2. SINGING SAW

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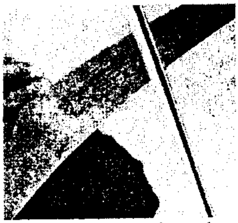
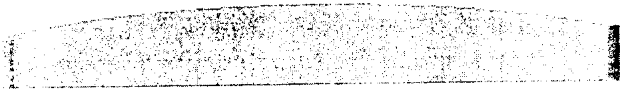
In order to receive sound we must have oscillating body. So to play on saw we must make him oscillate. We can do this with bow or mallet.



After beating saw with mallet or playing with bow in saw standing waves appear. If we will strew sugar or other dust on the surface of saw, and then will play, sugar will accumulate on point, which don't take place in oscillations. These points are called nodal points. Oscillations of saw make air oscillate and sound propagates. Loudness of sound is defined by amplitude of oscillations, so we can control on loudness of saw sound by controlling amplitude of oscillations of saw. Pitch of sound is defined by frequency of oscillations.

Let us define natural frequency of rectangle plate:



$$\zeta = \zeta_0(x, y)\cos(\omega t + \alpha)$$

$$\Delta(\zeta_0 - \zeta) - \chi^4\zeta_0 = 0\text{, where } \chi^4 = \omega^2\frac{12\rho(1-\sigma^2)}{h^2 E}$$

σ -is Puason's coefficient
E -is Eung's modulus
Δ -is Laplas's operator

By help of Relay-Ritz's method, we can define frequency of oscillations of plate with supported ends:

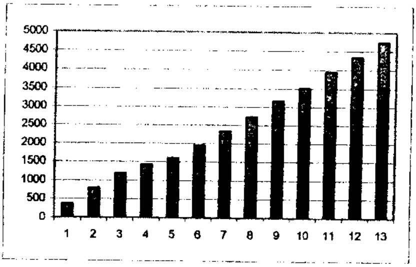
$$\zeta_0 = A\sin\frac{m\pi x}{b}\sin\frac{n\pi y}{a}$$

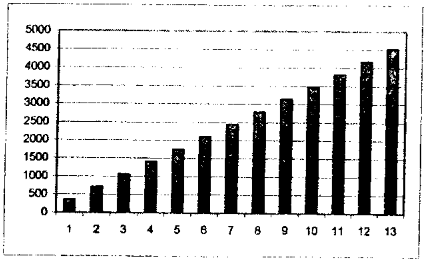
m and n are number of nodes along length and width
a -is length of the saw
b -is width of the saw

$$\omega = \frac{\pi^2 h}{2}\sqrt{\frac{E}{3\rho(1-\sigma^2)}}\left[\frac{n}{a^2}+\frac{m}{b^2}\right]$$

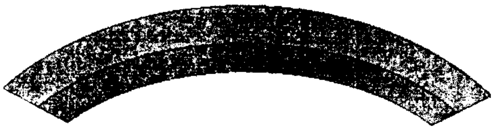
h -is thickness of saw

For iron: $\sigma = 0.25 - 0.3$ $E = (195-205)\cdot 10^9\,pa$ $\rho = (7.7-7.9)\cdot 10^3\,\frac{kg}{m^3}$

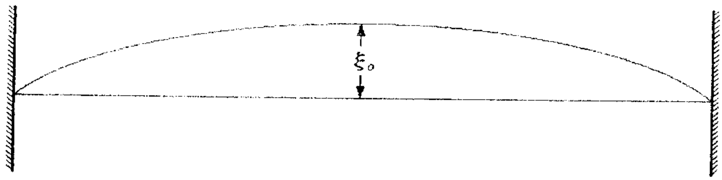




Experiments show that frequency of oscillations changes by deformation of saw.

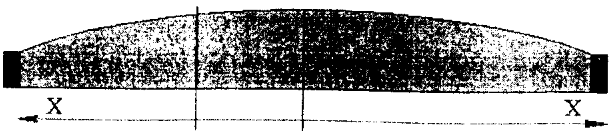


By deformation narrowed and expanded layers appear and stiffness of saw: $K = \frac{ES}{b}$ changes because, that quantities S and b change. That is the reason of change of frequency.



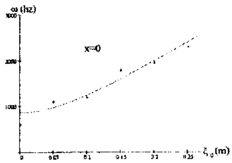
Potential energy of deformed saw: $U = U_0 + \frac{1}{2}K_0 \cdot \xi_0^{\,2} + \frac{1}{12}G\cdot \xi_0^{\,4}$

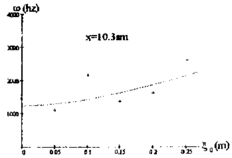
$$\omega^2(\xi_0) = \frac{U''}{m} = \frac{K_0}{m} + \frac{G}{m}\xi_0^{\,2}$$

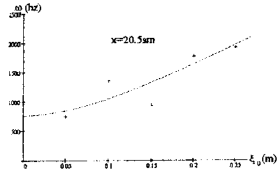


Comparison of Theory with Experiment

Saw No.1

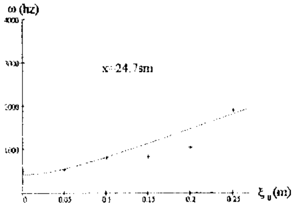






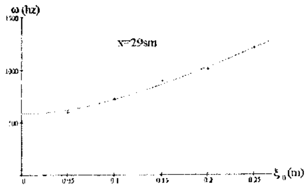
$$\frac{K_0}{m}=8.124\cdot10^7\sec^{-2}\quad \frac{K_0}{m}=4.889\cdot10^7\sec^{-2}\quad \frac{K_0}{m}=4.34\cdot10^7\sec^{-2}\quad \frac{K_0}{m}=5.303\cdot10^7\sec^{-2}$$

$$\frac{G}{m}=7.389\cdot10^5(m\cdot\sec)^{-2}\quad \frac{G}{m}=1.534\cdot10^6(m\cdot\sec)^{-2}\quad \frac{G}{m}=1.291\cdot10^6(m\cdot\sec)^{-2}\quad \frac{G}{m}=5.814\cdot10^6(m\cdot\sec)^{-2}$$



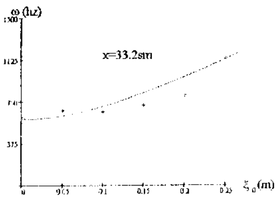
$$\frac{K_0}{m}=5.039\cdot10^7\sec^{-2}$$

$$\frac{G}{m}=1.789\cdot10^5(m\cdot\sec)^{-2}$$



$$\frac{K_0}{m}=1.789\cdot10^7\sec^{-2}$$

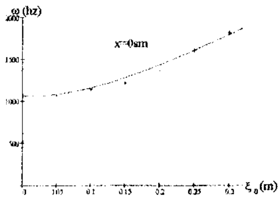
$$\frac{G}{m}=3.415\cdot10^5(m\cdot\sec)^{-2}$$



$$\frac{K_0}{m}=1.495\cdot10^7\sec^{-2}$$

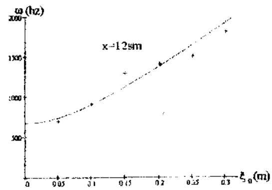
$$\frac{G}{m}=3.541\cdot10^6(m\cdot\sec)^{-2}$$

Saw No.2



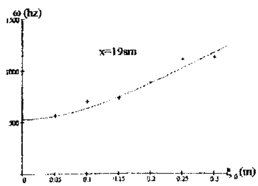
$$\frac{K_0}{m}=1.331\cdot10^7\sec^{-2}$$

$$\frac{G}{m}=1.119\cdot10^6(m\cdot\sec)^{-2}$$



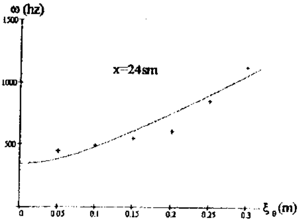
$$\frac{K_0}{m}=3.671\cdot10^7\sec^{-2}$$

$$\frac{G}{m}=4.531\cdot10^6(m\cdot\sec)^{-2}$$



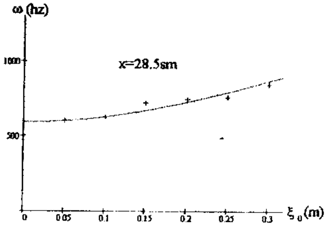
$$\frac{K_0}{m}=1.222\cdot10^7\sec^{-2}$$

$$\frac{G}{m}=2.786\cdot10^6(m\cdot\sec)^{-2}$$



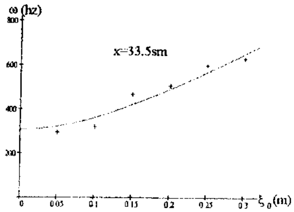
$$\frac{K_0}{m} = 1.120 \cdot 10^7 \sec^{-2}$$

$$\frac{G}{m} = 1.195 \cdot 10^6 (m \cdot \sec)^{-2}$$



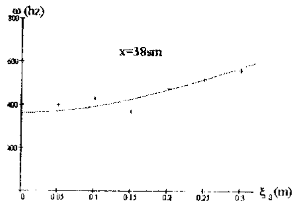
$$\frac{K_0}{m} = 4.481 \cdot 10^6 \sec^{-2}$$

$$\frac{G}{m} = 3.488 \cdot 10^6 (m \cdot \sec)^{-2}$$



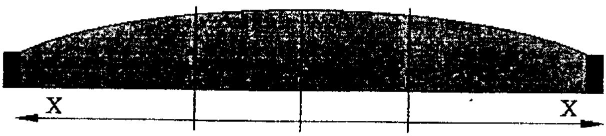
$$\frac{K_0}{m} = 3.611 \cdot 10^6 \sec^{-2}$$

$$\frac{G}{m} = 9.48 \cdot 10^4 (m \cdot \sec)^{-2}$$



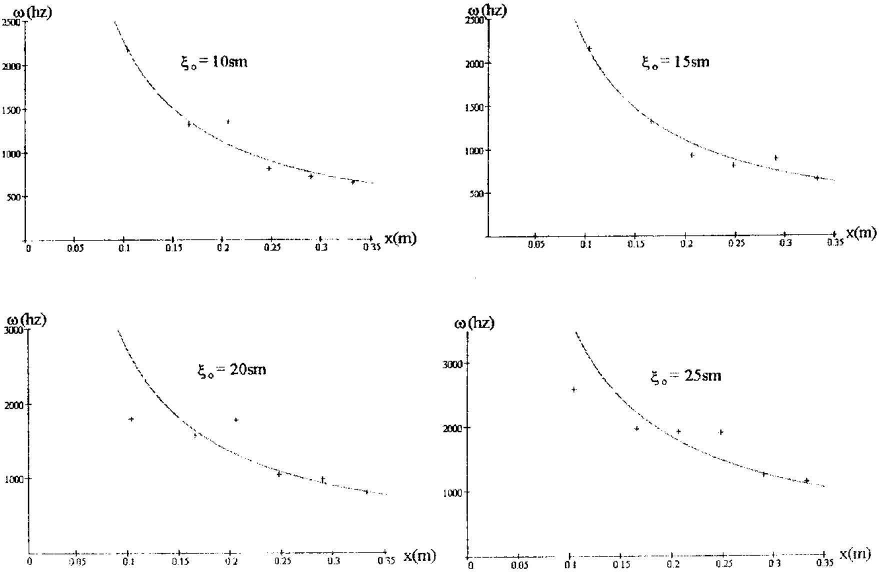
$$\frac{K_0}{m} = 2.169 \cdot 10^6 \sec^{-2}$$

$$\frac{G}{m} = 1.308 \cdot 10^5 (m \cdot \sec)^{-2}$$

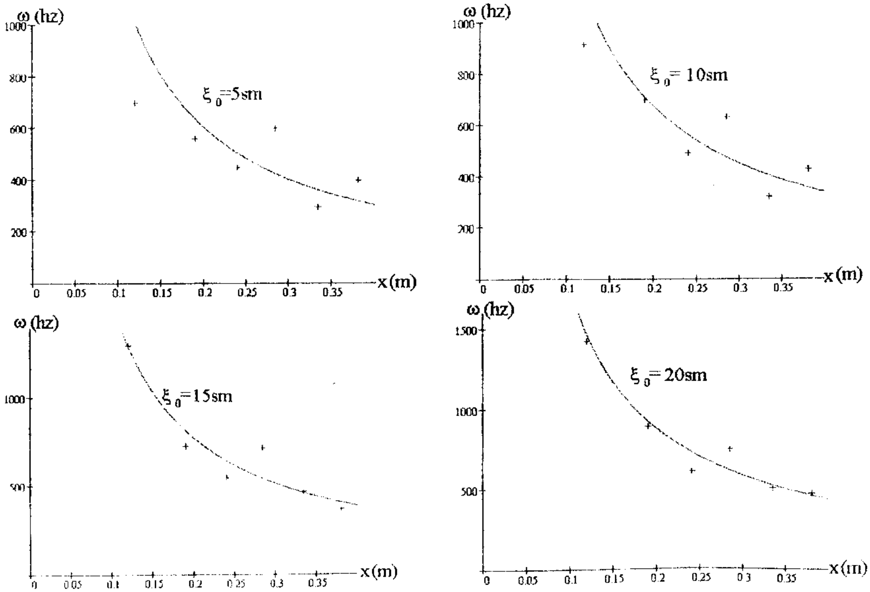


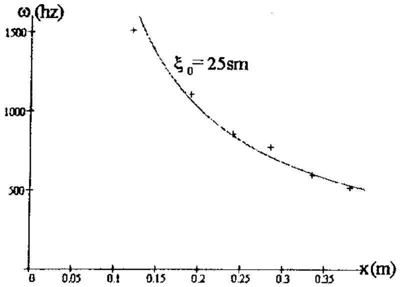
Quantity $\omega \cdot \lambda$ is velocity of propagation of wave on saw and is constant by given deformation. So we can find dependence of frequency of oscillations on the coordinate of point, on which we are playing.

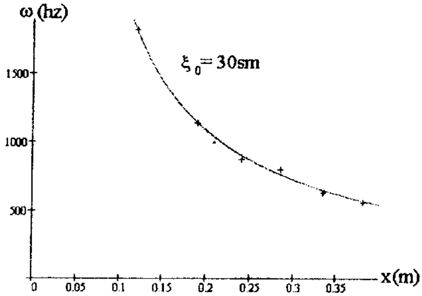
Saw No.1



Saw No.2







On figures experimental points and theoretical curves are shown.

1069.2hz	1228.8hz	1405.8hz	1417.4hz	1749hz	1798.1hz
C#6-9	C#6+31	F6+10	F6+25	A6-10	A6+37

1955.2hz.	2268.8hz	2334.6hz
B6-17	C#7+39	D7-10

Conclusion

We have made number of experiments, investigated oscillations of saw, showed method of discovering of nodal points, gave dependence of frequency of oscillations on deformation of saw and on coordinate of point, on which we are playing and showed which note correspond to given frequencies.

Acknowledgements

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