14. WEAT WAVES

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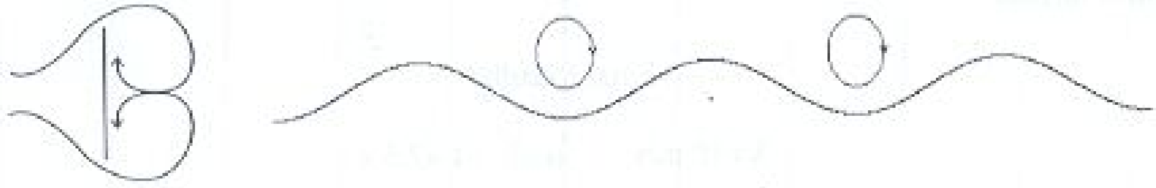
If wind blows an obstacle around it appear vortices. These vortices make wave-like structure. Because one comb behaves as an obstacle appear next vortex which flows on a surface of and makes depth area. (see pictures)



With certain amplitude stabile waves are large ones, because force acting on the wave from wind is proportional to A/λ, where A is amplitude and λ - wavelength. If the force is big the wave is not stabile. For decreasing of above force wavelength must be increased

Self organization mechanism

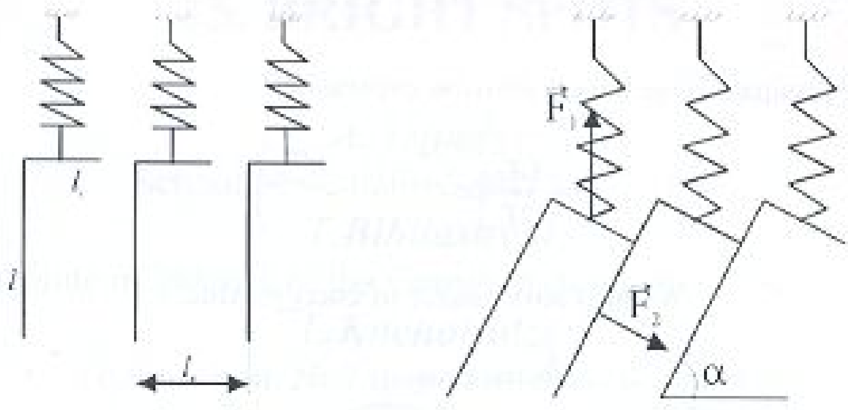
Self organization means that with different elasticity, masses and nonlinear distribution of concentration different wavelengths appear, but the system as a whole one behaves as a large wave. In this case appear areas with high density, which means that spikes are put on each other. This area behaves as a wall and it cuts waves because of which overtones take place.

Fourier component of amplitude is following:

$$A(\omega) \sim 1/\omega^2$$

it means that: λ is large.

Theoretical model



We characterize elasticity of spike by spring with elasticity k and this spring is deformed vertically. l_1 is initial deformation of spike caused by own mass. α is angle with which we have correlation among spikes, so we have correlated system were the wave can propagate.

F_1 is force caused by deformation and F_2 is force caused by wind.

$$F_1 = kA \qquad F_2 = \frac{1}{2}\rho S V^2 \sin 2\alpha \qquad A = \frac{(l_0 - l_1)}{l_0} l,$$

were F_1 is force caused by deformation, F_2 is force caused by wind, ρ is density of air, S – square of spike and V – velocity of wind.

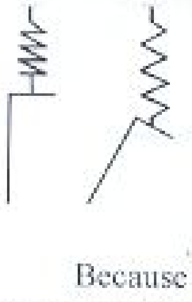
$$l_0 \sim 0.1 \text{ m}$$

$$l \sim 1 \text{ m} \qquad l_1 \sim 0.06 \text{ m} \qquad \sin\alpha = \frac{l}{l_0}$$

$$k \sim 10^{-1} \text{N/m}$$

$$F_1 = F_2 \Rightarrow \quad V \sim 3 \text{ m/s}$$

We write down equation of force balance and get result the minimal velocity of wind after which the wave propagates.



$$F_1 = -kx$$
$$F_2 = -\eta l V_x$$

Because of air friction spike looses its energy. Friction is caused by viscosity, F_2 is expression of a viscosity force and F_1 – elasticity one along vertical axe where η is an air viscosity.

Solution of spike motion equation gives us expression of amplitude fading:

$$A = A_0 e^{-\frac{\eta l}{m}t}$$

For energy dissipation we have following expression:

$$\overline{P_1} = \frac{kA^2}{2T}\left(e^{\frac{-2\eta l}{m}(t+T)} - e^{\frac{2\eta l}{m}t}\right)$$

We also have source of energy: wind.

Expression for force coming from wind action on the wave is following:

$$F = 4\gamma\rho(U-C)^2 s\frac{A}{\lambda}\pi^2 \qquad C = \frac{\omega}{\sqrt{n}}$$

$$P_2 = FV \qquad \omega = \sqrt{\frac{k}{m}}$$

$$V = \frac{AC}{\lambda}$$

where C is wave velocity, U – a wind one, n – spike concentration, λ - wavelength and A amplitude.

If we write down the balance equation for all energies we will get the wavelength expression:

$$\left|\overline{P_2}\right| = nS\left|P_1\right| \Rightarrow \lambda = (U-C)\frac{2\pi}{\omega}\sqrt{\frac{\gamma\rho C}{n\eta l}}$$

$\eta \sim 10^{-5}$Pa s $m \sim 10^{-3}$kg $\rho \sim 1$kg/m^3