6. FRACTAL DIFFRACTION

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Let's at first determine what is fractal and diffraction? It's well known that waves have ability to go around impediment, size of which is comparable with waves length. This phenomenon was called diffraction. It can be viewed on a simple experiment. If we will light on a thin cleft we will get alternation of light lines and dark ones. The diffraction can be described like interferention of secondary wave sources. So in the points where sum all waves we will be maximum we see light line. Also, if we take a thin ware in the monochromatic light on some distances we will not see geometric shadow, because the wave will go around the ware and will be also on the shadow.

Fig1.Example of the diffraction picture

For calculations, is very useful to use Fraungofer's diffraction. The main idea of it is to endlessness far screen. On practice this can get with lens, if we will look diffraction on the focal plane of the lens. In the calculations it is very simple to calculate phase difference between two points. And because the property of lens all waves, which are going by the same angle are going to the same point on the screen.

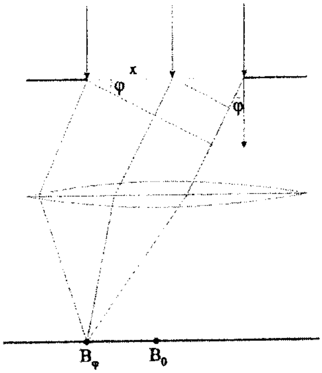


Fig2.Diffraction of the Frangoupher

Diffraction is used for investigating structure or size of matters, for example: diffraction of Roentgen's beam, with known wave length, is used for detecting of crystal's structure and measuring of this structure's size. Also such particles like quarks can be detected only by diffraction.

Let's think now what is fractal? In math fractal is variety with fractal dimension. Ideal fractals has property of self homo, this means that fractal consist of end or endless quantity of self homo structures, which are only dislarged copy of full picture. The dimension of line is 1, of area is 2, and fractal can has any value. It's dimension can be calculated by equation(1).

$$d = \dim A = \lim_{\varepsilon \to 0} \frac{\ln N(\varepsilon)}{\ln \frac{1}{\varepsilon}} \qquad (1)$$

Fig3.Cantor's fractal(bars)

For example math fractal: Cantor fractal -- is a fractal which can be got by deleting middle part of line, as shown on the picture. Number of such operations is called fractal's power. The area of this fractal decreases like $(2/3)^N$. The dimension of this fractal can be calculated by formula (2).

$$d = \lim_{n \to \infty} \frac{\ln 2^n}{\ln 3^n} = \frac{\ln 2}{\ln 3} \approx 0{,}630929 \qquad (2)$$

Now, when we said what is diffraction and fractal, we can ask now: what is diffraction on fractals? This is only diffraction on special diffraction grate: fractal. As I said diffraction is very useful for investigating the matters properties. Maybe some crystals have fractal structure. Then knowledge of the pictures, got by the diffraction on fractals can help to learn such crystals and their properties. Also this knowledge can help with learning some properties of fractals.

I already mentioned about Cantor's fractal. Let us at first look on the diffraction on it. For calculation of diffraction picture is better to use complex numbers, then calculations became more simple. Amplitude in any point of the screen can be calculated by the formula (3).

$$A_N = \frac{A_0}{b} \int_{-\infty}^{\infty} dx e^{ikx \sin \varphi} G(x) \qquad (3)$$

$$A_N = \frac{2 \sin\left(\frac{kb \sin \varphi}{3^N} \right)}{k \sin \varphi} \sum_{j=1}^{2N} e^{ikx_j \sin \varphi} \qquad (4)$$

Function G(x) is equal to 1 if in current place is hole, and 0 if there is screen. After some transformations we can get (4). As known, coordinates of Cantor bars can be got by next formulas by changing sign before every number.

$$\begin{aligned}
x_1 &= 2b\left(\frac{-1}{3} + \frac{-1}{3^2} + \ldots + \frac{-1}{3^{N-1}} + \frac{-1}{3^N} \right) \\
x_2 &= 2b\left(\frac{-1}{3} + \frac{-1}{3^2} + \ldots + \frac{-1}{3^{N-1}} + \frac{+1}{3^N} \right) \\
x_3 &= 2b\left(\frac{-1}{3} + \frac{-1}{3^2} + \ldots + \frac{+1}{3^{N-1}} + \frac{-1}{3^N} \right) \qquad (5) \\
x_{2N} &= 2b\left(\frac{+1}{3} + \frac{+1}{3^2} + \ldots + \frac{+1}{3^{N-1}} + \frac{+1}{3^N} \right)
\end{aligned}$$

$$\sum_{j=1}^{2N} e^{ikx_j \sin \varphi} = \prod_{m=1}^{N} 2 \cos\left(\frac{2kb \sin \varphi}{3^m} \right)$$

$$(6)$$

$$A_N = A_0 \left(\frac{2}{3} \right)^N \frac{\sin\left(\frac{kb \sin \varphi}{3^N} \right)}{\frac{kb \sin \varphi}{3^N}} \prod_{m=1}^{N} \cos\left(\frac{2kb \sin \varphi}{3^m} \right)$$

So according to this we can transform second part of this formula (6). With this formula we can calculate amplitude in any point of the screen. We made photo-tape, printed on the special printer with 4000 DPI with picture of the Cantor fractal with powers 4 and 5. You can see on the photos got by the laser on this tape, and near you can see theoretical results got by that formula on the computer (See color appendix). There are some interesting properties. 1st property – If wave's length is bigger than the smallest hole of the fractal, but smaller than distance between who holes, then wave goes through some holes like through one. 2nd property is that got picture is also fractal with the same power.

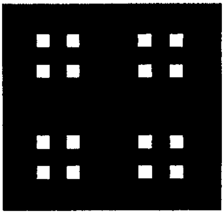


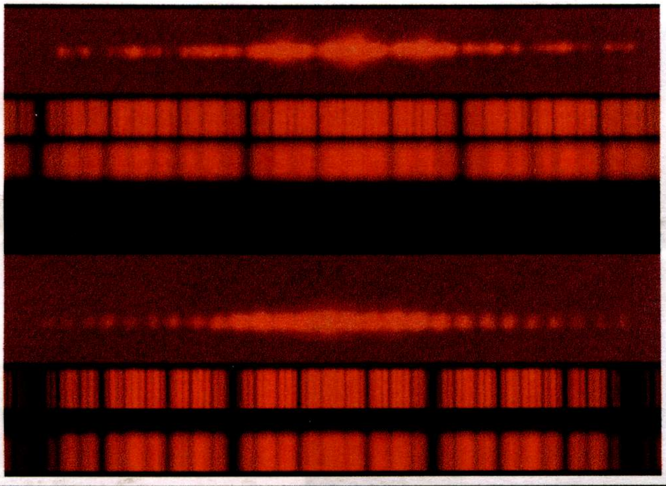
Fig3.Cantor's fractal(dust) 2th power

Let us try detect formula for another fractal. It can be simple done for fractal, called Cantor's dust. It looks like Cantor's fractal, but it is on the plane. Then formula for it is (7). From this formula we can see that max. Amplitude decreases like (2/3)2N, like the open part of fractal. Second multiplier is diffraction picture from one hole, and last is result of interferention from all fractal's holes. We made computer model for this fractal too. Now you can see experimental and theoretical pictures(See color insertion). On them is can be viewed better fractal structure of diffraction picture.

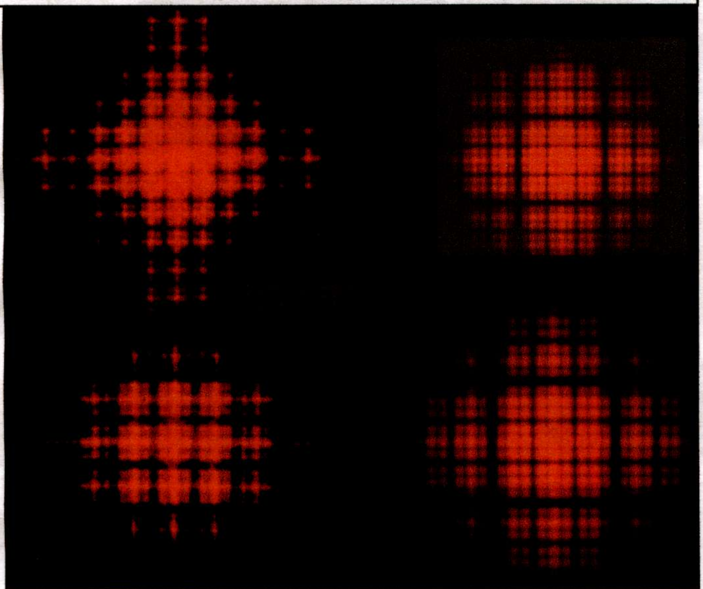
$$A_N = A_0\left(\frac{2}{3}\right)^{2N} \frac{\sin\left(\frac{kb\sin\varphi}{3^N}\right)}{\frac{kb\sin\varphi}{3^N}} \frac{\sin\left(\frac{kb\sin\gamma}{3^N}\right)}{\frac{kb\sin\gamma}{3^N}} \prod_{m=1}^{N} 2\cos\left(\frac{2kb\sin\varphi}{3^m}\right) \prod_{m=1}^{N} 2\cos\left(\frac{2kb\sin\gamma}{3^m}\right) \quad (7)$$

Summary

So in our researches we got formula for intensity for two different fractals, and made program which gives ability to calculate diffraction picture for fractals with different generations and different wave lengths. We got theoretical and experimental results, which are convergent. The next step is to spread of theory on other fractals, but problem happens when we are trying with not indent holes, then disfactorization is impossible, and it can be calculated by quantity methods.



Difraction pictures on Cantor bars with 4 power(top 3 lines) and 5 power (bottom 3 lines). First line is experimental results and under them theoretical.



Difraction pictures on Cantor dust with 4 power(top) and 5 power (bottom). Left pictures are - experimental and right-theoretical.