6. Singing Glass

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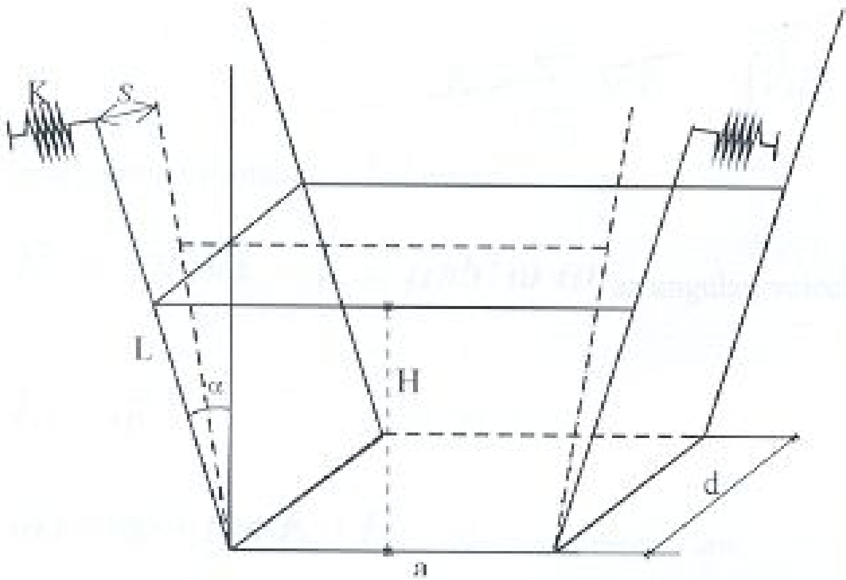
The main part of the problem is to find how depends frequency of the sound on some parameters. To find this dependence we must find a frequency, but the glass has very difficult form and we'll get too difficult mathematical equations. Therefore, we can solve this problem for a model of the glass. In this paper, I'll consider three model of the glass.

First model is the truncated pyramid, which you can see on the picture.



This model gave me best results because of this it's main model and I'll consider it more exact than others.

Now some comments about variables that are present here. Some of them are clear I think, but I want to talk about S and K. S is the little shift of the glass wall in some moment of the time. In real case(I mean in the glass) we have oscillations because of there is elasticity of the glass, so in model instead of glass elasticity I brought in spring with elasticity K.

So to find a frequency we must write the Low of Energy Conservation and after deriving it we'll get formula like formula (1). All energies which are present in this system:

Kinetic energy of the inner water

$$E_{Kw\_in} = \frac{\rho d \dot{S}^2 H^3}{12 L^2 \cos^4\alpha} \times \left(\frac{4a+3H\tan\alpha}{3} + \frac{4aH^2+3H^3\tan\alpha}{(a+2H\tan\alpha)^2} \right)$$

Kinetic energy of the outer water

$$E_{Kw\_out} = \frac{\rho d H^3}{12L^2\cos^4\alpha}\cdot\frac{(4D-3h\tan\alpha)}{6}\left(\frac{h^2}{4(D-h\tan\alpha)^2}+\frac{1}{36}\left(1+\frac{h\tan\alpha}{2(D-h\tan\alpha)}\right)^2\right)$$

Kinetic Energy of glass wall

$$E_{Kwall} = \frac{M\dot{S}^2}{3}$$

Potential energy of the spring

$$E_{Pspring} = KS^2 - S\left(\frac{Hgd}{2\cos^2\alpha L}\left(\rho_{in}H^2 - \rho_{out}h^2\right) + \frac{Mg}{\sin\alpha}\right) + C_1$$

Potential energy of the inner water

$$E_{Pw\_in} = \frac{\rho dgH^2S}{2L} \cdot \frac{a + H\tan\alpha}{a + 2H\tan\alpha}$$

Potential energy of the outer water

$$E_{Pw\_out} = C_2S$$

Potential energy of the glass wall

$$E_{Pwall} = \frac{1}{2}\frac{MHSg}{L\sin 2\alpha}$$

$$E_{Kw\_in} + E_{Kw\_out} + E_{Kwall} + E_{Pspring} + E_{Pw\_in} + E_{Pw\_out} + E_{Pwall} = const$$

$$\ddot{x} + \omega^2 x + C = 0 \quad (1)$$

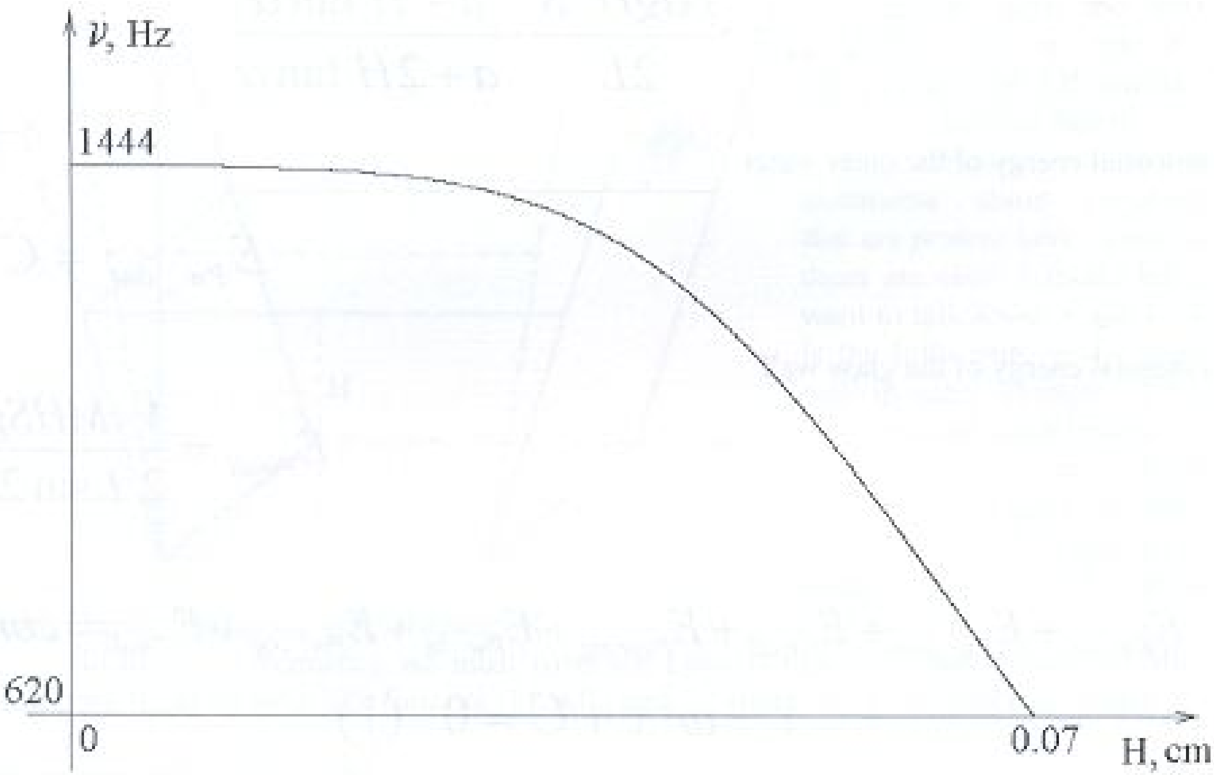
So here you can see frequency dependence on some water and glass parameters

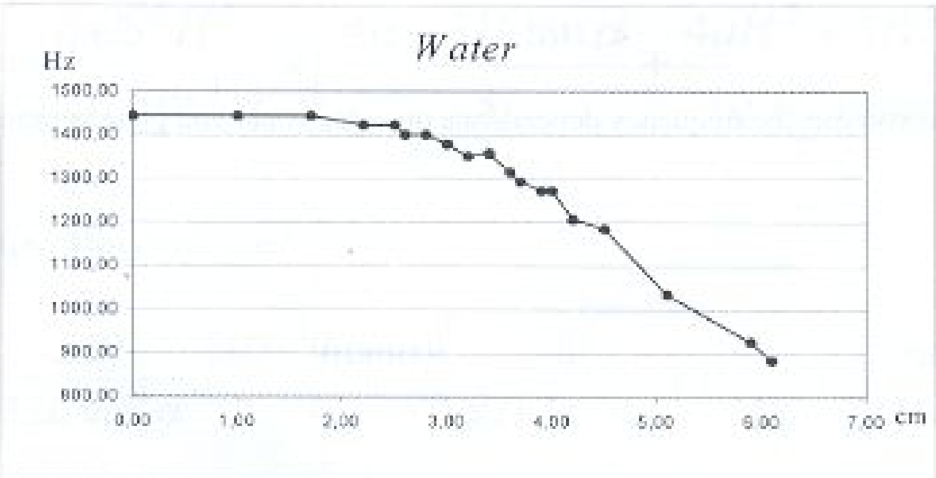
$$A = \frac{K}{\frac{\rho d h^3}{L^2 \cos^4 \alpha}\left(\frac{4D - 3h \tan \alpha}{6}\right)\left(\frac{h^2}{4(D - h \tan \alpha)^2} + \frac{1}{36}\left(1 + \frac{h \tan \alpha}{2(D - h \tan \alpha)}\right)^2\right) + \frac{M}{3}}$$

$$B = \frac{K}{\frac{H^3 \rho d}{12 L^2 \cos^4 \alpha}\left(\frac{4a + 3H \tan \alpha}{3} + \frac{4aH^2 + 3H^3 \tan \alpha}{(a + 2H \tan \alpha)^2}\right)}$$

$$\omega = \sqrt{A + B}$$

Graph of the Sound frequency dependence on the inner water's height

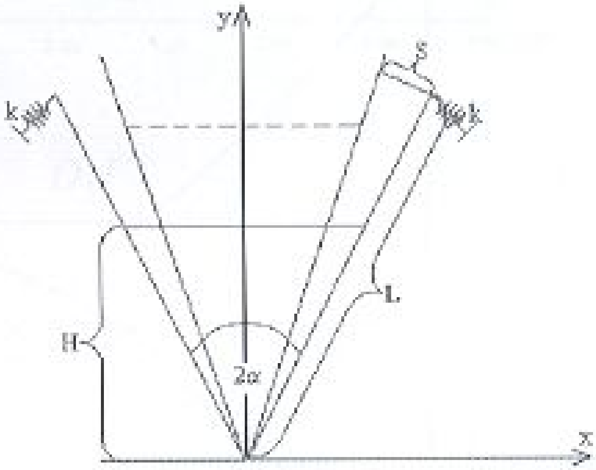


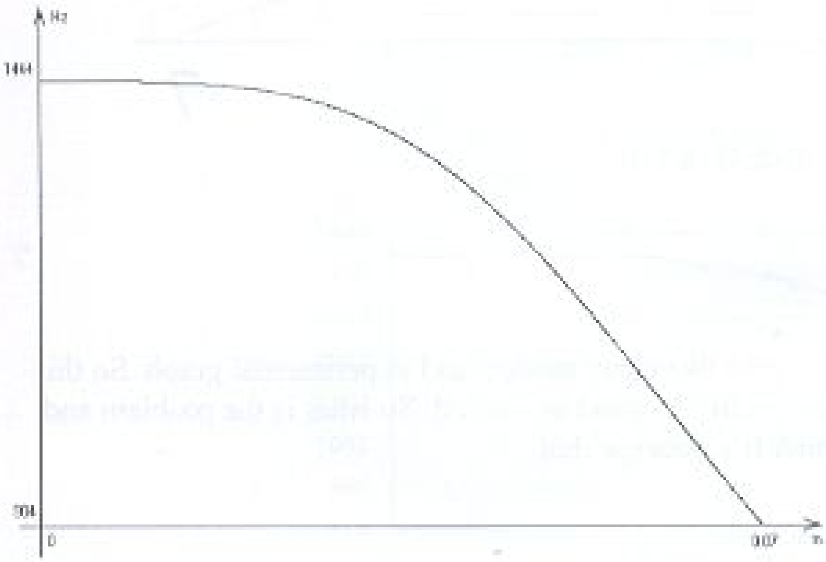


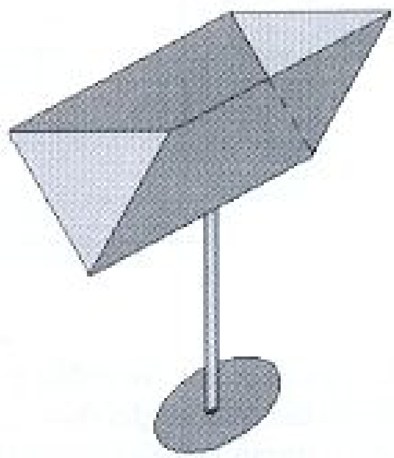
How you can see from these theoretical and experimental graphs they are close to each other, so we can say that this theory explains reality good enough.

Now lets talk about other models which I had considered. Second model is full pyramid you can see it on the picture and also here are formula of frequency for this model and graph of this formula.

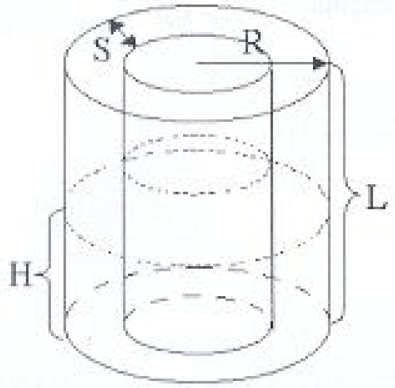
$$\omega = \sqrt{\frac{2K}{\frac{\rho d tg\alpha H^4}{2L^2 \sin^2 2\alpha}\left(\frac{tg^2\alpha}{3}+1\right)+\frac{2M}{3}}}$$



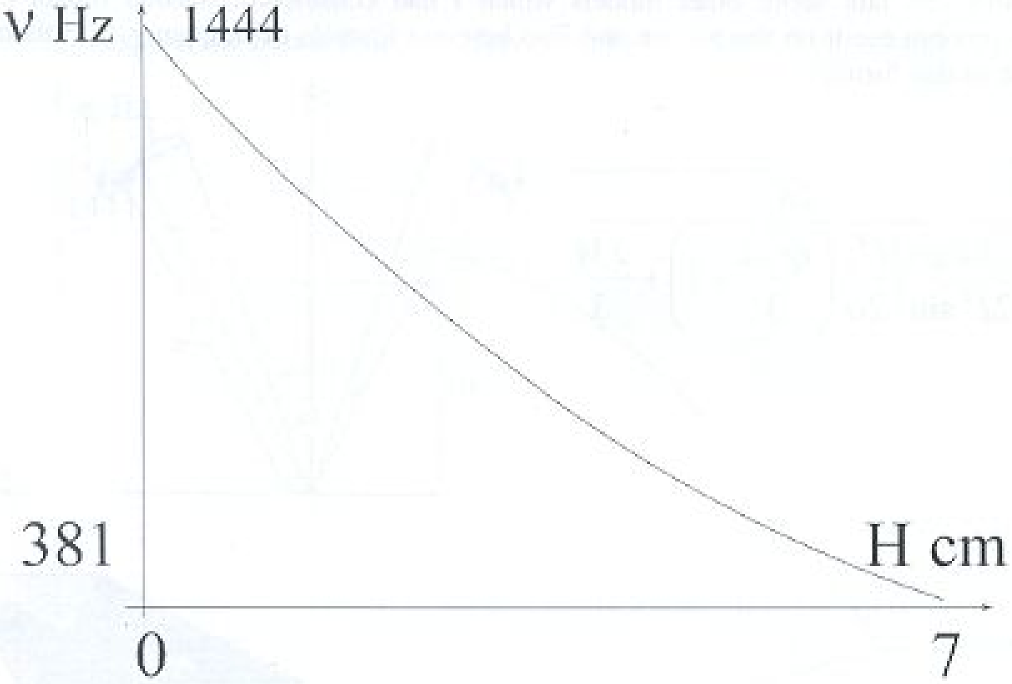




Here you can see third cylindrical model of the glass. You also can see dependence and graph of this dependence.

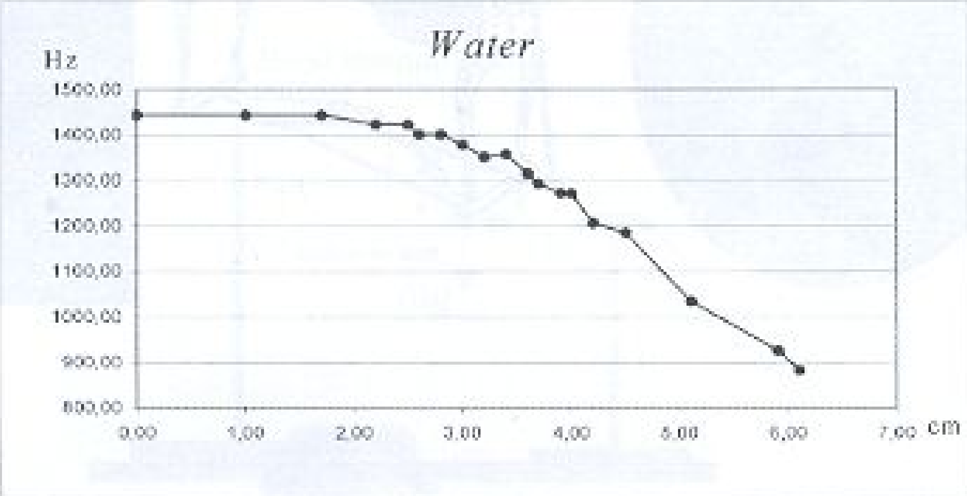


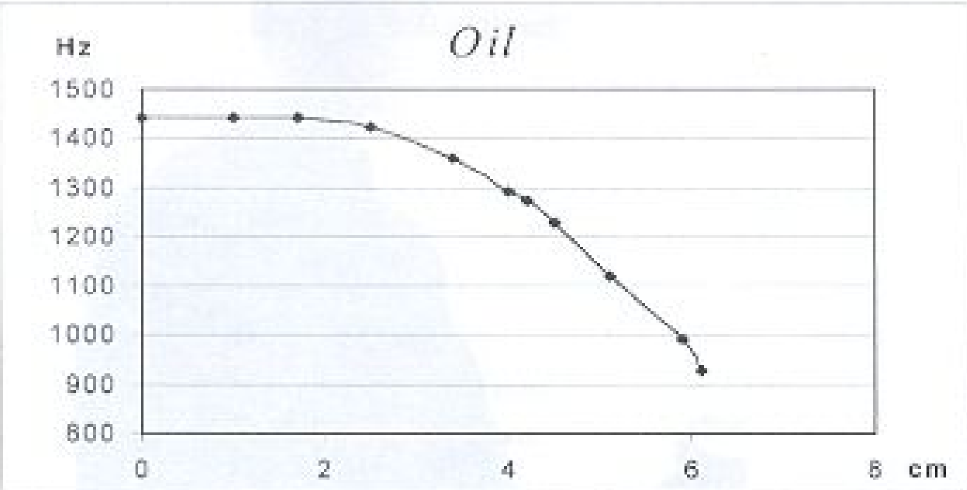
$$\omega = \sqrt{\frac{24k}{12\rho_k RLd + \rho H(3R^2 + 8H^2)}}$$

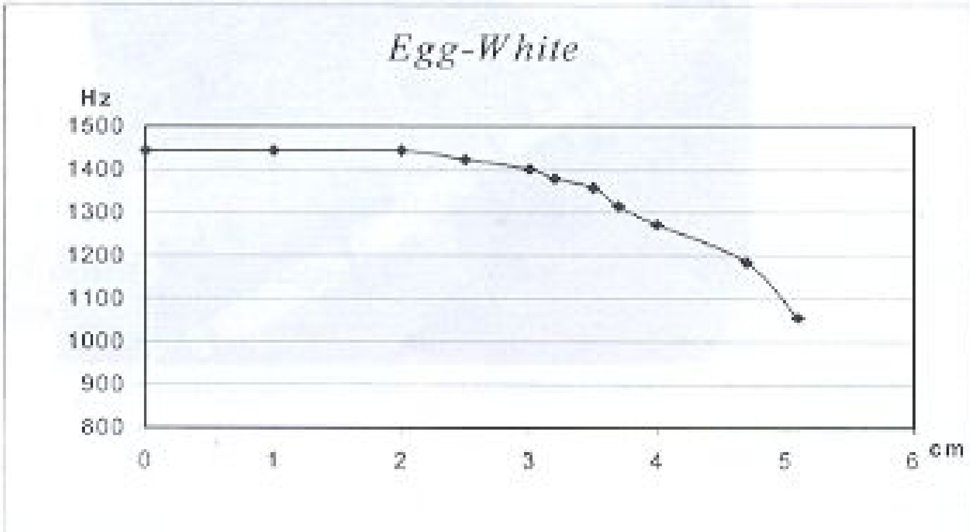


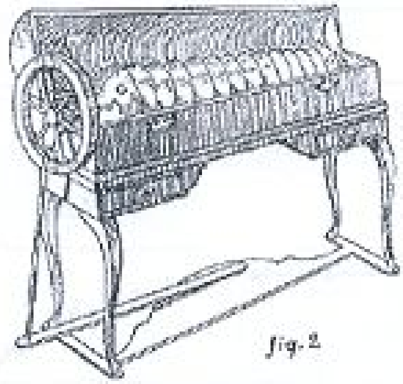
How you can see from this graph it is not like other models and experimental graph. So this means that this model don't explains reality as good as needed. So what is the problem and why it doesn't explains reality. I think it's because that

Experimental Data









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