2. IONIC MOTOR

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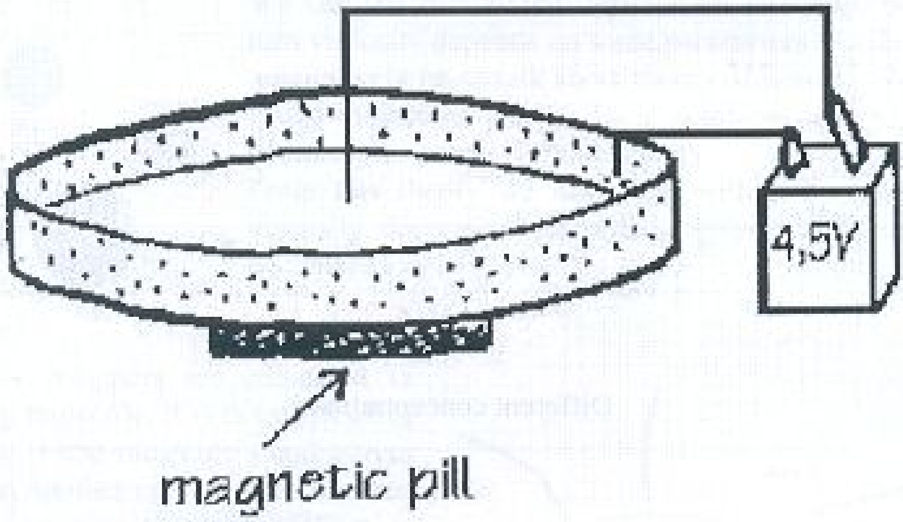
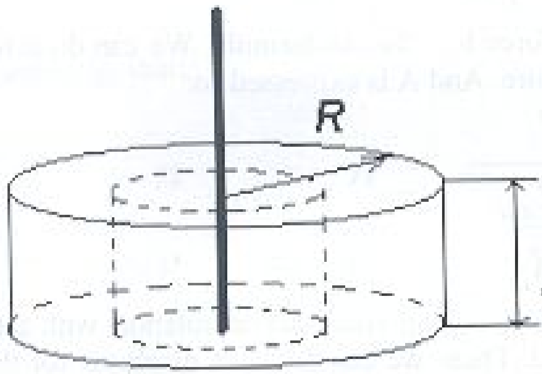
School № 42 named after I.N. Vekua



If we make experiment we can see rotating liquid in the Petri dish. At first I want to explain this phenomenon qualitative and then go to the theoretical description.
Every body moving in the liquid "feels" resistance of it. In our case movement of small particles in the electrolyte is affected by liquid – liquid attends to stop these bodies but from the Newton's third law we know that in this case these bodies will act on the liquid with same force. And liquid posesess movement.

Now let us try to describe this phenomenon by some mathematics.
Before we start I want to say, that I assume electric and magnetic field uniform.

To calculate field in our system we can consider it like cylindrical condenser (fig. 1) and write Gauss theorem for the cylindrical surface:



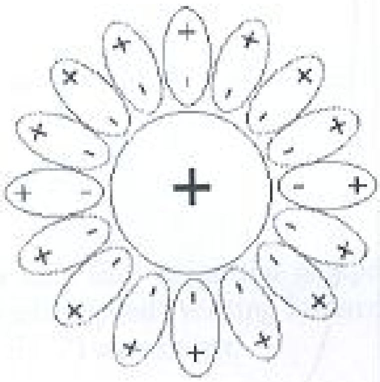
$$ES = \frac{q}{\varepsilon\varepsilon_0}$$

Expressing q and C:

$$q = CU; \quad C = \frac{2\pi\varepsilon\varepsilon_0 H}{\ln\frac{R_{out}}{R_{in}}}$$

Fig. 1

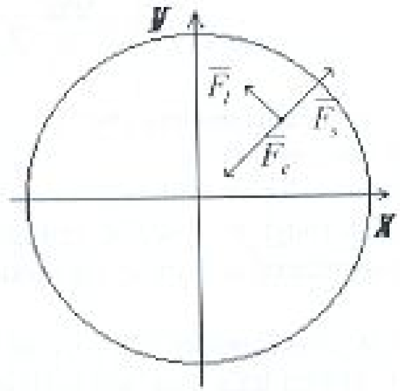
So we can get field tension:



$$E(r) = \frac{U}{\ln\frac{R_{out}}{R_{in}}}\frac{1}{r}$$

Fig. 2.

Now our target is to describe motion of single ion, for this we have to know that in electrolytic solvates we have not ions but more complex particles – solvions. Solvions appearance can be explained by existence of electric interaction between ions and solvent molecules, see figure 2. And from the solvatation chemistry we know that solvions can be considered like small elastic balls. Let's write Newton's second law in both vector and projections form for the moving solvion:



$$m\,\vec{a} = \vec{F}_s + \vec{F}_e + \vec{F}_l$$

Fig. 3.

On the figure 3 you can see choused coordinate system and forces vectors. And in projections one can write:

$$m\ddot{x} = -6\pi r_s \eta \dot{x} - \frac{Aq}{r^2} x - q\dot{y}B$$

$$m\ddot{y} = -6\pi r_s \eta \dot{y} - \frac{Aq}{r^2} y + q\dot{x}B$$

Here you can see that we've expressed resistance force by Stocks formula. We can do it for this case ("microworld") it's known from the literature. And A is expressed so:

$$A = \frac{U}{\ln \frac{R_{out}}{R_{in}}}$$

So we get differential equations, which we can solve using computer simulation with any numerical method, for example Runge-Kutt method. Then we can get such diagrams for the solvion trajectory (fig. 4) and velocity (fig. 5):

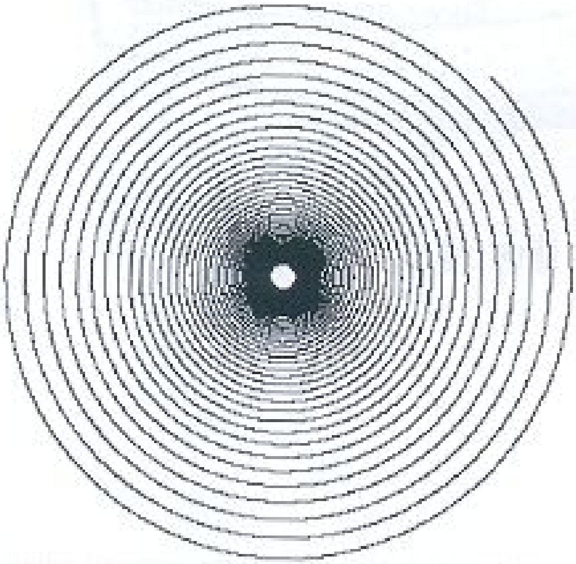


Fig. 4.

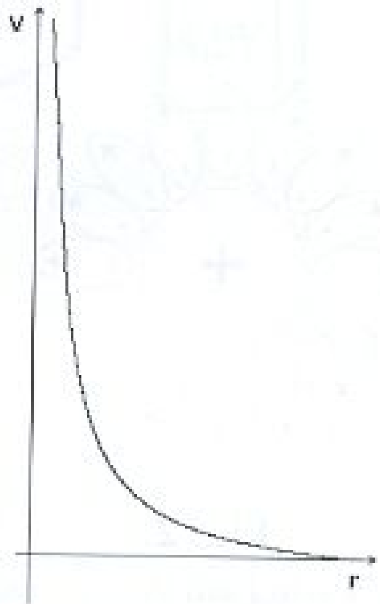


Fig. 5.

Now we can see that character of the trajectory is right but velocity dependence doesn't satisfy us, because we know from the hydrodynamics that near the surface of body immersed in the liquid velocity of the liquid flow is zero, but we didn't get this from our model. So this model is too rough to describe velocity of the ion. Let's go to our second model – in it we describe dynamics of not single ion motion but of the whole water.

Now let's consider that solvion velocity consists of two components: tangential and radial (fig. 6). At first let's find radial component.

One can write expression for the current flow:

$$j = n_{+}qv_{+} + n_{-}qv_{-}$$

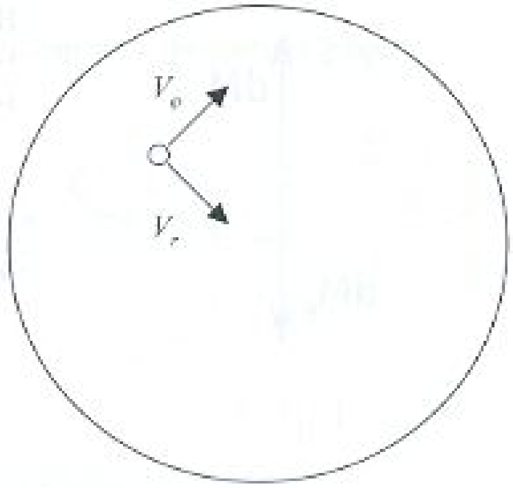


Fig. 6.

Assuming, that:

$$n_{+} = n_{-} = n$$

$$v_{+} = v_{-} = v_{r}$$

and

$$I = jS \; ; S = 2\pi rh \; ; n = \frac{N}{V}$$

So, radial velocity is expressed as follows:

$$v_r = \frac{MIR}{8emN_A}$$

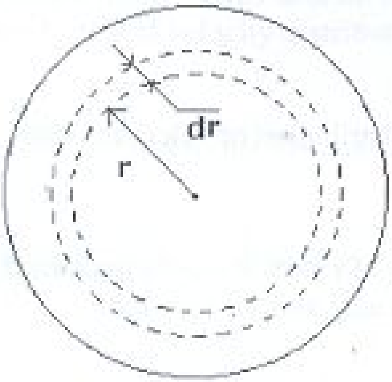
Now let's find tangential component of solvion velocity(this will be equal to water rotatio tangential speed).Writing Lorenz force moment for the thin layer of the liquid(with a thicknes dr – fig. 7) we can get:

$$\left.\begin{aligned} dM_l &= dqBv_r r \\ dq &= nq_0 dV \end{aligned}\right\} \Rightarrow dM_l = \frac{BI}{R} r^2 dr$$

For the moment of the viscous resistance force we have:

$$dM_R = \left(\left(-\eta S \left.\frac{dv_\varphi}{dr}\right|_{r+dr}\right) - \left(-\eta S \left.\frac{dv_\varphi}{dr}\right|_{r}\right)\right) r =$$

$$= -2\pi h \eta \frac{d}{dr}\left(r^2 \frac{dv_\varphi}{dr}\right) dr$$



Because we are observing established rotation, one can equate these two moments (figure 8.):

Fig. 7.

$$dM_l = dM_R \Rightarrow$$

Here we've got expression for the solvion tangential velocity which is equal to water rotation tangential velocity.

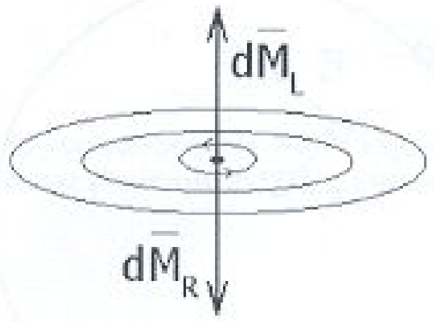


Fig. 5.

$$v_\varphi = \frac{C}{6}(R^2 - x^2) + \frac{Cr}{6x}(R-r)(R-x)$$

where

$$C = \frac{BI}{2\pi\eta Rh}$$

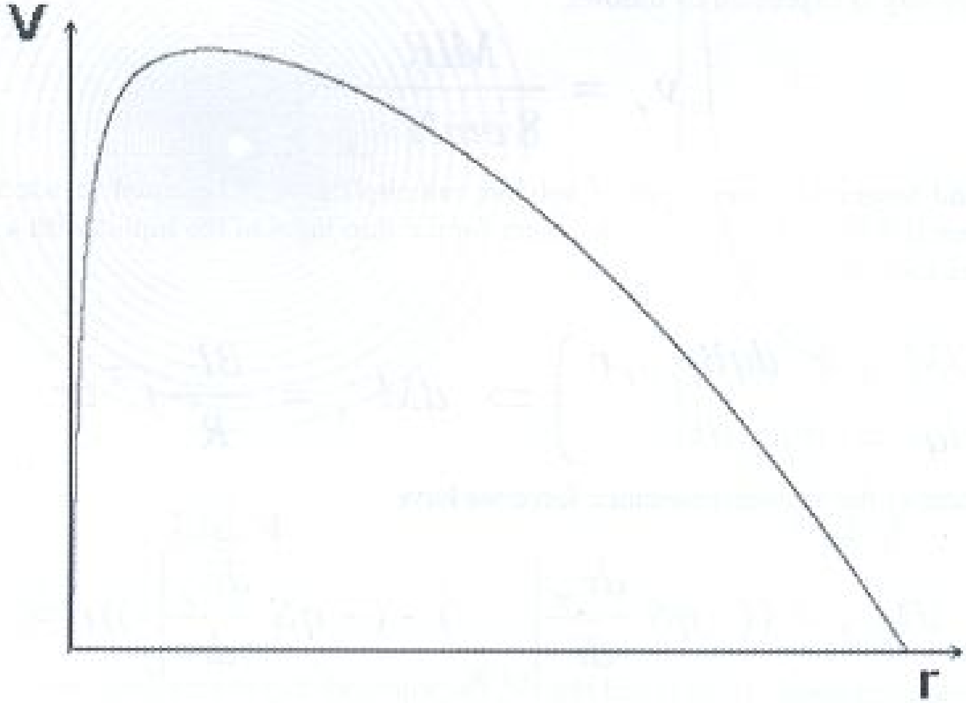
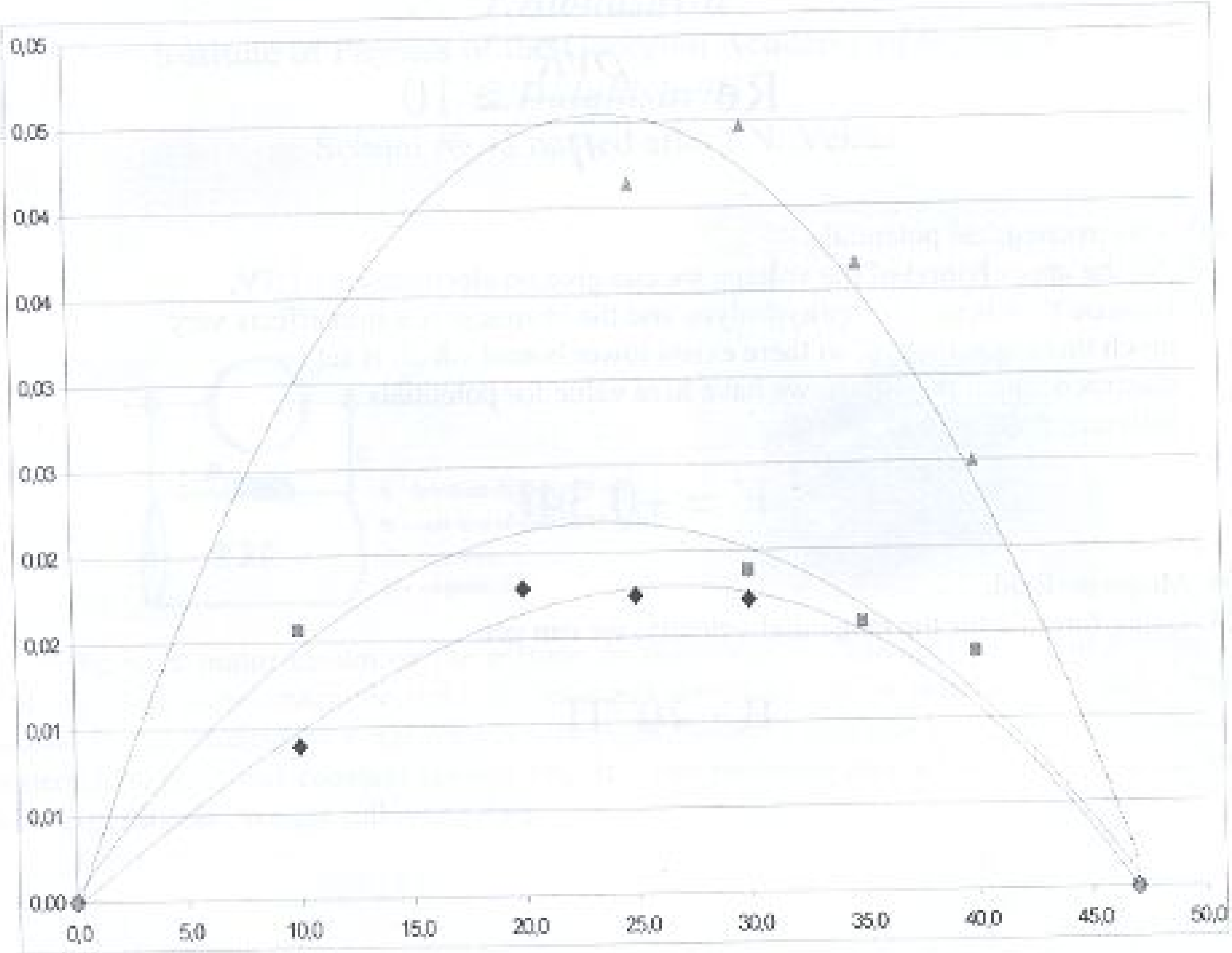


Fig. 9.

Tangential velocity dependence on distance from center – distribution of velocities onto radius.

Here we can see that our graph is more "compressed" near zero of r axis, it can be explained if we remember that viscous force depends on surface square and near zero of r we have relatively small surface of inner electrode.

Now let's see some experimental data. Here you can see experimental graphs.



I want to clarify some unclear moments in this problem. At first about radial component of water velocity – does the liquid in our system poses radial velocity? The answer is "no" and it's because the both types of ions move along radius but in opposite directions, when one ion "tries" to move water in one direction , other "tries" to move water in opposite direction so the water stays stationary. But if we throw in electrolyte small bodies (chalk dust for ex.) they doesn't stay on fixed distance from inner electrode. It can be explained by existence of velocity, and correspondingly pressure gradient. And because of pressure gradient there appears force (which acts on our small body) and this force makes small body moving to the peak point of velocity distribution.

Additional questions that may appear in this problem.

Q1: Lifetime of this system.
A1: We know Faradays formula :

$$M = KIt$$

And if we put in I=1A, M=10g we can get that time after which all electrolyte will exhaust is t ~ 3 hours.

Q2: Reynolds number.
A2: R here is Petri dish radius.

$$\mathrm{Re} = \frac{\rho v R}{\eta} \approx 10$$

Q3: Electrochemical potentials.
A3: So the upper bound of the voltage we can give on electrodes is ~1,1V, because then begins water hydrolyze and the chemical reaction affects very much this phenomenon, so there exists lower bound which is set by electrochemical potentials, we have here value for potentials difference V :

$$V \approx +0.34V$$

Q4: Magnetic field.
A4: Using formula for the tangential velocity, we can get:

$$B \sim 10^{-5} \mathrm{Tl}$$