11. BILLIARD

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I think that the main part of the problem is to understand what is the chaos therefore we must define it. So I think that we have chaotic distribution of some particles on the plane or in the dimension if we have no priorities in the particle's positions. Or we can say this such a way: if the probability that particle will be in some point of the plane (or dimension) is similar for all points than those distribution is chaotic.

Ok lets go back to the Billiard. At the beginning, I want to consider very simple and very rough model of the Billiard. It is rough so it has some idealizations.

1) After the first collision, the energy of the 16^{th} ball will be distributed uniformly for all balls. So we have no energy losses.
2) We have no more collisions between balls after the first collision.
3) All balls are identical. All collisions are elastic.

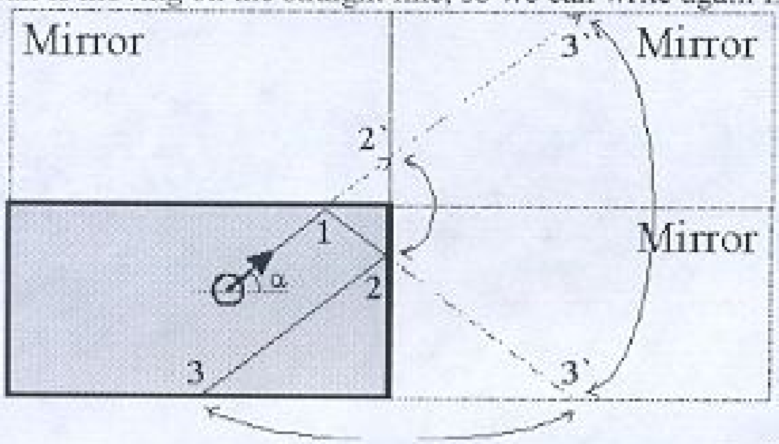
By idealization (1) and Low of Energy Conservation we can calculate velocity of the balls after the collision.

$$V_1 = \frac{7}{20}V$$

Now we can absorve movement of one ball because we have no more collisions between balls.

Let's assume that after the first collision one of the balls was directed by any angle but with known velocity. Our problem is to find coordinates of this ball, when it will be stopped by friction with table. For this we must follow it's way and it's every collision with walls. But I think this too difficult so to do this more easily I want to bring in mirror reflections of the table (pic. 1).

On this picture ball is moving on the straight line, so we can write again Low of Energy



conservation and find full L way which this ball would make. $L = \dfrac{mV_1^2}{2F_{fr}}$

But we wan to find coordinates of this ball in the table coordinate system so we must convert L to the X and Y of this system. Here you can see formulas by which we can make this convert.

$$X = \begin{cases} a \cdot \left\{ \dfrac{X_0 + L_x}{a} \right\}, \left[\dfrac{X_0 + L_x}{a} \right] = 2k \\ a - a \cdot \left\{ \dfrac{X_0 + L_x}{a} \right\}, \left[\dfrac{X_0 + L_x}{a} \right] = 2k + 1 \end{cases}$$

$$Y = \begin{cases} b \cdot \left\{ \dfrac{Y_0 + L_y}{b} \right\}, \left[\dfrac{Y_0 + L_y}{b} \right] = 2k \\ b - b \cdot \left\{ \dfrac{Y_0 + L_y}{b} \right\}, \left[\dfrac{Y_0 + L_y}{b} \right] = 2k + 1 \end{cases}$$

It is very interesting that we get fraction parts in these formulas, because there is the formula that is generating accidental numbers.

$$a_{n+1} = \{a_n k\}, \quad a_n \in (0;1), k > 1$$

It gives us numbers and probability that it will give us some number is similar for all numbers and this like our definition of chaos. So our ball's final coordinates are chaotic. But we must add one inequality which u can see here. It is because that if we had no collisions with balls than we will have distribution like this on picture and it's not chaotic. So after solving this inequality we will get final answer: To get finally after stopping of the balls chaotic distribution on the table we must impact 16[th] ball with velocity not less than 17 m/s.

$$\left. \begin{matrix} L_x > a \\ L_y > b \end{matrix} \right\} \Rightarrow L > \sqrt{a^2 + b^2} \Rightarrow \qquad V>17 \text{ m/s}$$

Now lets consider more exact model of the Billiard. This model is with the help of the computer. The final answer that this model will give us is the velocity of the first impact, angle by which was made this impact. Idealizations of this model are not too serious but I must say about them.

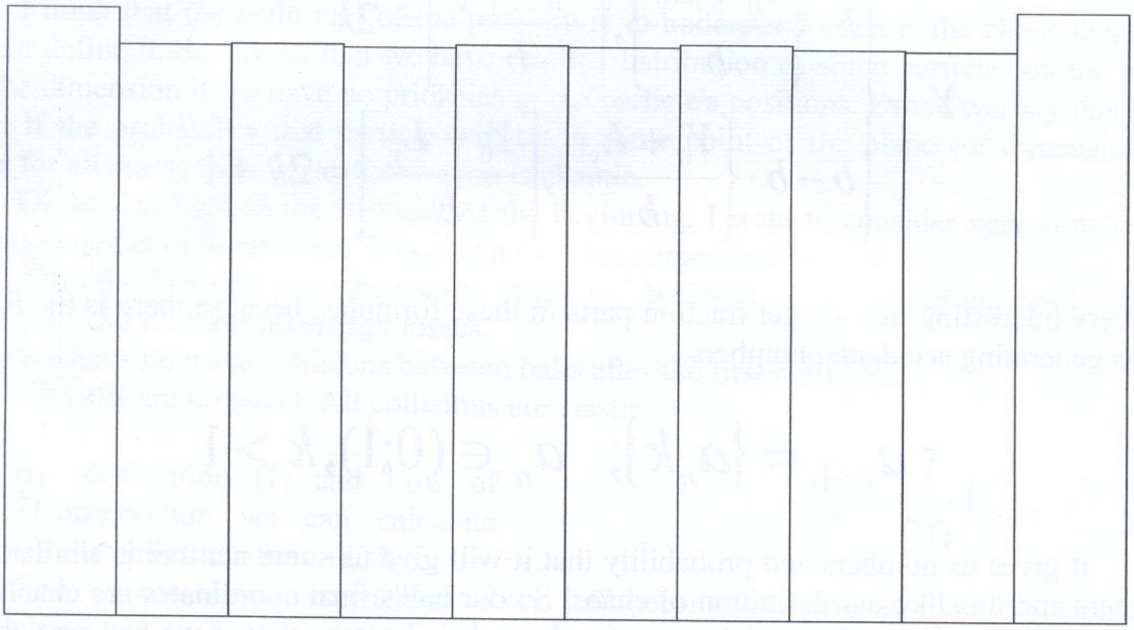
1) The friction force is constant and we have only rolling friction.
2) All collisions are absolutely elastic and central.
3) Balls are moving only by straight lines and we have no jumps.

Now lets talk about model and what is doing computer. Computer "knows" initial distribution of the balls (pic.1). We must input angle and velocity of 16[th] ball and then it will follow each ball movement. Computer is calculating balls position every 0.00001-sec. and I think this is good exactness. So computer "knows" each ball position at any moment of the time.

Now lets go back to the chaos definition. We define chaos for the plane. Yes our Billiard table is a plane, but it is not infinity it has borders. Therefore, these borders must take effect and we must understand what is a real chaos for the Billiard table. For this computer removes friction (to watch balls motion for a long time) and shoots 16[th] ball with any velocity and angle. Then after every $\Delta t=0.1$ sec. time it divides the table on the 10×10 cell and looks how many balls are there in each cell, remembers this number.

Computer makes those calculations 400000 times and adds all 400000 tables to each other. I think is sufficient big statistic. Well finally we will get 10×10 table which shows us distribution of the balls during approximately 22 hour of balls moving. Here you can see the histogram of this distribution in 2-D view.

Function of chaotic distribution



I want to say that if in the future I will get that distribution on the table than I'll say that this distribution is chaotic.

To prove this definition I can say that when this distribution is established it is not changing any more.

How you can see from the histogram, balls are more often near the walls than in the other parts of the table. This is very interesting and I think it's because we have more collisions near the walls then in other parts, because here we have not only collisions between balls we have also collisions with the walls.

Well now lets go back to the model. After definition of the chaos computer returns friction to the model and begins loop.

Loop:

1) Get angle in the range $\alpha \in [o; 90)$
2) Get velocity in the range $V \in (0; +\propto)$
3) Shoot the 16^{th} ball with taken velocity and angle.
4) After stopping all balls computer divides table 10×10, then creates table of numbers and writes in this table of numbers how many ball is in each cell of Billiard table.
5) Compare table of numbers with table of chaotic distribution and if these two tables are similar then last distribution is chaotic and computer remembers this velocity and angle and makes everything again for next angle. Back to (1)
   Else if they are not similar computer changes only velocity. Back to (2)
6) Finding minimal velocity from list of all velocities that gave us chaos.

So after this loop we will get a final answer. But at first I want to talk about comparing. Computer makes this comparing by method X^2 (Greek). X^2-is a number and for table 10×10 it must be less than 35 after comparing and in this case we must say that two compared tables are similar with exactness 99%.

Principally by one picture we can not say is it chaotic or no. Therefore for each angle and velocity computer gets $\alpha \pm 1\%\alpha$, $V \pm 1\%V$ regions and makes experiments 10000 times in this range. Then it adds these 10000 tables and compares it with the table of chaotic distribution.

Well it's time to say about final answer on the problem. After all these experiments we got next answer:

Final Results:

$V \approx 10$ m/s $\quad \alpha \approx 2^{\circ} \quad t \approx 7.5$ s