# 4. SOAP FILM

***Z. Osmanov, A. Razmadze***
School № 42 named after I.N. Vekua

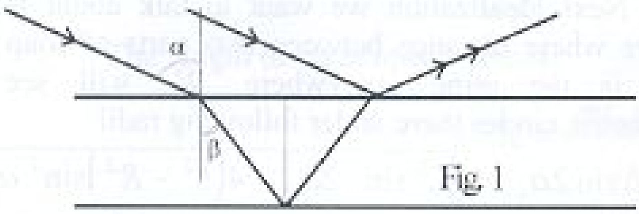


Fig 1

In soap film color appearance is caused by interference. Incident wave divides on two parts: reacted from the upper and lower sides. Film must be thin enough for these waves to meet each other. They are brought together and interfered to each other. This gives us possibility to watch interference picture.

It is obvious that these two parts have passed different ways. If reacted wave falls behind the reflected one by whole number of waves, then they will meet each other in phase and will happened increasing of summer wave, we will see increased correspondent color. If reacted wave will fall behind by odd number of halfwaves will happened decreasing of summer wave, we will not see correspondent color.

Maximum and minimum distribution in interference picture depends on light waves length, on reflection index, angle of incidence and films thickness. Dependence of interference from these parameters is given by path length's difference:

$$\Delta r = 2d\sqrt{n^2 - \sin^2\alpha} + \frac{\lambda}{2}$$

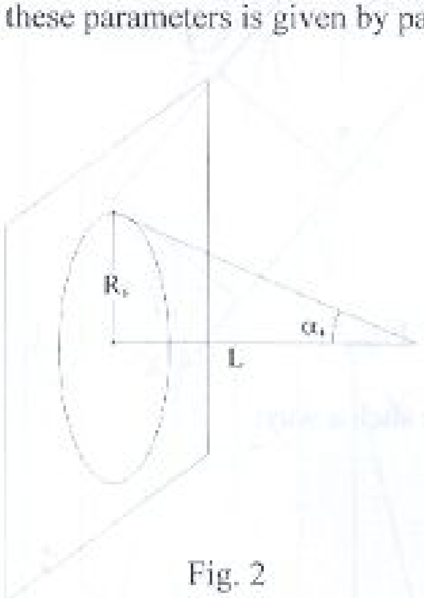


Fig. 2

Real soap film is quit difficult object of observation. In this article we consider it as totality of some idealizations. They are rough, but considering them we can better understand what happens in reality. The first idealization we want to talk about is plane-parallel plate. It is obvious that the only parameter changing in the expression of path length's difference for different color is angle of observation (it correspondents to the angle of incidence). The angle under which we observe increased color with wavelength λ is:

$$\alpha_K = \arcsin\left(\sqrt{\frac{\lambda nk}{d} - \frac{\lambda^2 n^2 k^2}{4d^2}}\right),$$

where k is number of maximums.

It is not difficult to understand that passing from plane picture (Fig. 1) to space one (Fig. 2) we will see concentric circles of different colors. These circles are with following radii:

$$R_x = Ltg\ \alpha_k,$$

where L is distance between eye and plate.

Speaking about colors we must mention about intensities with which we see them. This is the expression for intensity at interference:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\frac{2\pi\Delta r}{\lambda}\right)$$

The intensity of the wave that was reflected from the lower and upper sides is respectively:

$$I_1 = I_0(1-R)R(1-R) \qquad I_2 = I_0 R$$

We got them from Frenel's formulas. In them reflection number is following:

$$R = \frac{1}{2}\left[\frac{\sin^2(\alpha-\beta)}{\sin^2(\alpha-\beta)} + \frac{tg^2(\alpha-\beta)}{tg^2(\alpha+\beta)}\right]$$

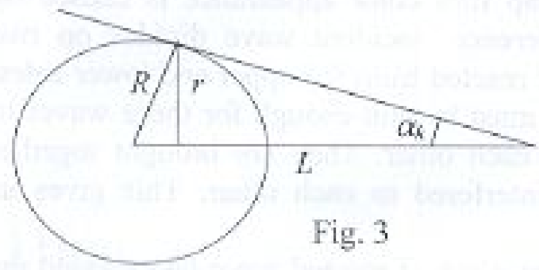



Fig. 3

Next idealization we want to talk about is sphere where distance between two parts of soap film is the same everywhere. We will see concentric circles there under following radii:

$$r = L\sin 2\alpha_k - \sqrt{L^2\sin^2 2\alpha_k - 4\left(L^2 - R^2\right)\sin^2\alpha_k}$$

L is distance between our eye end center of soap bubble.

How we know under the action of gravitating force soap film changes its thickness (liquid in it flows). To bring in proposed solution times factor we consider liquids flowing. For small element of liquid we write Newton's II law:

$$0 = ma = mg_x - \eta S\frac{\Delta V}{\Delta z}, \text{ ( flow has settled ).}$$

Having made some transformations we have:

$$Sd\rho g_x \sim \eta S\frac{V}{d}$$

From this we get middle velocity of flowing:

$$V \sim \frac{\rho g}{\eta} d^2$$

After this from Law of mass conservation we have:

$$-Vd\Delta t = l\Delta db$$

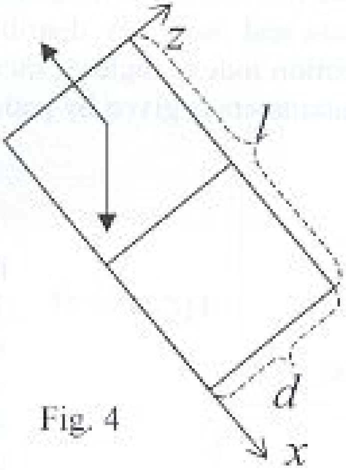



Fig. 4

From this we can get, that film thickness depends on time such a way:

$$d(t) = \frac{1}{\sqrt{\frac{2\rho g_x t}{\eta l} + \frac{1}{d_0^2}}}$$

(We assumed that thickness decreases uniformly everywhere.)

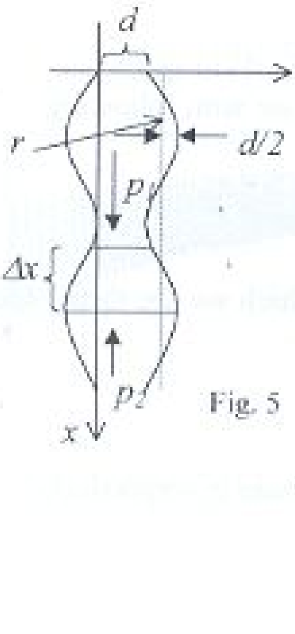



Fig. 5

Flow is laminar when $\mathrm{Re} \in [20;30]$

When $\mathrm{Re} \in [30;50]$ appears regime of waviness flow. Under this regime surface tension forces became comparable with others. In cause of it appears capillary wave, which is undamped in time.

Newton's II law in this case is:

$$\Delta ma = -\eta \Delta S V' + \Delta m g_x - \frac{\Delta p}{\Delta x} \Delta x S_1$$

Middle velocities in both regimes are approximately equal:

$$\langle V_W \rangle \approx \langle V_L \rangle \sim \frac{\rho g}{\eta} d^2$$

The length of undamped wave is:

$$\lambda \approx \frac{\rho g_x d^2}{\eta} \sqrt{\frac{2\sigma d}{\rho}}$$

This wave moves with velocity $c \approx 3V$

The frequency of given wave is:

$$f = \frac{1}{v^2} \sqrt{\frac{2\sigma d}{\rho}}$$

For the function, which expresses thickness of the film we have:

$$z(t) = d\left(\frac{1}{2}\cos(\omega t - kx) + 1\right)$$

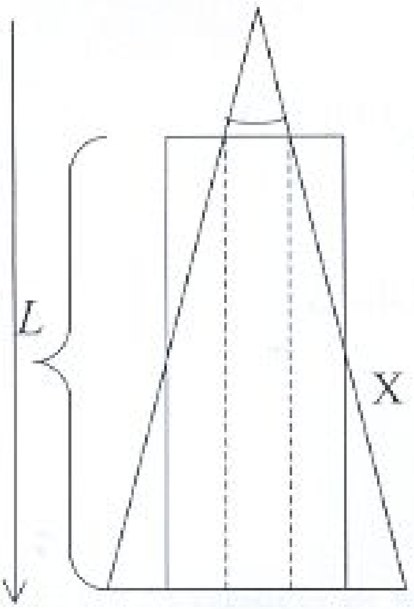



Fig. 6

In cause of flowing film takes wedgeble shape. During Δt time soap film's upper part decreases by Δd:

$$\Delta d = \frac{\rho g d^3}{\eta l} \Delta t$$

For the angle of opening we have:

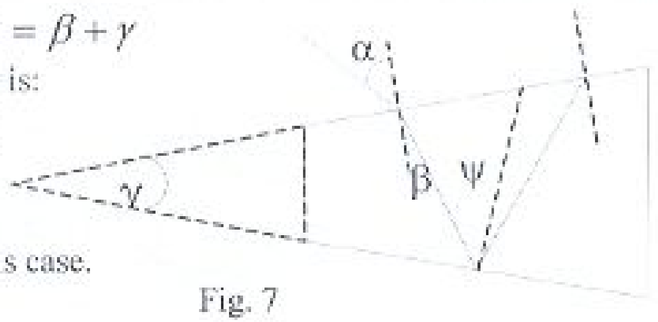
$$\gamma(t) = \frac{1}{l}\left[d - \left(\frac{2\rho g t}{\eta l} + \frac{1}{d_0^{\,2}}\right)^{-\frac{1}{2}}\right]$$

It's the thickness along x-axis $d(x) = d(t) + 2\gamma(t)x$

$$\psi = \beta + \gamma$$

The path length difference for wedge is:

$$\Delta r = 2d(x) n \cos\psi + \frac{\lambda}{2}$$




Now we can judge about interference in this case.

Fig. 7

On the other hand the film is membrane, it joins into resonance with frequencies, coinciding with its natural frequency.

The rough estimation of soap film's natural frequency is:

$$\nu \sim \sqrt{\frac{\sigma}{m}}$$

The soap film mass is:

$$m \approx 4\pi R^2 \rho d$$

$$d \in \left(10^{-7};10^{-5}\right)m$$

Theoretical frequencies which we got are following:

$$\nu \in (100;1000)Hz$$

It is in good agreement with experiment.