15. BRIGHT SPOTS

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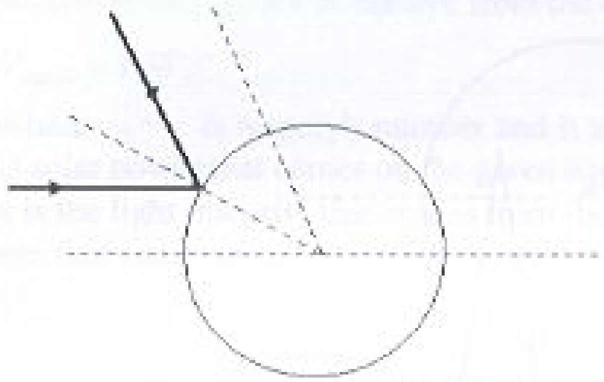
Let us start with the definition of dewdrops:

Dewdrops are little water drops, in which surface tension force is much more than the gravity force and we can consider it as the dominant force applied to the drops. By its influence dewdrop takes the sphere shape, so we will solve the problem for the sphere drops.

At first let us consider the case of direct reflection. A dewdrop is shown on the picture 1, on which a sunbeam falls by an φ angle. In this case our eye is located by the angle $\theta=2\varphi$ and we see the direct reflection spot.

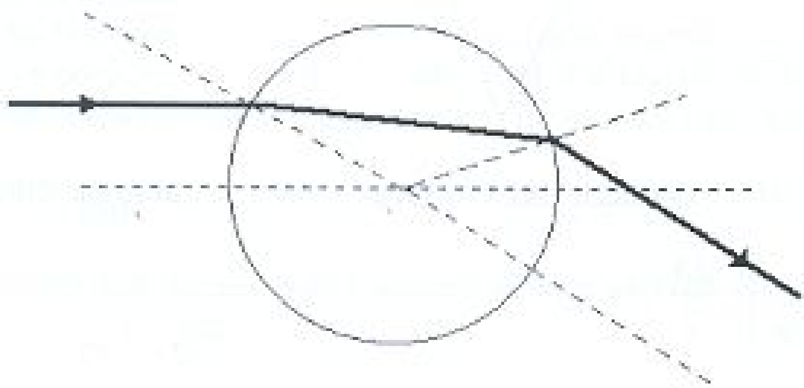
Pic. 1



$$\vartheta(\phi)=2\phi$$
$$\alpha(\phi)=\phi$$

Now let us talk more generally about other spots. Let us consider the case when the sunbeam enters the drop by the angle φ, is refracted by the angle β and leaves the drop by the angle α. In this case our eye is situated by the angle θ and we see the bright spot (see picture 2).

Pic. 2



If we write refraction rule, from the picture we can see that it is easy to find θ angle:

$$\vartheta(\phi) = 2\phi + \left(\pi - 2\arcsin\left(\frac{\sin\phi}{n}\right)\right)N$$

Where (N-1) is the internal reflection number and n is the refraction coefficient (in our case it approximately equals to 4/3).

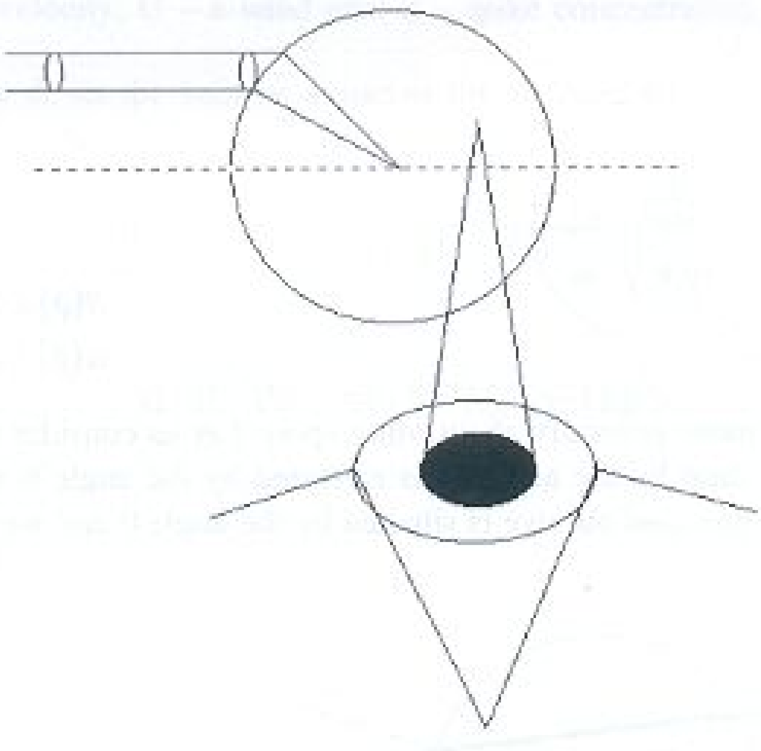
As the problem is about spots' location, so it is important to find α angle. It is clear from the picture that in the case of absence of internal reflections we have α=θ-φ, but the refraction may take place also in points of internal reflection. In this case as we count angle α from horizontal axis, then we will have:

$$\alpha(\phi) = 2\pi\left\{\frac{\phi + \left(\pi - 2\arcsin\left(\frac{\sin\phi}{n}\right)\right)N}{2\pi}\right\}$$

Now let us consider the case, when the bunch of beams with the cross section ΔS falls on the dewdrop. We can define ΔS by the following way: $\Delta S \approx \frac{\pi}{4}(r\cos\phi d\phi)^2$

The bunch enters by the angle φ and an eye sees spots in the angle $d\theta = d_{eye}/L$, where d_{eye} is diameter of eye and L is distance from dewdrop to observer (see picture 3).

Pic. 3



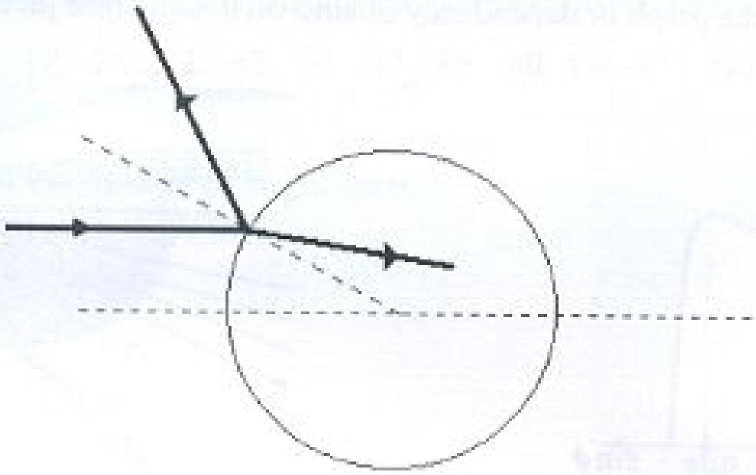
We need to know by which dφ angle must the beams enter to fall in the dθ angle:

$$d\phi = \left(1 - \frac{N\cos\phi}{\sqrt{n^2 - \sin^2\phi}}\right)^{-1}\frac{d\vartheta}{2}$$

Now let us talk about power that comes in our eye:
Picture 4 shows the sunbeam that is reflected from the dewdrop and refracted inside it. The coefficient of reflection is expressed by following formula:

$$R = \frac{1}{2}\left(\frac{\sin^2(\phi-\beta)}{\sin^2(\phi+\beta)} + \frac{tg^2(\phi-\beta)}{tg^2(\phi+\beta)}\right)$$

Pic. 4



So, power that comes in our eye from the direct reflection is equal to:

$$W_{eye,0} = RW_0e^{-\alpha l}$$

Where $e^{-\alpha l}$ is Napery's number and it shows how the medium absorbs the light and W_0 is solar power that comes on the given square on earth equals to $W_0 = \omega\Delta S$
Where ω is the light intensity that comes from the sun.
And power that comes after (N-1) internal reflection equals to:

$$W_{eye,N} = R^{N-1}(1-R)^2 W_0 e^{-\alpha l}$$

Below some literature data are given, which are important in our case:

$d_{eye} \approx 0{,}5$ sm. $\quad S_{eye} = \frac{\pi}{4}d^2_{eye} \quad \omega = 0{,}139$ w/sm^2

It's known that an eye sees the light if the power that comes from the object (in our case dewdrop) is 1.5% or more of the power that comes from the surface on which the object is located.

As the problem is about dewdrops, and naturally dewdrops are on the grass or plants, we considered a green surface. And it's known from literature, that per unit of time green surface reflects 7% of intensity that comes from sun, so:

$$\omega_G \approx \frac{7}{100}\omega$$

Power that comes from the green surface into our eye, equals to:

$$W_G = \omega_G S_{eye} e^{-\alpha l}$$

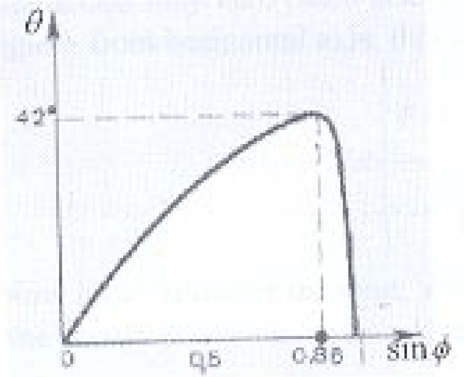
So if the following equation is correct, then we will see the spot:

$$\frac{W_{eye,i}}{W_0} > 1,5\%$$

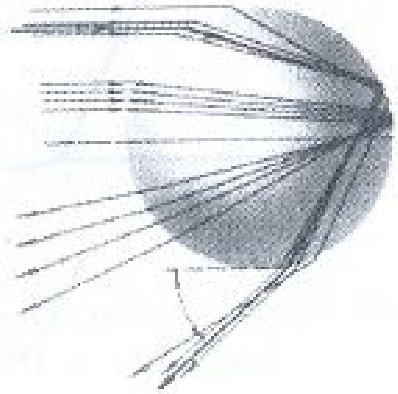
Now let us talk about the kind of spots we are able to see on the dewdrop: it can be direct reflection spots, spots produced by the way shown on picture 3, or spots produced in the extreme point, where sunbeams leave the spot.

We have constructed the graph of dependency of sinφ on θ angle (see picture 5).

Pic. 5



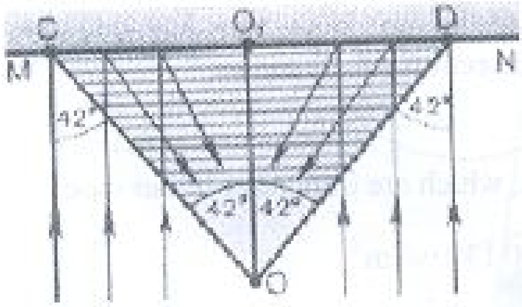
Pic. 6



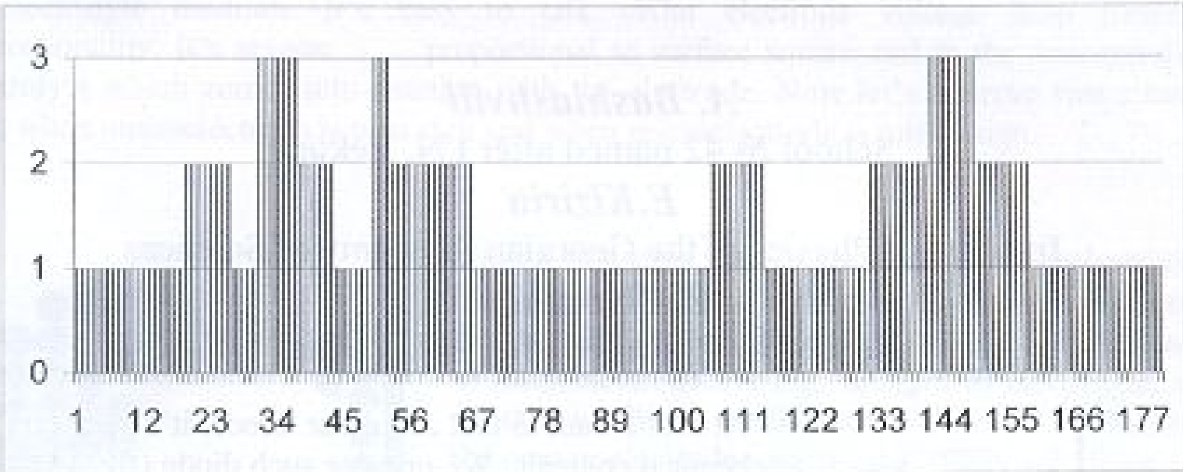
Picture 6 shows the spot produced in the extreme point.

If there are several drops located on the surface we will see the picture something like rainbow. On picture 7 several drops are located on the MN surface, observer is in the point O and he sees the picture on the CD distance. Most intensively he sees the spots located near D and C points.

Pic.7



We have mainly investigated the phenomenon with one dewdrop and wrote a computer program. We give the computer the value of θ angle and it draws all the possible spots that are on the drop when we observe it at the given angle. Below is given the table of dependence of θ angle (θ is from 1° to 90°) on number of spots (there are given all possible spots):



Vertical axis: number of bright spots.
Horizontal axis: angle θ.