9. POURING OUT

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It's very important in this problem how we define technical device. We assume that all devices, which human uses, are technical devices and human himself isn't technical device.

Shape of bottle is significant in pouring out process. We have made number of experiments with champagne bottle filled with water. We choose champagne bottle because, it has good shape in comparison with others. Its neck is like cone-shape, which makes easier motion of liquid in this part of bottle. In made experiments bottle moved on different interesting trajectories. $\Delta t = 0.3\,\text{sec}$–is inaccuracy of measurements, which consists of man's reaction time and inaccuracy of stopwatch.

1) In turned over position

$t = (11.2 \pm 0.3)\,\text{sec}$

2) Rotating

In this experiment we have vortex of water, that's why crater appears and air enters intensivly.
$t = (5.1 \pm 0.3)\,\text{sec}$

3) Declined in such way, when gurgling doesn't begin

$t = (10.4 \pm 0.3)\,\text{sec}$

4) Motion of bottle up and down

$t = (10.4 \pm 0.3)\,\text{sec}$

5) Period of gurgling is connected with motion of bottle up and down

$t = (8.5 \pm 0.3)\,\text{sec}$

6) To empty beating on the bottom bottle by hand

$t = (8.3 \pm 0.3)\,\text{sec}$

7) Rotation of bottle around its vertical axis left right by hand

$t = (5.9 \pm 0.3)$ sec

8) Motion of turned over bottle

$t = (5.9 \pm 0.3)$ sec

9) Motion of bottle in horizontal plane left and right (top view is shown on figure)

$t = (8.1 \pm 0.3)$ sec

10) Motion of bottle up and down in vertical plane (side view is shown on figure)

$t = (6.8 \pm 0.3)$ sec

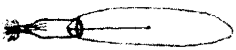
11) Motion of turned over bottle in circle

$t = (6.7 \pm 0.3)$ sec

12) Motion of bottle in circle in horizontal position

In this experiment we have pressure, caused by rotation in circle.

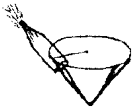
$t = (4.7 \pm 0.3)$ sec



13) Motion of declined bottle in circle

$t = (4.3 \pm 0.3)$ sec

14) Motion of bottle above head in circle

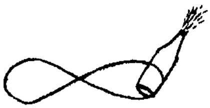


$t = (4.9 \pm 0.3)$ sec

15) Motion in circle with small rotation of bottle

$t = (5.1 \pm 0.3)$ sec

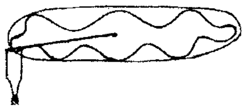
16) Motion of bottle on trajectory shown on figure



$t = (3.7 \pm 0.3)$ sec

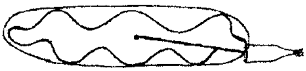
17) Motion of bottle on trajectory shown on figure

1) In turned over position



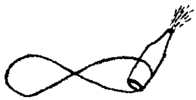
$t = (6.5 \pm 0.3)$ sec

2) In horizontal plane



$t = (3.9 \pm 0.3)$ sec

The shortest period of pouring out of bottle in our experiments is 3.5 sec, which was fixed in the following experiment:

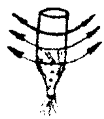


The explanation of shortest time of pouring out of bottle is the fact, that in this case besides vortex we also have pressure, caused by rotation.

We have made experiments on liquids, with different viscosity and saw that bottles filled with liquids with big viscosity get empty faster by different methods. For example bottle filled with honey, oil and other liquids with big viscosity pours out by the methods

"To empty bottle beating on the bottom by hand "

"Rotation of bottle around its vertical axis by hand"



During pouring out of turned over bottle, we will see phenomenon of gurgling.

Lowering down of water level is accompanied by extension of air in bottle, that's why air pressure decreases. As a result of the fact, that this pressure is less then atmosphere pressure the last one pushes the water. On the surface of water Relay-Tailors disturbances appear and air goes into bottle, and this is the reason of gurgling.

fig.1

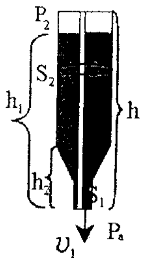
We can determine that all devices which are putting into the bottle aren't external devices. For example, there exist external and internal modems.

Let us consider bottle with tube shown on fig.2.

This method is convenient because, more air goes in tube and we don't have phenomenon of gurgling, which hinders pouring out of bottle. As I sad, vortex makes that we don't have gurgling, and tube plays the same role in this case.

Tube wholly is inside bottle and its the second end finishes with neck. In the tube air moves freely.

fig.2



Cross-section areas of bottle $S_1 < S_2$, and cross-section areas of tube $S_3 < S_4$

We wrote down Bernoulli's equation for water

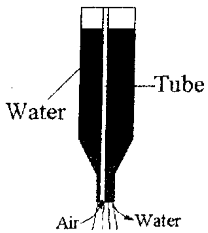
$$p_2 + \rho g h + \frac{\rho \upsilon_2^2}{2} = p_a + \frac{\rho \upsilon_1^2}{2} \quad (1)$$

and continuity condition

$$\upsilon_2 S_2 = \upsilon_1 (S_1 - S_4) \quad (2)$$

$$\Delta p = p_a - p_2$$

$$(1),(2) \Rightarrow \Delta p = \rho g h + \frac{\rho \upsilon_2^2}{2}\left(1 - \frac{S_2^2}{(S_1 - S_4)^2}\right) \quad (3)$$



The volume of air, entering the bottle equals to the volume of water, coming out of the bottle. that's why

$$\upsilon_2 S_2 = \upsilon_3 S_3 \quad (4)$$

Let us write down Poiseuille's equation for air

$$\upsilon_3 S_3 = \frac{\pi \Delta p}{8 \mu_1 l} R_3^4 \quad (5)$$

$$S_3 = \pi R_3^2 \quad (6)$$

where l is length of tube

$$(4),(5),(6) \Rightarrow \qquad \Delta p = \frac{8\mu_1 h \pi S_2 \upsilon_2}{S_3^2} \qquad (7)$$

Let us take designates

$$a = \frac{S_2^2}{(S_1 - S_4)^2} - 1 \,, \qquad b = \frac{8\mu_1 h \pi S_2}{S_3^2}$$

$A = \dfrac{b}{a\rho_w}$, $B = \dfrac{2g}{a}$ find υ_2 from Bernoulli's and Poiseuille's equations

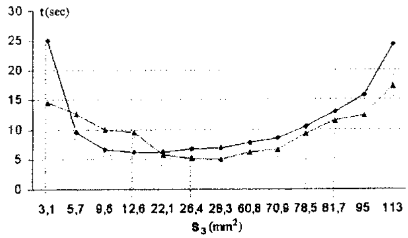
$$\upsilon_2 = -A + \sqrt{A^2 + Bh}$$

$$\upsilon_2 = -\frac{dh}{dt}$$

From these equations we can calculate time, which is needed by water to lower down from h_1 to h_2 in cylinder-shape part.

$$t_c = \frac{2A}{B}\left[\sqrt{1+\frac{Bh_1}{A^2}} - \sqrt{1+\frac{Bh_2}{A^2}} + \ln\left(\frac{\sqrt{1+\frac{Bh_1}{A^2}}-1}{\sqrt{1+\frac{Bh_2}{A^2}}-1}\right)\right]$$

We have made experiments on big coca-cola bottle filled with water with tubes, with different cross-section areas. In made experiments we have measured time of pouring out of bottle with different tubes, thickness of them was the same. By this way we choose tube with optimal cross-section area, by which bottle is pouring out faster. Here theoretical and experimental graphs are shown. On these graphs we can see how time of pouring out of bottle depends on internal cross-section area of tube. Thickness of tubes is equal to 1.5mm.



Periods of time, during which coca-cola bottle with optimal tube pours out:

Theoretical $t = (6.9 \pm 0.2)\,\text{sec}$

Experimental $t = (5.0 \pm 0.3)\,\text{sec}$

Cross-section areas of optimal tube are

Internal $S_3 = 36.0mm^2$

External $S_4 = 90.3mm^2$

Experimental quantities:
$\eta_l(20^0) = 1.8 \cdot 10^{-7}\, pa \cdot \sec$
$\eta_w(20^0) = 1.0 \cdot 10^{-3}\, pa \cdot \sec$
$l = 32 sm$
$d_1 = 2.1 sm$
$d_2 = 8.0 sm$
$h_1 = 28.8 sm$
$h_2 = 8.5 sm$

Champagne bottle pours out:

With optimal tube $t = (2.7 \pm 0.3)\sec$

With vortex $t = (5.1 \pm 0.3)\sec$

We also have considered pouring out of bottle without bottom. In this case $\Delta p = 0$,

$$a_1 = \left(\frac{S_2}{S_1}\right)^2 - 1$$

$$\upsilon_2 = -h'$$

and write Bernoulli's equation for this case

$$\rho g h = a_1 \frac{\rho \upsilon_2^2}{2}, \quad \upsilon_2 = \sqrt{\frac{2gh}{a_1}}$$

$$t_{open} = \sqrt{\frac{2a_1}{g}}\left(\sqrt{h_1} - \sqrt{h_2}\right)$$

For coca-cola bottle

Theoretical $t_{open} = 1.6 \sec$

Experimental $t_{open} = (1.5 \pm 0.3)\sec$

Besides that, we have made experiments on pouring out of different bottle with and without optimal tube.

Experimental Data

Bottle	Volume (liter)	Time with tube (sec)	Time without tube (sec)
Coca-cola (big)	1.5	5.3 ± 0.3	21.0 ± 0.3
Wine	0.75	3.2 ± 0.3	11.9 ± 0.3
Champagne	0.75	2.7 ± 0.3	11.2 ± 0.3
Oil	0.6	2.6 ± 0.3	10.3 ± 0.3
Lemonade	0.5	2.6 ± 0.3	10.2 ± 0.3
Borjomi	0.35	1.2 ± 0.3	5.3 ± 0.3
Beer	0.35	1.2 ± 0.3	5.0 ± 0.3
Coca-cola (small)	0.25	1.1 ± 0.3	6.1 ± 0.3

You see how tube makes faster pouring out of bottle.

Experiments show, that the higher temperature of liquid the faster pouring out of bottle.

Time of pouring out of champagne bottle at different temperature of liquid

Temperature $[^0C]$	Time (sec)
92	7.6 ± 0.3
80	7.8 ± 0.3
50	9.6 ± 0.3
40	10.0 ± 0.3

Reynolds's number (with the best tube)

1) Flow of water in cylinder-shape part

$$\upsilon_2 = 0.1 m/\sec \ , d_2 = 8sm$$

$$\mathrm{Re} = \frac{\upsilon_2 d_2 \rho}{\eta_w} = 8000 > 1000$$

2) Flow of water in neck of bottle

$$\upsilon_1 = 1.8 m/\sec , d_1 = 2.1 sm$$

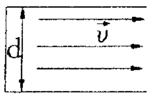
$$\mathrm{Re} = \frac{\upsilon_1 d_1 \rho}{\eta_w} = 37800 > 1000$$

Because of fact, that we have turbulent regimes, let's refine how useful is Bernoulli's equation.

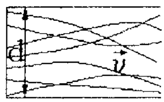
1) $\eta_w(20^0) = 10^{-3} pa \cdot \sec$, that's why work of viscosity is small;

2) We have fluctuations of velocity and pressure. But we can find mean velocity and pressure by diameter d and write conservation low of energy like Bernoulli's equation.

υ, P



$\overline{\upsilon}, \overline{P}$



Lacks: We haven't taken into account work of viscosity in Bernoulli's equation, but we estimated it and maximal inaccuracy, which it gives is $0.2 \sec$. Our Lack is the fact, that we haven't considered unsteady of water flows and haven't estimated acceleration of water in this case.

Conclusions: 1) We have made number of experiments on pouring out of champagne bottle, which moved on different trajectories.
2) We have discovered those methods by which bottle filled with big viscosity liquid is pouring out maximally fast.
3) We have made experiments on liquids by different temperature.
4) We have used tube method and received coincidence of theoretical and experimental results.
5) We have investigated how to pour out bottle maximally fast in different conditions.

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