Problem No.16 – The Moon and the Sun

Solution:

In this problem we have to determine, when it's possible to see the Moon and the Sun at the same moment. There are two different possibilities in understanding the word "to see". We can imagine an observer with many astronomical devices or another one without any devices. In the first possibility we need only to compute if the Moon and the Sun are over the horizon i.e. it is possible to observe it in principle. In the second possibility we need also determine if the observer can see both of the Moon and the Sun, which means for example if the Moon isn't too near to the Sun. The second possibility is more problematic, because it isn't possible to determine the weather one year ahead and so to say if the Moon or the Sun will or won t be behind some cloud.

Firstly I'll talk about computing the position of the Sun.

We use Kepler s equation for determining the position the Earth on its orbit. The form of this equation is

$$2(t - t_0)/T = E - e \cos E$$

t ... current time
t_0 ... time of the last perihelium
T ... revolution period
e ... numerical eccentricity of Earth's orbit
E ... eccentric anomaly (an angle describing the position on an orbit, see Fig.30).

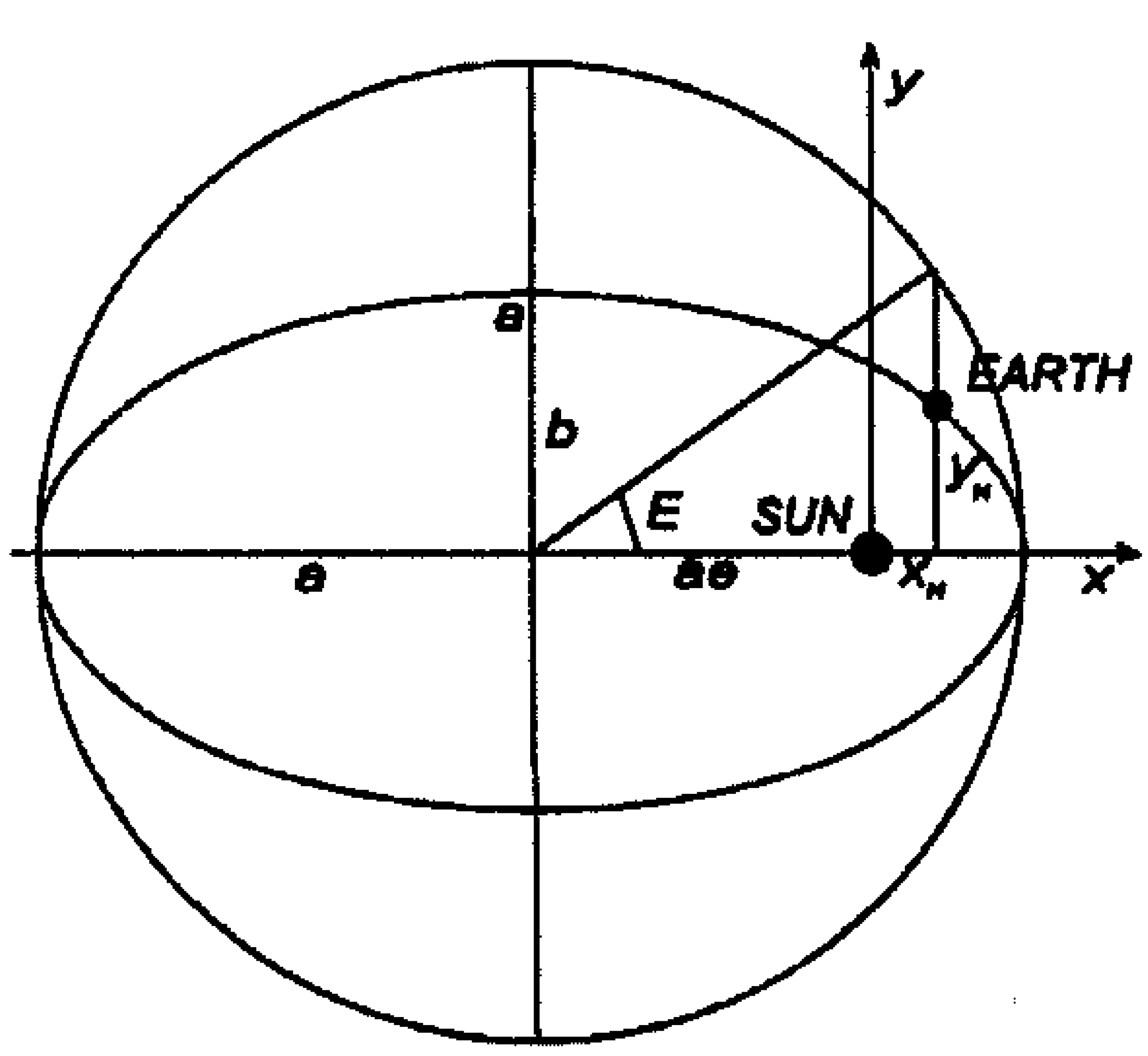

Fig.30 Explanation of parameters used in the solution

We regretably cannot express from Kepler's equation as an eccentric anomaly (which is an angle we are interested in), but we can easily compute it numerically by using an iterative method. We write the equation in another form:

$$E = 2(t - t_0)/T + e \cos E,$$

and put $E_0 = 0$. We compute the E by this way:

$$E_{i+1} = 2(t - t_0)/T + e \cos E_i$$

$E \approx E_i$, for sufficiently big i

This method has the advatage that it is very fast – i.e. the sequence E_i convergates very quickly to the exact value.

From eccentric anomaly we can simply compute coordinates of the Earth in heliocentric coordinate system, which means in coordinate system with origin in the Sun and x-axis directed to the perihelium (see Fig.30)

$$x_H = a\,(\cos E - e)$$

$$y_H = b \sin E = a\sqrt{1-e^2}\,\sin E$$

$$z_H = 0$$

Now we need to determine the coordinates of the Sun in the horizontal coordinate system, which has origin in the observer and the x-axis directed to the south. For this we use rotation matrices.

If we have a point with coordinates x, y, z, then after rotating of the coordinate system around the x axis for angle α counter-clock-wise (i.e. from y axis toward z axis) we get the new coordinates of the point x', y', z' of this point from equation:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = R_X(\alpha)\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

analogical for R_y, R_z.

By using these matrices we can express the horizontal coordinates of the sun in this way:

$$\begin{pmatrix} x_S \\ y_S \\ z_S \end{pmatrix} = R_X(90^\circ - \varphi)\, R_Z(\vartheta - 90^\circ)\, R_X(\varepsilon)\left(-R_Z(-\omega Z)\begin{pmatrix} x_H \\ y_H \\ z_H \end{pmatrix}\right),$$

φ ... geographical latitude
ϑ ... stellar time
ε ... angle between ecliptic and Earth equator
ωZ ... angular distance of Earth's orbit's perihelium from spring point

We know that horizontal coordinates of the zenith are $x_Z = 0$, $y_Z = 0$, $z_Z > 0$. From the coordinates of the Sun and the zenith we can simply get the angle between them and so also the angular height of the sun. We use the well known formula $\boldsymbol{ab} = |\boldsymbol{a}||\boldsymbol{b}|\cos\alpha$ ($\boldsymbol{a}$, $\boldsymbol{b}$ are vectors and α is angle between them). So angular height of the sun is

$$\pi/2 - \text{angle between sun and zenith} = \pi/2 - \arccos(\boldsymbol{s}\,.\,\boldsymbol{z}/(|\boldsymbol{s}|\,.\,|\boldsymbol{z}|)) =$$

$$= \pi/2 - \arccos(z_s/|\boldsymbol{s}|),$$

where $\boldsymbol{s} = (x_s, y_s, z_s)$ is vector to the sun, and $\boldsymbol{z} = (0, 0, 1)$ is vector to the zenith.

This gives us the angular height of the middle of the Sun, supposing there is no atmosphere on the Earth. To determine whether the Sun is visible or not we need to considere Sun s diameter and refraction. Sun s diameter is permanent (the changes caused by an elliptic orbit of the Earth can be neglected). Near to the horizon we can also suppose a constant refraction. So the sun is visible when its angular height is greater than $-49' = -(34' + 15')$ (i.e. the refraction plus Sun's perimeter).

Now to the moon coordinates. The computation is similar as the one by the Sun, the formula for coordinates converting in this case is

$$\begin{vmatrix} x_S \\ y_S \\ z_S \end{vmatrix} = R_X(90^\circ - \varphi)\, R_Z(\vartheta + 90^\circ)\, R_X(\varepsilon)\, R_Z(-\Omega)\, R_X(i)\, R_Z(-\omega M) \begin{vmatrix} x_H \\ y_H \\ z_H \end{vmatrix},$$

Ω ... astronomical longitude of the Moon's orbit's perigee
ωM ... angular distance of Moon's orbit's perigee
i ... inclination of Moon's orbit to orbit

But the moon's orbit is influenced by the Sun. It causes for example that the angular line rotates slowly, the elipse which is the lunar orbit rotates in the orbit plane and so on. (Angular line is the line of intersection of the lunar orbit plane and the ecliptic.) There is many of these interferences. Some of these were included in our solution. We found out the formula describing for example angle of the angular line on dependence on the time, and then counted with this value instead of the constant in computing Sun coordinates.

Now we are able to decide if at a given time the Sun and the Moon are over the horizon. The second step in our solution is to find out times of Moon s and Sun s rises and sets from it.

We used the numerical method – a very suitable method for this purpose. For Sun rises and sets we know an interval within which is exactly one rise or set. In our geographical latitude it is an interval between the noon and the midnight for every day. For Moon s rise and set is the situation more complicated because we don't know any interval which would contain exactly one Moon s rise or set and we firstly need to determine these intervals. We found out the Moon s angular height during all the year at one hour intervals. There is a Moon s set in the interval where at the beginning the Moon is over the horizon and at the end under the horizon. In the interval where the situation is inverse there is a Moon s rise. So we established all the Sun and Moon rises and sets during this year.

Finally we have to make a table of instances of the time when both of the Moon and the Sun are over the horizon. But it is very easy now. We know the times of Sun and Moon rises and sets so we only overlap it and get the desirable table. There is the first part of this table in appendix. Then we just consider if the Sun and the Moon are over the horizon and we are ready (see Fig. 31).

Also we solved the other possibility of understanding this problem. We found out that with eyes only it is possible to see the moon first 36 hours after or before the newmoon. The time of the newmoon can be easily determined from astronomical tables or using our program. Then we cancel the 36 hour interval before and after each newmoon from our table. But we insert again the time of the eclipse there. This year could be seen an annular eclipse of the sun. It was at the time approximately from a quarter to eight to half past eight in the evening of May the 10.

So we can, in this way, make our table a little more accurate. But it must been said that this is NOT accurate enough – there is a very large influence of clouds (according to weather). Because of it, it is better to get only the first table as a result, with warning that it determines only where it is possible to observe (not only by visual methods) both of the Moon and the Sun.

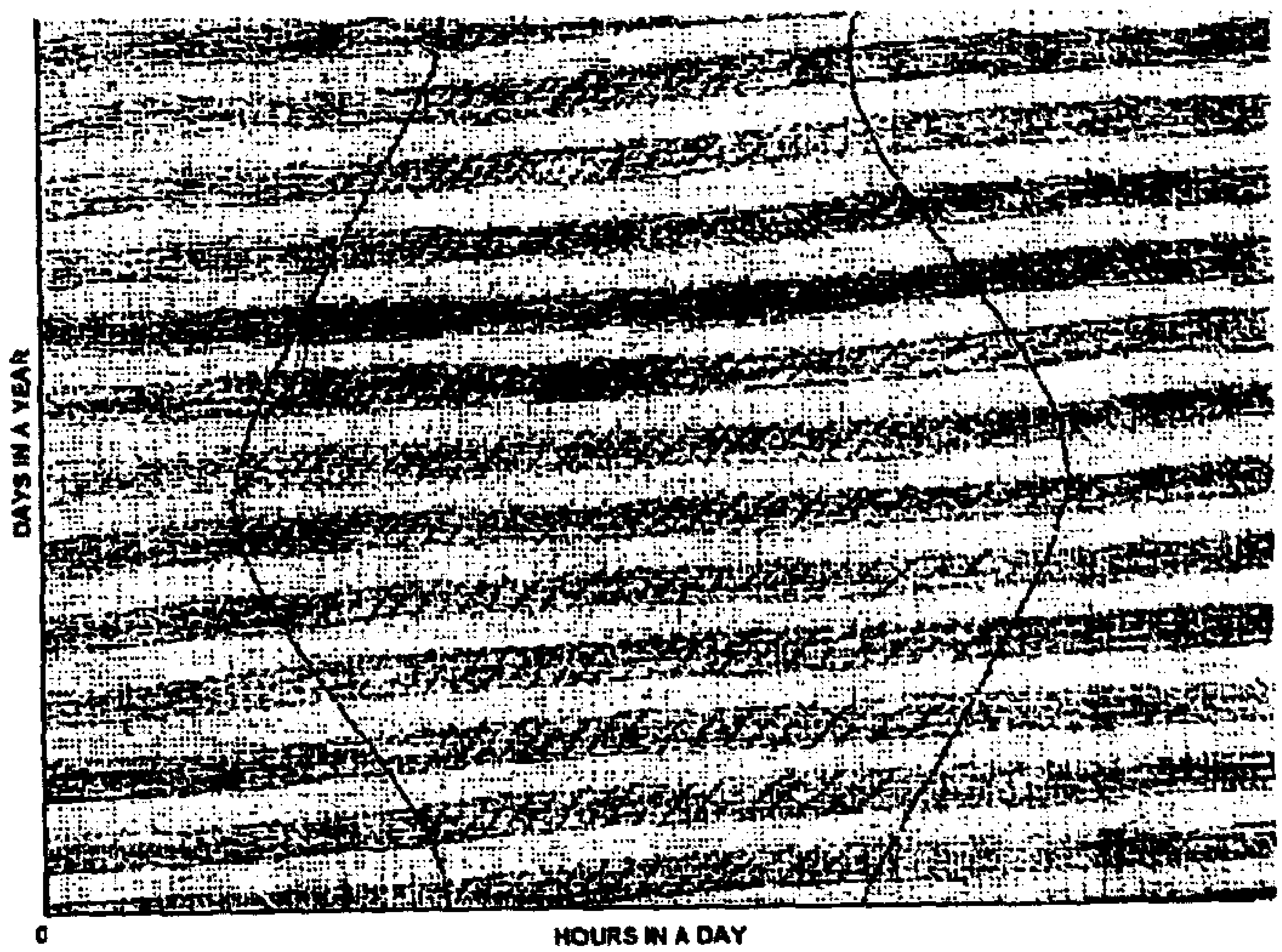

Fig.31 The result – the vertical curve stands for the ranges of visibility of the Sun, the horizontal curves for the Moon's ones

Problem No.17 – Straw

This problem belongs to the most interesting, but most difficult ones of the tournament. The question may be very important in real life, but it's really difficult to solve it in the exact terms of the physics.

First of all, there's a question of defining a "safe fall". We understand that only a fall which doesn't cause any injuries can be called "safe", but what physical qualiies are related to it? We did a lot of research in medicine, biophysics etc. and finally we found an old Czechoslovak norm for rescue-nets, which says that maximum dynamical force acting on one's body should not exceed 10 kN. We took this as a basis for our further work.

In order to express some fundamental physical properties of straw, we got some samples and did several experiments. It's obvious that the straw is compressed under an outer force in some way, resisting this force and effectively slowing down any motion. When the force (pressing down on the straw) is small, it s not difficult to find out the relation of the contraction to the pressure (it's obvious that the contraction doesn't depend on the area of the straw used). We found out soon that the results can be expressed in the form $s = hc(1 - e^{bp})$ where "s" is the contraction, "h" original height of the straw, "c" is the coefficient of maximum contraction and "b" another coefficient of the "rigidity" of the straw. We found approximate values $c = 0{,}85$; $b = 3{,}4.10^{-4}$ Pa^{-1} using experimetal means. This formula is quite accurate under realistic conditions.

As the straw acts with a force on the falling body, it absorbs energy. If one is to fall safely, all his/her energy must be absorbed by the straw (some remnants, equivalent to a fall from the height of several centimeters, can be neglected), while the retarding force must remain within given limitations. This is