tremble.

Summary

The problem was posed in a very imprecise way – it wants us to find the causes of a phenomenon which – in reality – does not occur. We showed (and consequently verified by a practical experiment) that the aspen leaf requires a very light air flow. On the other hand it has very good conditions for movement (these are mentioned above). They are quite visible in the photo below and in the real leaf.

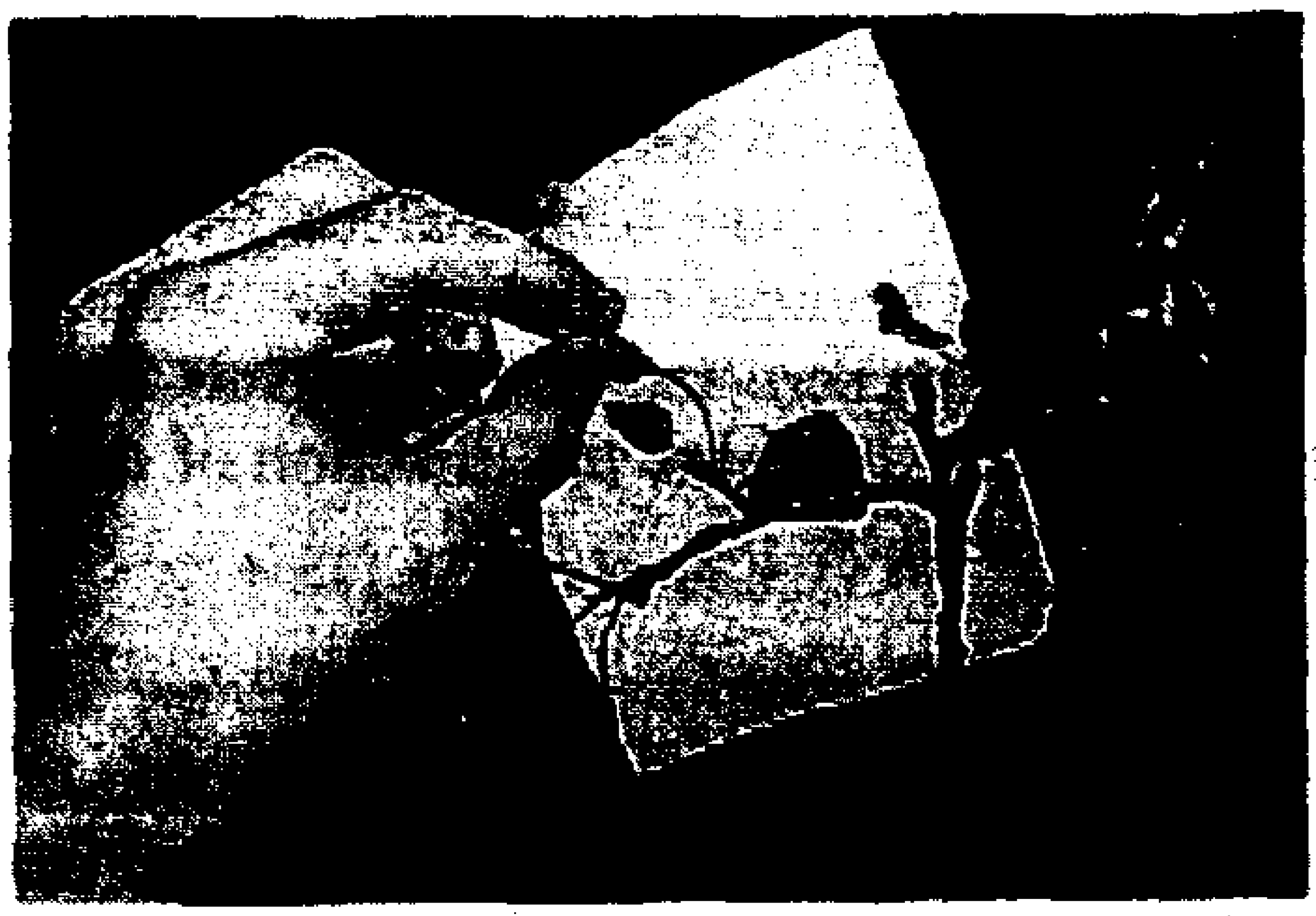

Fig. 14 Detailed photo of an aspen leaf

Problem No.8 - Superball

INTRODUCTION

This task seems to be one of the most interesting. We solved it theoretically – we produced two physical models of the situation.

The ball has mass $m = 8.23$ g, diameter $R = 2.6$ cm and we dropped it from the height $h_0 = 5$ cm.