Problem No.11 – Siphon

Our solution focused on those models which – due to the practical setting of the problem – correspond best to the reality:

a) removal of the siphon from the lower vessel.

b) removal of the siphon from the upper vessel.

The water flows through the hose due to the different water pressures in the individual sectors of the siphon. This pressure difference is determined by the position of the bubble. It is therefore evident that the water can flow from the higher vessel into the lower one only when the water column in the siphon (in the bubble-lower section) is higher that the height of the column in the bubble-upper vessel section.

I. A.B.

If we want the water to keep flowing, we must make sure that during the whole time the siphon is removed, at least some quantity of water in the bubble-upper vessel section is kept under the surface level of the water contained in the upper vessel.

II.

In the second part of the solution, two cases may occur: the bubble is too long (due to the long time during which the siphon was pulled out) and static equilibrium sets in when the bubble stops moving to equalise the hydrostatic pressures caused by the water columns in the individual sections of the hose. In the other case when the length of the bubble is under a certain value, there doesn t exist a position when the water column heights (and thus also the pressures) would be the same – the pressure of the water column in the section delineated by the bubble and the lower vessel is higher. The exciting pressure difference keeps the water (and the bubble) in motion until the bubble "escapes" from the water contained in the lower vessel.

II.A

We chose as the first model example the situation shown in Fig.19a. The siphon consists of two vertical straight sections which change into a circual curve in the upper section.

1.) Let us assume that the height difference of the water surfaces in both vessels equals to δh. We will then suck in a bubble of the exact length of δh. In the initial position (the bubble has just closed) the pressure is definitely higher in the bubble-lower vessel section. The bubble will start moving. At first, the pressure will go up in both sections, both ends of the bubble will move upwards. From the moment when the "front" end of the bubble floats over the top of the curve, the following happens: the pressure in the bubble-lower vessel section starts going down, the pressure in the other section will continue to go up for a little while. Equilibrium, however, will only set at the moment when the "rear" end of the bubble reaches point W (see Fig. 19a).

Proof: The sum of the water levels difference and the length of the straight section in the upper vessel i.e. $\delta h + x$, equals to the length of the other straight section. It is evident that the column height, i.e. the pressures, are the same and the system is indeed in a state of equilibrium.

2.) Let us assume that the length is shorter than the level difference $l < \delta h$, i.e. pressure p_2 is evidently higher in the bubble-lower section of the hose. When the bubble floats down in the other section of the siphon both pressures will go down, but due to the straigtness of this section, they will keep dropping always by the same value.

The pressure difference will therefore not change and the bubble (as well as the water) will move upwards toward the vessel 2. The movement of the bubble in the upward direction need not be considered here, because this would increase pressure p_2 by a certain value, but pressure p_1 would go up by a lower value (due to the hose curvature), which would cause an increase in the pressure difference, i.e. the energy of the system. For this reason, the bubble will start going down.

3.) The above gives a definite position of the bubble ends which typifies a state of equilibrium of a certain kind. By the position of the ends we mean the height above the water level in appropriate vessel. It is therefore evident that there exists another place for the position of the bubble end corresponding to vessel 1, which is point W where the same equilibrium is established as in the previous case. The corresponding length of the bubble is $l = \pi r + \delta h$. If we prolong the bubble by length b, then – in order to recover the equilibrium – the surface levels will drop by $\frac{1}{2}b$ in both straight sections of the hose. The transport of the water will cease. $l \geqq \pi r + \delta h$.

4.) $\pi r + \delta h > 1 > \delta h$. Let us assume that in this case the "rear" end of the bubble will stop in the curve at angel β (see Fig. 19b). We can then apply these relationships to the whole length of the hose:

a) according to the individual sections:

$$z = x + \pi r + x + \delta h \tag{1}$$

b) according to the length of the bubble:

$$z = x + \beta r + l + h_2 \tag{2}$$

(β in radians)

An equilibrium has been reached, which gives a rise to $h = h_1 = \quad = h_2$. This height of the water column in vessel 1 corresponds to angle $\alpha = \pi - \beta$:

$$h_1 = x + r \sin(\pi - \beta)$$

$$h = x + r \sin(\beta) \tag{3}$$

By combining equations (1), (2), (3) we will get the following relationship for β:

$$2x + \pi r + \delta h = x + \beta r + l + x + r \sin(\beta)$$

$$\beta + \sin(\beta) = (\pi r + \delta h - l) : r \tag{4}$$

We can see that angle β is not dependent on x, because if the angel changes then the water column heights change in the same way – and so do the pressures – the equilibrium position of the bubble will therefore not change.

II.B

In the other model example (see Fig. 19c) the siphon is formed by two straight sections of the hose.

1) Let us assume that the surface level difference equals to σh and α, β, is the inclination of the siphon sections toward the vertical axis (see Fig. 19c). We will now suck in a bubble of the exact legth of $l = \sigma h/(\sin (90^\circ - \beta) = \sigma h/\cos (\beta)$. We will place the bubble in this position the upper end of the bubble point W, the lower end of the bubble surface level of vessel 2. It is then evident that the pressures acting on the bubble are zero. When the bubble moves upwards, then due to the straighlness of section 2, there will be an increase of the pressures which are caused by the water columns. The increments in pressure will be the same, i.e. the equilibrium will not be disturbed. We can deduce then that any position of the bubble in section is an equilibrium position. Since the bubble is moving from vessel 1 to vessel 2, it will take the first possible position corresponding to the state equilibrium: the "rear" end of the bubble will stop at point V.

2) We will suck in a bubble of the length of $l < h : \cos \beta$. We will place the bubble in this position: "rear" end of the bubble-point W. Pressure p_2 is now evidently higher than pressure p_1 (cf. 1) and the bubble will be set in motion towards vessel 2. Due to the straightness of section 2, however, there will be no pressure change and the bubble will "travel" as far as vessel two and the transport of the water will continue. Here we do not have to consider the movement of the bubble in the opposite direction, because it would cause an increase of pressure p_2 and pressure p_1 would drop due to the hose bend at point W. This would increase the pressure difference and the energy of the system would increase. The bubble will therefore move toward vessel 2.

3) Now all that remains is to solve the problem for $a + b \geqq 1 > \delta h : \cos \beta$ (see Fig. 19c). After a state of equilibrium sets in, the heights of the water columns in the individual sections of the siphon must be the same (h). Two equations may be applied to these heights:

$$\sin (\pi - \beta) = h : (b - (1 - x)) \tag{1}$$

$$\sin (\pi - \alpha) = h : (a - x) \tag{2}$$

$$h = (b - 1 + x) \cos \beta \tag{1a}$$

$$h = (a - x) \cos \alpha \tag{2a}$$

By combining (1) and (2) adjusting equations (1) and (2), we find a relationship for x:

$$x = (a \cos \alpha + (1 - b) \cos \beta) : (\cos \beta + \cos \alpha) \tag{3}$$

By substituting this value into (2a), we get a relationship for the water column height h:

$$h = \cos \alpha \, (a - (a \cos \alpha + (1 - b) \cos \beta) : (\cos \beta + \cos \alpha)).$$

The value demonstrates well the position of the bubble in the siphon.

Conclusion:

In the first case, when we pull the hose out from the lower vessel, the same rule applies, i.e. to ensure continuing transport of the water, we must make sure that at least a certain quantity of the water in the hose is kept all the time a little under the surface level of the upper vessel. As regards the other case, i.e. determination of the ultimate position of the equilibrium state from which the transport of the water can continue, it is not possible to define a generaly applicable rule, the problem has to be solved individually for each respective case, taking into account the shape and the dimensions of the siphon. As a general rule, the position of the bubble (if there is any static equilibrium at all) may be determined on the basis of a zero pressure difference (i.e. the heights) in the individual sections of the siphon.

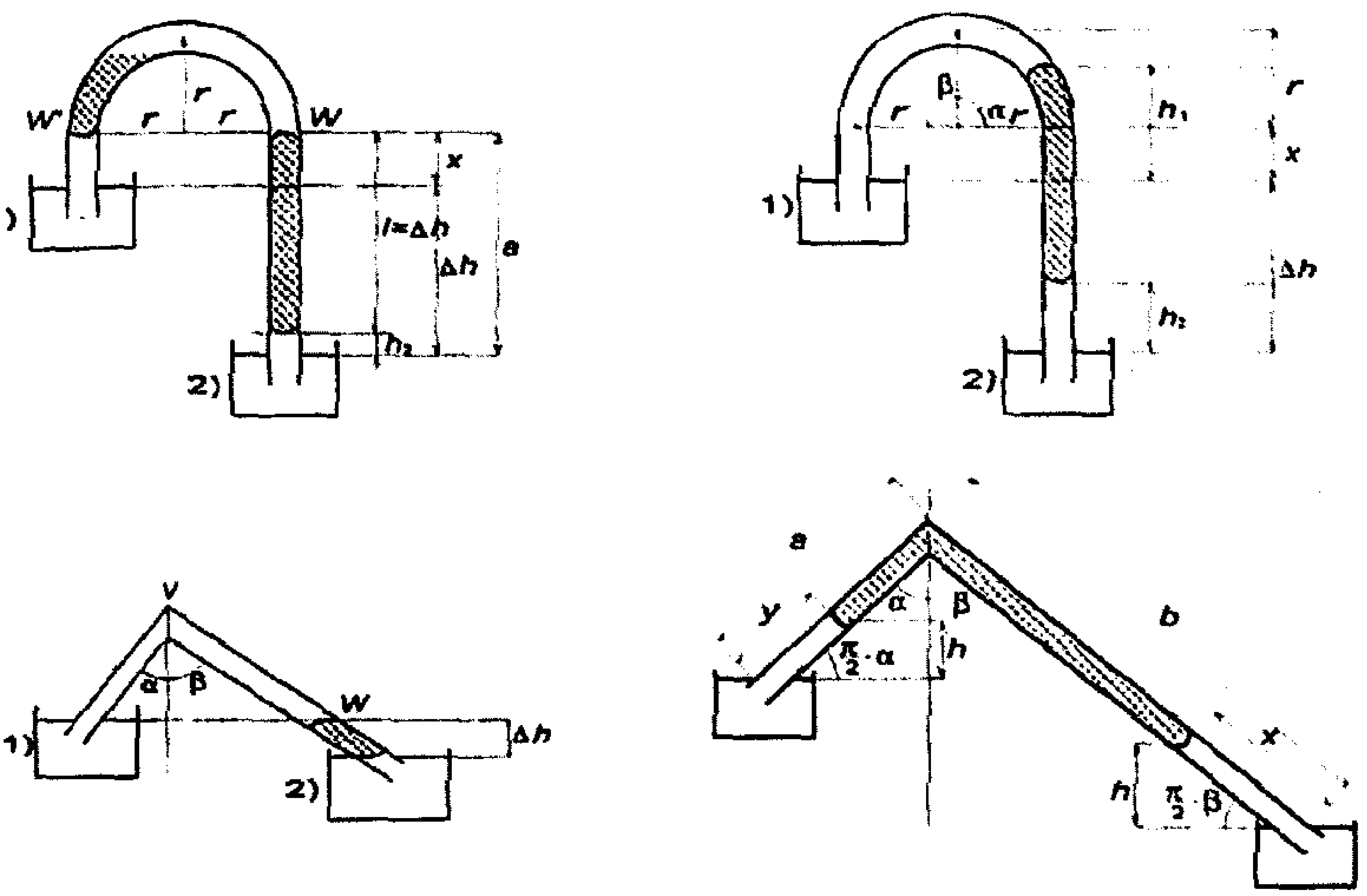

Fig. 19(a-d) Various possible positions of the bubble in the hose

Problem No.12 – Boiling

This problem was solved experimentally as well as theoretically.

Experimental part:

We inserted metal balls into a vessel containing water heated to 90 – 100°C. These balls had different radiuses and were heated to temperature 150 – 200°C. The balls were heated in an electric kiln with a stabilized temperature controlled by a switch mercury thermometer. The values of evaporation intensity we determined with the aid of a digital balance with graduations of 0.01 g. We plotted on a graph dependence of the measured evaporation intensity (mg/s) on the time. The evaporation intensity was uniformly decreasing. After about 15 seconds, the evaporation intensity rapidly increased and within the following 4 seconds it dropped almost to zero.

Theoretical analysis:

In the theoretical part of the solution, we supposed uniform heat conduction from the middle to surface of the metal ball, where it's transfered to water. The ball is composed of an infinite number of spheres of negligible width. The average surface area of the ball is S. The sum of heat that will be transferred to the liquid was divided into n parts. When the first part of the heat is abstracted, the temperature of the metal ball goes down, and the next part of the heat Q/n is abstracted for a longer time. From the known value of abstracted heat we determined the mass of evaporated water. Then we derive the theoretical dependence