IYPT 2010

№13 SHRIEKING ROD

GEORGIAN Team

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A metal rod is held between two fingers and hit. Investigate how the sound produced depends on the position of holding and hitting the rod?

Presentation Plan

a) **Wave types in rod**

- Longitudinal Waves
- Bending Waves
- Torsion waves

b) **Longitudinal waves**

- Wave equation
- Solution of Wave equation
- Frequency modes ; Node positions

c) **Experiments with Longitudinal waves**

d) **Bending waves**

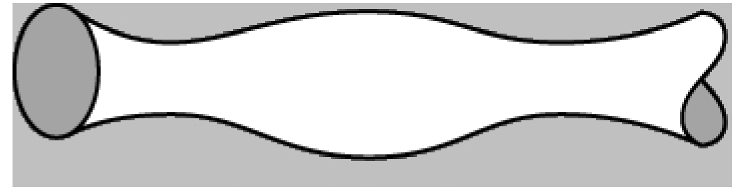
- Wave equation
- Solution of Wave equation
- Frequency modes ; Node positions

e) **Experiments with Bending waves**

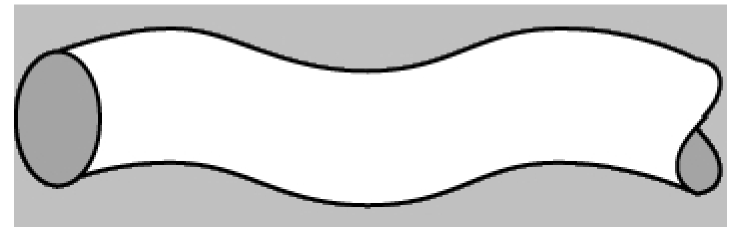
f) **Conclusion**

Wave types in rod

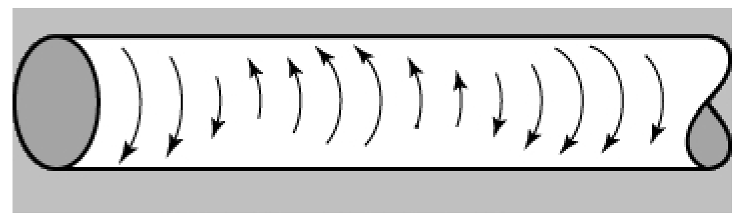
(A) Quasi-longitudinal Compression waves in a thin rod
Quasi longitudinal due to transverse strains– as rod stretches, it grows thinner

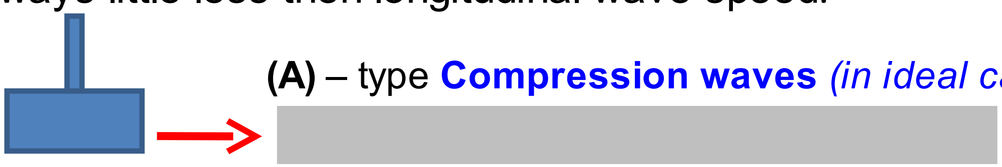


(B) Bending waves in a thin rod
Bending waves involve both compression and share strains. Their velocity depends on frequency - they are ***DISPERSIVE***



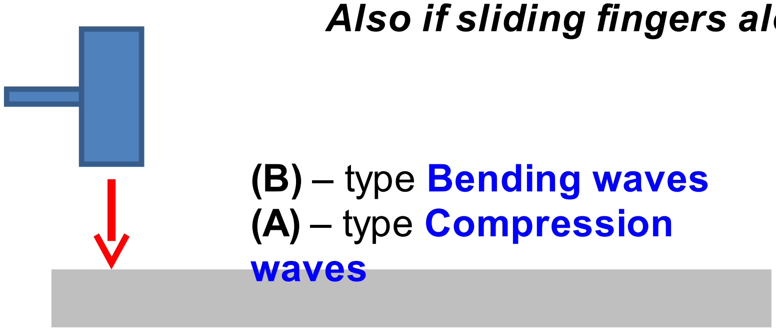
(C) Transverse Torsion waves in a thin rod
A lateral displacement χ which varies with x gives rise to a ***shear strain***.
In a thin rod torsion shear waves travel at a speed which is always little less then longitudinal wave speed.



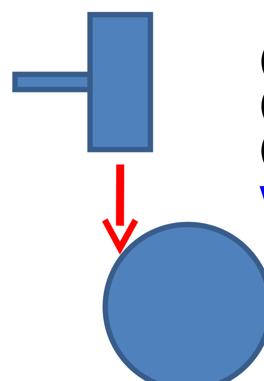


(A) – type **Compression waves** *(in ideal case)*

Also if sliding fingers along the rod



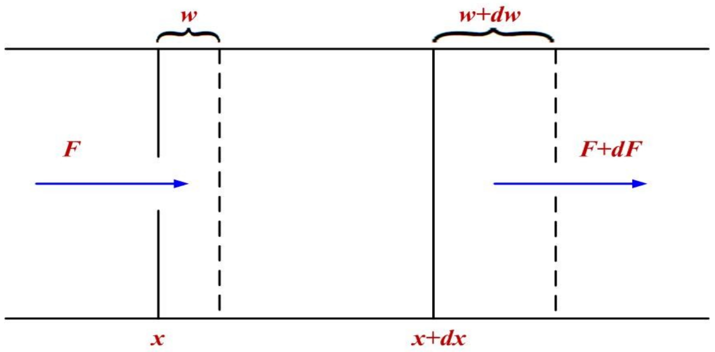
(B) – type **Bending waves**
(A) – type **Compression waves**



(C) – type **Torsion waves**
(B) – type **Bending waves**
(A) – type **Compression waves**

Longitudinal Waves

Wave equation



- Short segment of length dx
- Cross-section area S
- Force $F(x)$
- The plane at x moves a distance w to the right
- The ***Stress*** F/S
- The ***Strain*** (change in length per unit of original length) $\partial w / \partial x$
- The Young's module E

The **Hooke's law**:

$$\frac{F}{S} = E\frac{\partial w}{\partial x} \qquad (1)$$

The net force:

$$dF_{net} = F(x+dx) - F(x) = \frac{\partial F}{\partial x}dx = SE\frac{\partial^2 w}{\partial x^2}dx$$

The **Newton's second law**:

$$dF_{net} = a \cdot dm = \rho S dx\frac{\partial^2 w}{\partial t^2} = SE\frac{\partial^2 w}{\partial x^2}dx$$

or

$$\frac{\partial^2 w}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 w}{\partial x^2} = c_L^2\frac{\partial^2 w}{\partial x^2} \qquad (2)$$

- one-dimensional wave equation for waves with a **velocity**

$$C_L = \sqrt{E/\rho} \qquad (3)$$

Solution of Wave equation

Let us search the **harmonic standing wave modes** in the form: $w = f(x) Cos\,(\omega t - \varphi)$

Substituting in (2) gives: $$\frac{d^2 f(x)}{dx^2} = -\frac{\omega^2}{c_L^2} f(x)$$

This yields: $$f(x) = ACos\,(kx) + Bsin(kx)$$

Where the wave number $k = \omega / c_L = 2\pi / \lambda$. **(4)**

Boundary conditions for both ($x=0$; $x=l$) ends free: $F = 0 \rightarrow \frac{\partial w}{\partial x} = 0$

This gives : $B = 0$; $Sin\,(kl) = 0 \rightarrow k = \frac{\pi n}{l} \rightarrow \omega = \frac{\pi n c_L}{l}$.

i.e.

Frequency modes: $$\nu_n = \frac{\omega}{2\pi} = n\frac{c_L}{2l} \quad ; n = 1, 2, 3, \ldots$$ **(5)**

Wave lengths: $$\lambda_n = \frac{c_L}{\nu_n} = \frac{2l}{n} \quad ; n = 1, 2, 3, \ldots$$ **(6)**

Wave solution: $$w = A\,Cos(\frac{\pi n}{l} x) \cdot Cos\,(\frac{\pi n c_L}{l} t - \varphi) \quad ; \quad C_L = \sqrt{E/\rho}$$

Standing wave modes

The First Harmonic:

Wave length: $\lambda_1 = 2l$

Frequency: $\nu_1 = \dfrac{\sqrt{E/\rho}}{2l}$

Nod positions: $l/2$

$$C_L = \sqrt{E/\rho}$$

The Second Harmonic:

Wave length: $\lambda_2 = l$

Frequency: $\nu_2 = \dfrac{\sqrt{E/\rho}}{l} \quad = 2\nu_1$

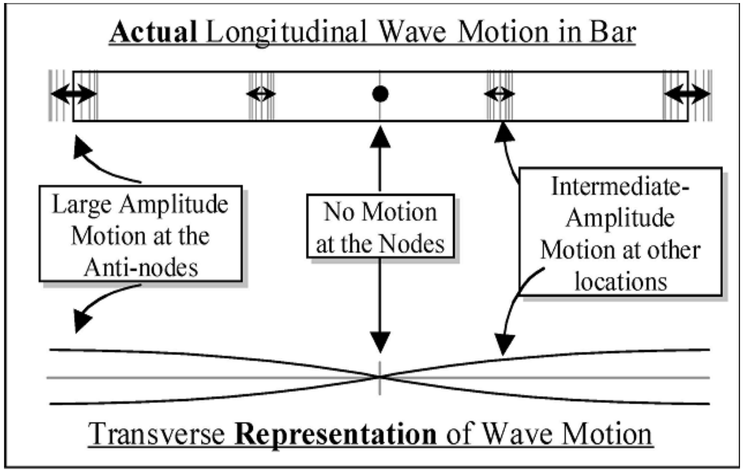
Nod positions: $l/4$; $3l/4$

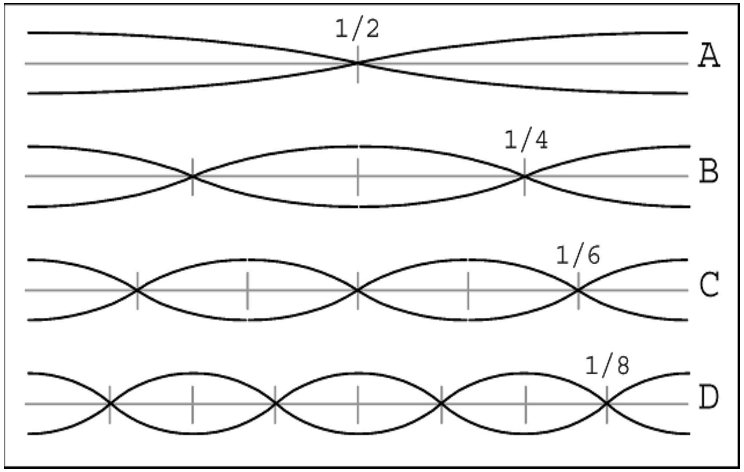
The Third Harmonic:

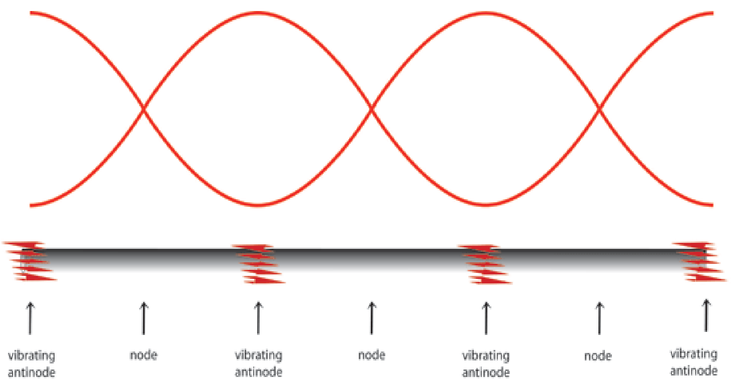
Wave length: $\lambda_3 = \dfrac{2l}{3}$

Frequency: $\nu_3 = \dfrac{3}{2}\dfrac{\sqrt{E/\rho}}{l} \quad = 3\nu_1$

Nod positions: $l/6$; $l/2$; $5l/6$





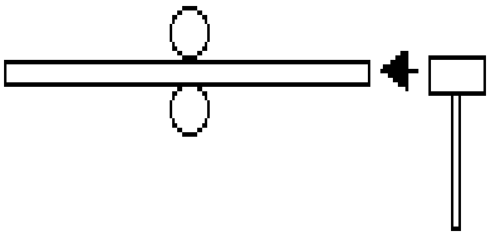


The modal frequencies, wavelengths, and the locations of nodes and anti-nodes for the first nine harmonics associated with a vibrating rod of length, *L*

Harmonic Mode #, n	Frequency f (Hz)	Wavelength λ (m)	Node Locations, (m)	Anti-Node Locations, (m)
1	f_1	$\lambda_1 = 2L$	0	$\pm{}^1/_2L$
2	$f_2 = 2f_1$	$\lambda_2 = {}^1/_2\lambda_1 = L$	$\pm{}^1/_4L$	$0, \pm{}^2/_4L = \pm{}^1/_2L$
3	$f_3 = 3f_1$	$\lambda_3 = {}^1/_3\lambda_1 = {}^2/_3L$	$0, \pm{}^2/_6L = \pm{}^1/_3L$	$\pm{}^1/_6L, \pm{}^3/_6L = \pm{}^1/_2L$
4	$f_4 = 4f_1$	$\lambda_4 = {}^1/_4\lambda_1 = {}^1/_2L$	$\pm{}^1/_8L,\ \pm{}^3/_8L$	$0, \pm{}^2/_8L = \pm{}^1/_4L, \pm{}^4/_8L = \pm{}^1/_2L$
5	$f_5 = 5f_1$	$\lambda_5 = {}^1/_5\lambda_1 = {}^2/_5L$	$0, \pm{}^2/_{10}L = \pm{}^1/_5L, \pm{}^4/_{10}L = \pm{}^2/_5L$	$\pm{}^1/_{10}L,\ \pm{}^3/_{10}L, \pm{}^5/_{10}L = \pm{}^1/_2L$
6	$f_6 = 6f_1$	$\lambda_6 = {}^1/_6\lambda_1 = {}^1/_3L$	$\pm{}^1/_{12}L,\ \pm{}^3/_{12}L = \pm{}^1/_4L, \pm{}^5/_{12}L$	$0, \pm{}^2/_{12}L = \pm{}^1/_6L, \pm{}^4/_{12}L = \pm{}^1/_3L, \pm{}^6/_{12}L = \pm{}^1/_2L$
7	$f_7 = 7f_1$	$\lambda_7 = {}^1/_7\lambda_1 = {}^2/_7L$	$0, \pm{}^2/_{14}L = \pm{}^1/_7L, \pm{}^4/_{14}L = \pm{}^2/_7L, \pm{}^6/_{14}L = \pm{}^3/_7L$	$\pm{}^1/_{14}L,\ \pm{}^3/_{14}L, \pm{}^5/_{14}L, \pm{}^7/_{14}L = \pm{}^1/_2L$
8	$f_8 = 8f_1$	$\lambda_8 = {}^1/_8\lambda_1 = {}^1/_4L$	$\pm{}^1/_{16}L,\ \pm{}^3/_{16}L, \pm{}^5/_{16}L, \pm{}^7/_{16}L$	$0, \pm{}^2/_{16}L = \pm{}^1/_8L, \pm{}^4/_{16}L = \pm{}^1/_4L, \pm{}^6/_{16}L = \pm{}^3/_8L$
9	$f_9 = 9f_1$	$\lambda_9 = {}^1/_9\lambda_1 = {}^2/_9L$	$0, \pm{}^2/_{18}L = \pm{}^1/_9L, \pm{}^4/_{18}L = \pm{}^2/_9L, \pm{}^6/_{18}L = \pm{}^3/_9L = \pm{}^1/_3L, \pm{}^8/_{18}L = \pm{}^4/_9L$	$\pm{}^1/_{18}L,\ \pm{}^3/_{18}L, \pm{}^5/_{18}L, \pm{}^7/_{18}L, \pm{}^9/_{18}L = \pm{}^1/_2L$

All modes (*n = 0, 1, 2, 3, 4, 5, …) of vibration of the rod all have the same longitudinal* speed of propagation of sound in the rod $c_L = \sqrt{E/\rho}$ $= v_i\lambda_i$

Experiments with Longitudinal Waves



To make rod sound clearer and louder, we shall:

3. Hit the rod as fast as possible
4. Hit it not very hard, because harmonicity of waves will be violated.
5. Hit the rod with thing, that doesn't produce good sound (e.g. Ebonite Rod), not to interrupt Main one.
6. Try to hit it vertically or horizontally, and not intermediate.

- Also **sliding of hand** down the length of the rod.
- Addition of **Rosin** makes **fingers more sticky.**
- The **pitch** of the sound can be **varied** by changing **holding places of the Rod** or by changing the **length of the rod** itself.

Ringing The Rod

Experiment

Experiments with Longitudinal Waves

1. **Aluminium rod** (ρ_{AL} =2,7·10³ kg/m³ ; E_{AL} =70×10⁹ N/m² ; $C_{L(Al)}$ = 5082.4 m/s)

Length of the **rod 1**: $l = 1{,}2m$ $\nu_1 = \dfrac{\sqrt{E/\rho}}{2l}$ $C_L = \sqrt{E/\rho}$ $\lambda_n = \dfrac{2l}{n}$ $\nu_n = n\dfrac{c_L}{2l}$

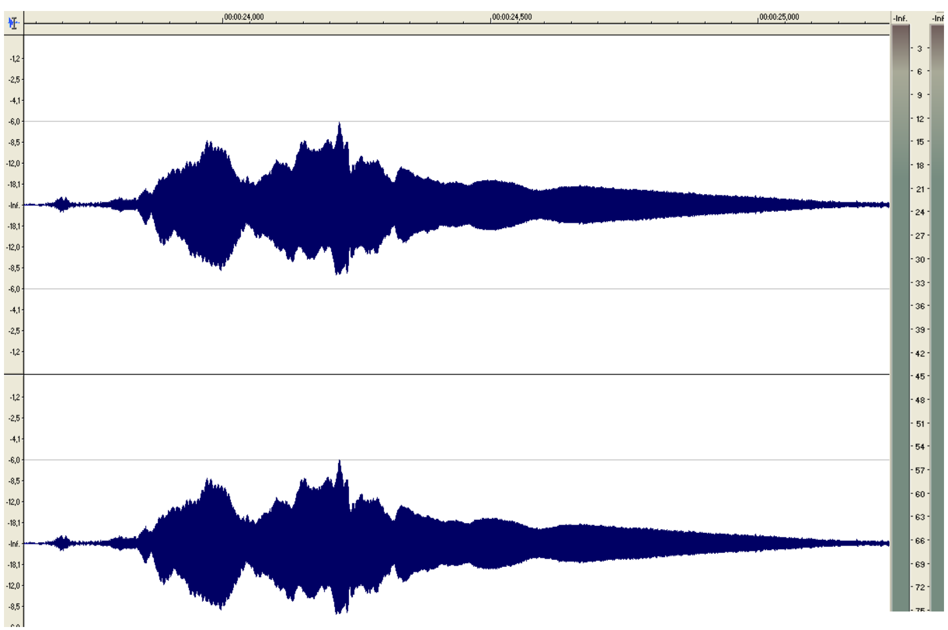
The first harmonic: $\lambda_1 = 2.4m$; $\nu_1 = 2117\ Hz$; *nodes: 0.6m*

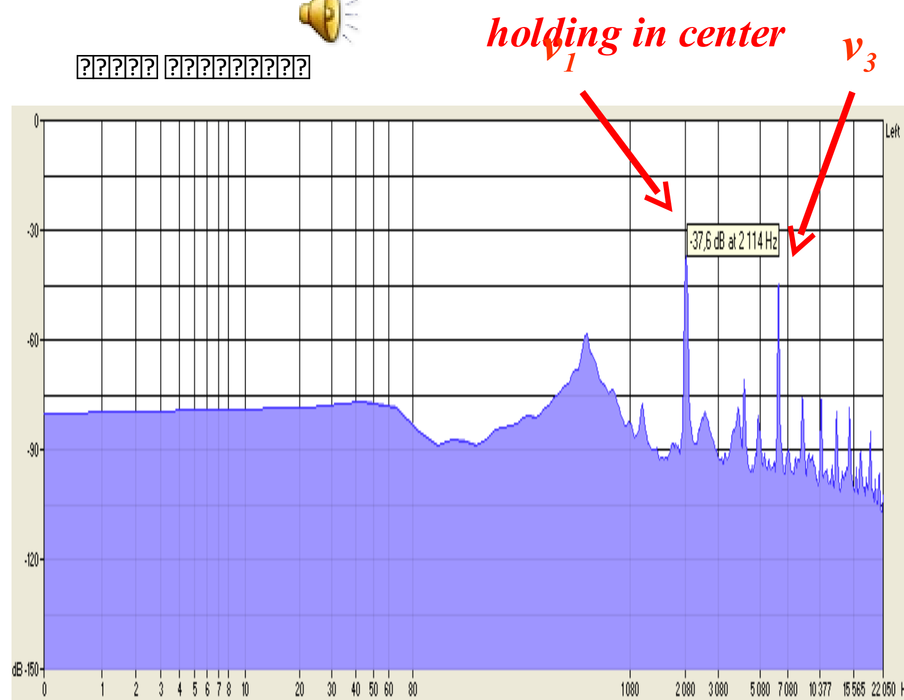
The second harmonic: $\lambda_2 = 1.2m$; $\nu_2 = 4234\ Hz$; *nodes: 0.3m , 0.9m*

The third harmonic: $\lambda_3 = 0.8m$; $\nu_3 = 6351\ Hz$; *nodes: 0.2m , 0.6m , 1.0 m*

If hold the rod in the place where nods of several modes are placed all these modes will occur.

Touching the rod at the ends will stop the sound .





Experiments with Longitudinal Waves

2. **Aluminium rod** (ρ_{AL} =2,7·10^3 kg/m^3 ; E_{AL} =70×10^9 N/m^2 ; $C_{L(Al)}$ = 5082.4 m/s)

Length of the **rod 2**: l= 0.75m $\nu_1 = \frac{\sqrt{E/\rho}}{2l}$ $c_L = \sqrt{E/\rho}$ $\lambda_n = \frac{2l}{n}$ $\nu_n = n\frac{c_L}{2l}$

The first harmonic: λ_1=1.5m ; ν_1=3388 Hz ; nodes: 0.37m

The second harmonic: λ_2=0.75m ; ν_2=6776 Hz ; nodes: 0.19m , 0.56m

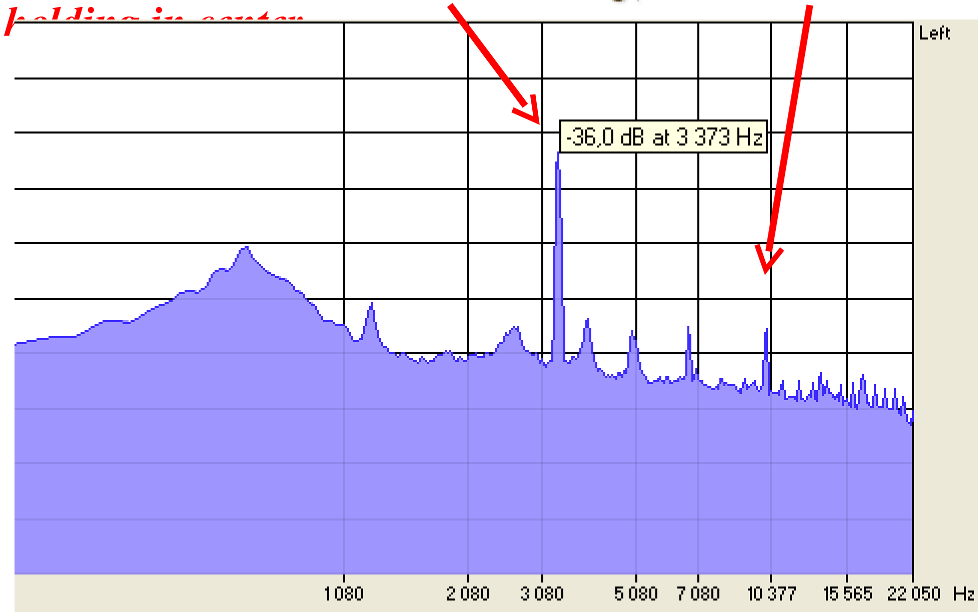
The third harmonic: λ_3=0.5m ; ν_3=10 164 Hz ; nodes: 0.12m , 0.37m , 0.63 m

If hold the rod in the place where nods of several modes are placed all these modes will occur.

Touching the rod at the ends will stop the sound .

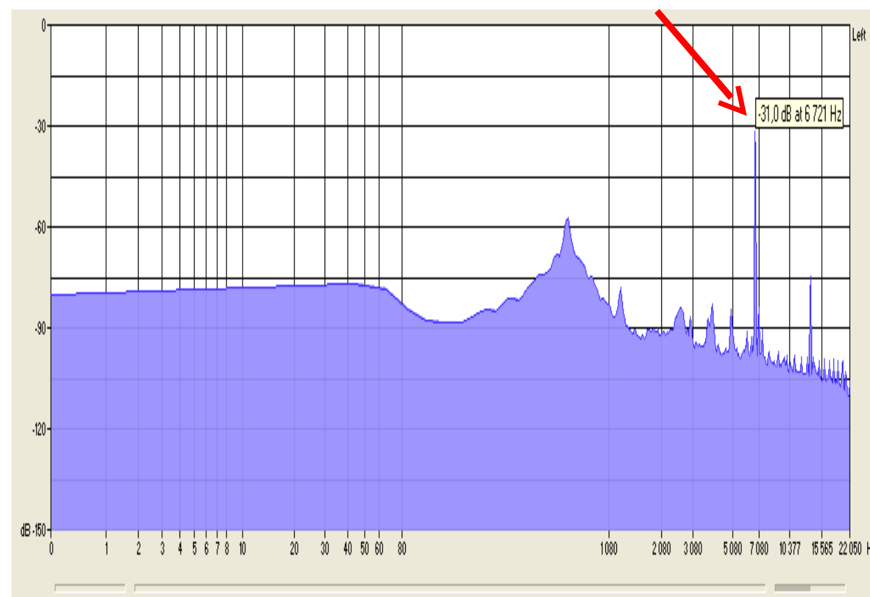
holding in center

ν_1 ν_3



holding at l/4

ν_2



Experiments with Longitudinal Waves

3. **BRASS rod** (ρ_{Br} =8,5·10³ kg/m³ ; E_{Br} =95×10⁹ N/m² ; $C_{L(Br)}$ = 3480 m/s)

Length of the **rod 3**: l= 0.4m $\nu_1 = \dfrac{\sqrt{E/\rho}}{2l}$ $C_L = \sqrt{E/\rho}$ $\lambda_n = \dfrac{2l}{n}$ $\nu_n = n\dfrac{c_L}{2l}$

The first harmonic: λ_1=0.8m ; ν_1=4350 Hz ; nodes: 0.2m

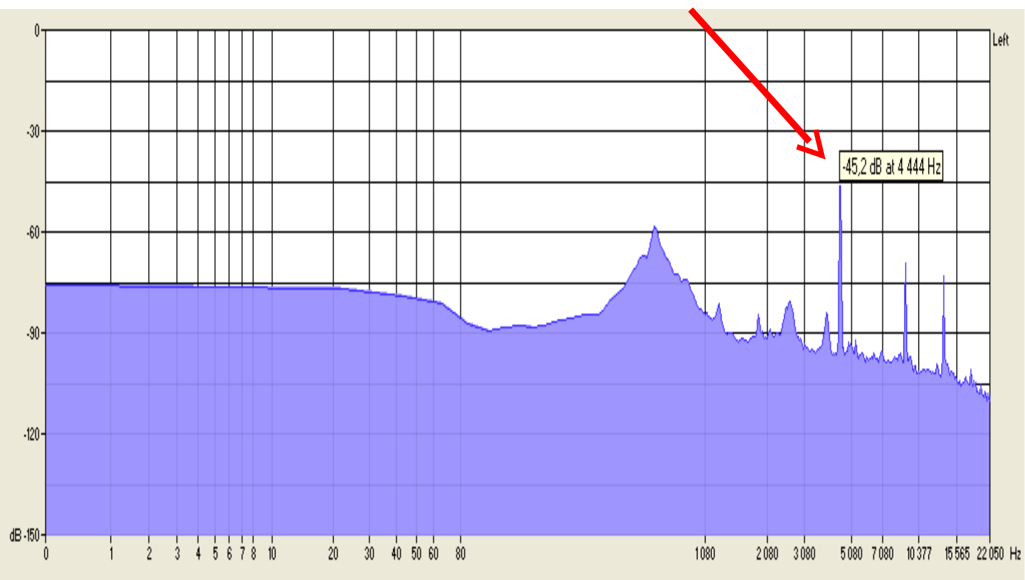
The second harmonic: λ_2=0.4m ; ν_2=8700 Hz ; nodes: 0.1m , 0.3m

The third harmonic: λ_3=0.27m ; ν_3=13 050 Hz ; nodes: 0.07m , 0.2m , 0.33 m

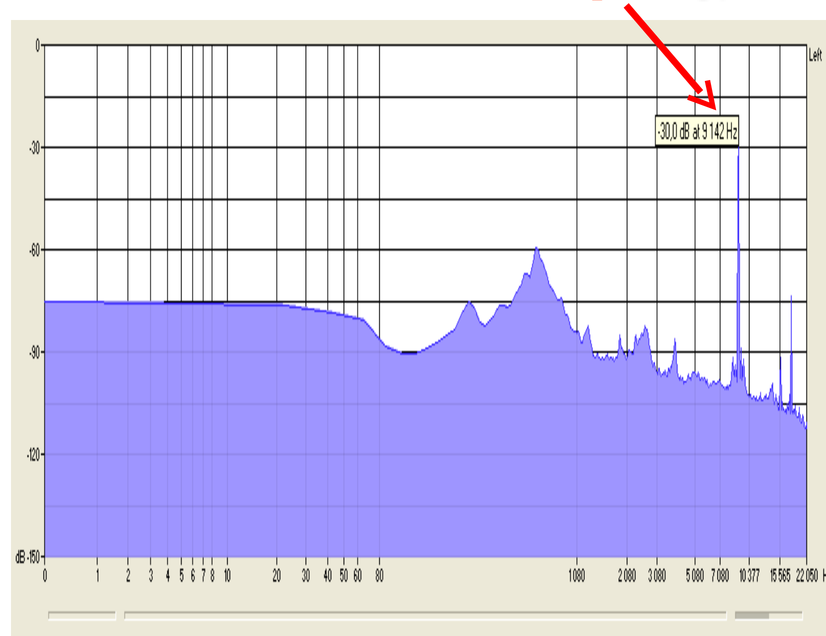
If hold the rod in the place where nods of several modes are placed all these modes will occur.

Touching the rod at the ends will stop the sound .

holding in center ν_1



holding at 1/4 ν_2



Experiments with Longitudinal Waves

4. **STEEL rod** (ρ_{St} =7,8·10³ kg/m³ ; E_{St} =200×10⁹ N/m² ; $C_{L(St)}$ = 5150 m/s)

Length of the **rod 4**: $l = 0.6m$ $\quad \nu_1 = \dfrac{\sqrt{E/\rho}}{2l}$ $\quad C_L = \sqrt{E/\rho}$ $\quad \lambda_n = \dfrac{2l}{n}$ $\quad \nu_n = n\dfrac{c_L}{2l}$

The first harmonic: λ_1=1.2m ; ν_1=4291 Hz ; nodes: 0.3m

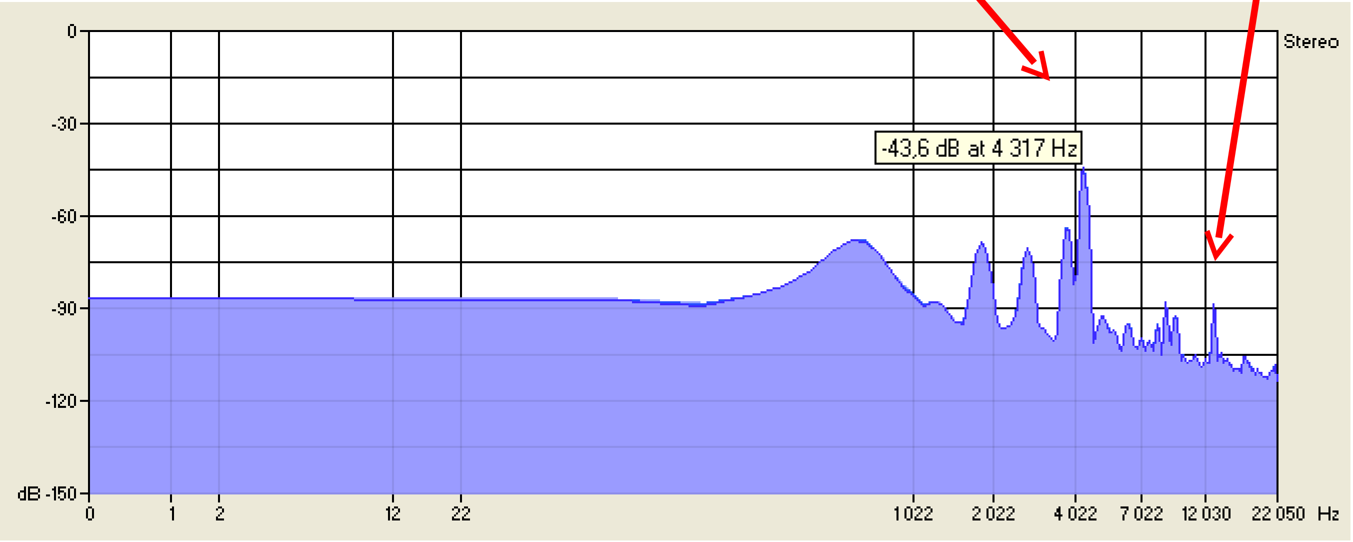
The second harmonic: λ_2=0.6m ; ν_2=8582 Hz ; nodes: 0.15m , 0.45m

The third harmonic: λ_3=0.4m ; ν_3=12 873 Hz ; nodes: 0.1m , 0.3m , 0.5 m

If hold the rod in the place where nods of several modes are placed all these modes will occur.

Touching the rod at the ends will stop the sound.

holding in center ν_1 ν_3

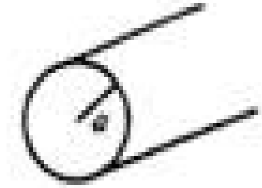


BENDING Waves

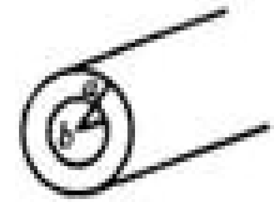
Wave equation

Eyler-Bernoulli beam theory equation of motion

$$\frac{\partial^2 y}{\partial t^2} = -\frac{EK^2}{\rho} \cdot \frac{\partial^4 y}{\partial x^4}$$



$K = a/2$



$K = (\sqrt{a^2 + b^2})/2$

$$K = \frac{\sqrt{(inner\ \ radius)^2 + (outer\ \ radius)^2}}{2}$$

Where y is displacement normal to rod axis.

"**Radius of Gyration**": $$K^2 = \frac{1}{S}\int z^2\, dS$$

z is distance from central axis of rod.

Here shear deformations and rotary inertia are neglected.

Harmonic solutions are of the following form:

$$y(x,t) \equiv Y(x)e^{i\omega t}$$

$$Y(x) = \cos(\omega t + \varphi) \cdot [A\cosh(kx) + \ B\sinh(kx) + \ C\cos(kx) + \ D\sin(kx)]$$

$k = \omega / v$ is the wave (propagation) number.

Velocity is dependent on frequency. So here is ***dispersion***. $$v = \sqrt{2\pi\nu K c_L}$$

Rossing, Fletcher "Physics of Musical Instruments"

Frequency Modes of Standing Bending Waves

Frequency modes depend on the end conditions

For our task we consider FREE end - no torque and no shearing force

They give restrictions on standing wave frequencies:

$$\frac{\omega l}{2v} = \frac{\pi}{4} \cdot [3.011\,;5\,;7\,;\ldots;(2n+1)\ldots]$$

$$v = \sqrt{2\pi\nu K c_L}$$

$$\nu_n = \frac{\omega_n}{2\pi} = \frac{\pi K}{8l^2}\sqrt{\frac{E}{\rho}} \cdot [3.011^2; 5^2; 7^2; \ldots; (2n+1)^2 \ldots]$$

And wave lengths:

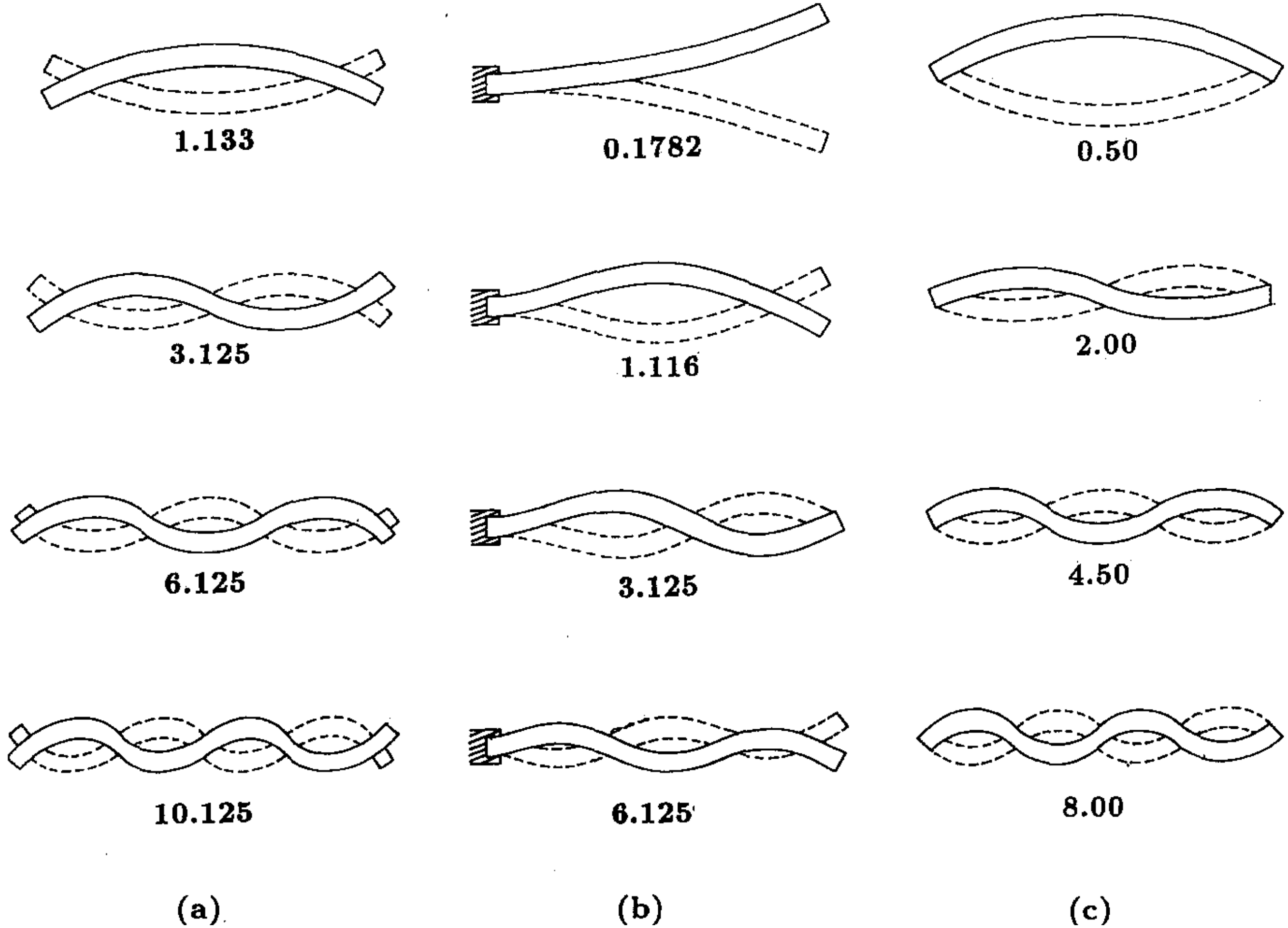
$$\lambda_n = \frac{4l}{[3.011\,;5\,;7\,;\ldots;(2n+1)\ldots]}$$

Table I. Characteristics of transverse vibrations in a bar with free ends.

Frequency (Hz)	Wavelength (m)	Nodal positions (m from end of 1-m bar)
$f_1 = 3.5607\ (K/L^2)\sqrt{E/\rho}$	$1.330L$	0.224, 0.776
$2.756\,f_1$	$0.800L$	0.132, 0.500, 0.868
$5.404\,f_1$	$0.572L$	0.094, 0.356, 0.644, 0.906
$8.933\,f_1$	$0.445L$	0.073, 0.277, 0.500, 0.723, 0.927

To obtain actual frequencies multiply by

$$\frac{\pi K}{l^2}\sqrt{\frac{E}{\rho}}$$



Note: Frequencies for bending waves are sufficiently lower than for the the longitudinal waves due to extra coefficient $\frac{K}{l}$.

Higher modes are vanishing very rapidly.

Waves are damped for supported or clamped ends

Experiments with BENDING Waves

1. **Aluminium rod** (ρ_{AL} =2,7·10³ kg/m³ ; E_{AL} =70×10⁹ N/m² ; $C_{L(Al)}$ = 5082.4 m/s)

FREE Ends

$$\nu_n = \sqrt{2\pi\nu_n K c_L} \quad ; \quad K = \frac{1}{2}\sqrt{a^2+b^2} \quad ; \quad \nu_n = \frac{\pi K}{8l^2}\sqrt{\frac{E}{\rho}} \cdot [3.011^{\,2}; 5^2\,; 7^2\,; \dots; (2n+1)^2 \dots]$$

$$\lambda_n = \frac{4l}{[3.011\,; 5\,; 7\,; \dots; (2n+1) \dots]}$$

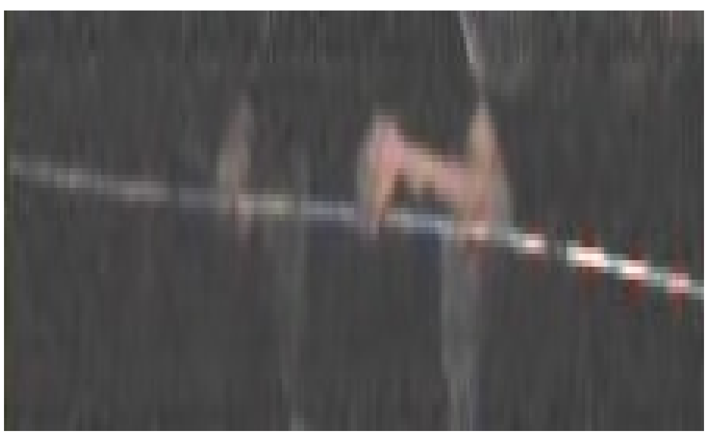
Length of the **rod 1**: *l= 1,2m ; Radii: a=0.005m, b=0.004m ; K=0.0032m*

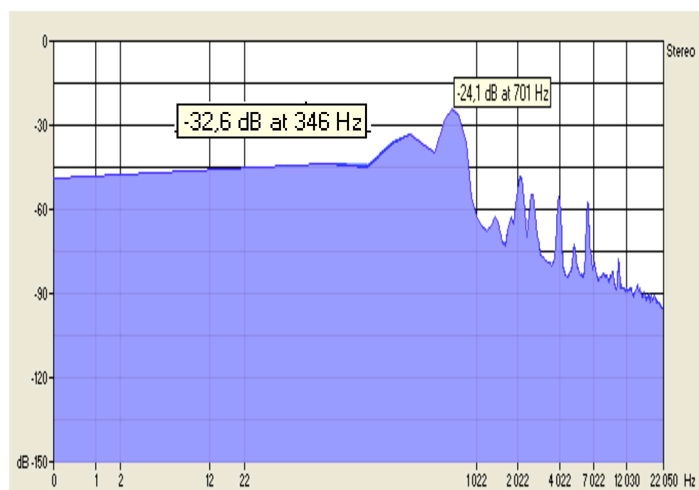
The first harmonic: λ_{B1}= 1,596m ; ν_1= 40 Hz ; nodes: 0.27m , 0,93m

The second harmonic: λ_{B2}= 0,96m ; ν_2=110 Hz ; nodes: 0.16m , 0.6m , 1.04m

The third harmonic: λ_{B3}= 0.69m ; ν_3=215 Hz; nodes: 0.11m, 0.43m, 0.77m

1.





holding in center

Experiments with BENDING Waves

4. **STEEL rod** (ρ_{St} =7,8·10³ kg/m³ ; E_{St} =200×10⁹ N/m² ; $C_{L(St)}$ = 5150 m/s)

FREE Ends

$$v_n = \sqrt{2\pi v_n K c_L} \quad ; \qquad K = a/2 \quad ; \qquad v_n = \frac{\pi K}{8 l^2}\sqrt{\frac{E}{\rho}} \cdot [3.011^{\,2} ; 5^2 ; 7^2 ; \dots ; (2n+1)^2 \dots]$$

$$\lambda_n = \frac{4l}{[3.011 ; 5 ; 7 ; \dots ; (2n+1) \dots]}$$

Length of the **rod 4**: *l= 0.6m ; Radius a= 0.007m ; K=0.0035m*

The first harmonic: λ_{B1}= 0.8m ; v_1= 190 Hz ; **nodes: 0.13m , 0.47m**

The second harmonic: λ_{B2}=0.48m ; v_2= 525Hz ; **nodes: 0.08m , 0.3m , 0.52m**

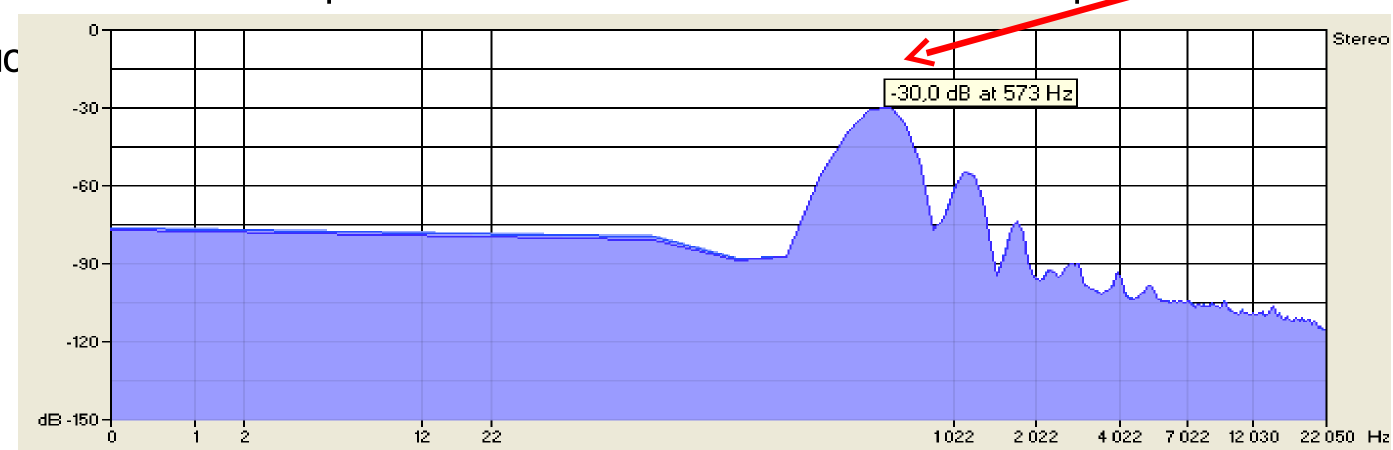
The third harmonic: λ_{B3}=0.34m; v_3= 1029Hz; **nodes: 0.056m, 0.21m, 0.39m, 0.544m**

If hold the rod in the place where nods of several modes are placed all these modes will occur.

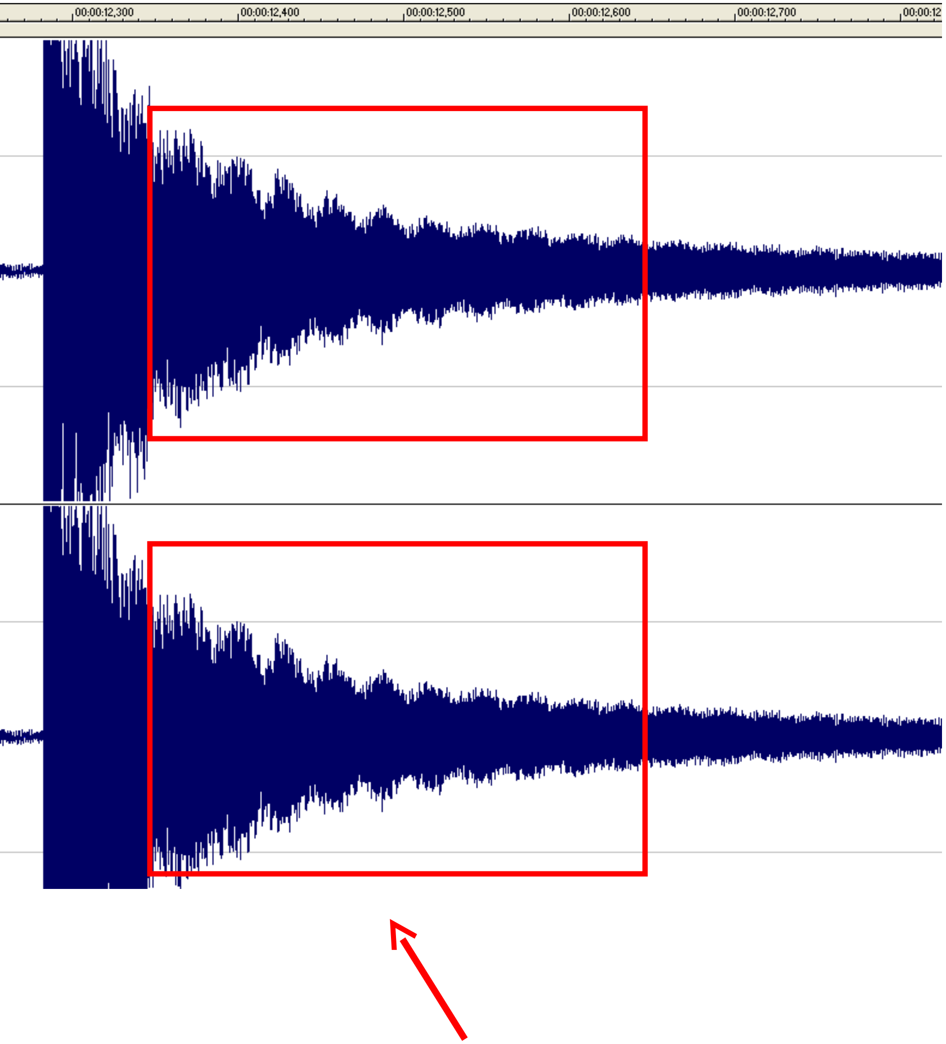
v_2

holding at 0.3m

Touc



Beats



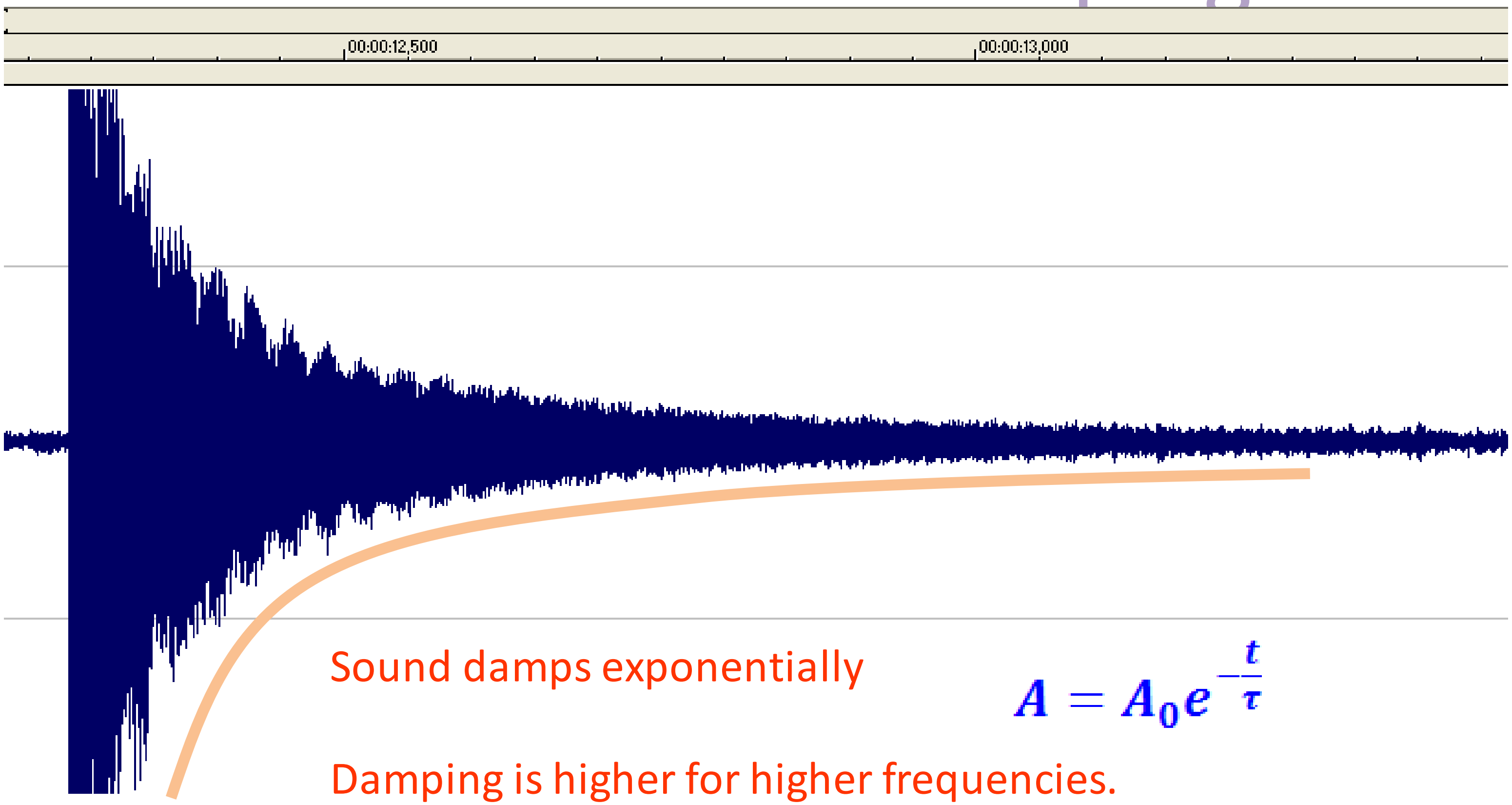
Our experimental Beats

Sound in Experiments sometimes became stornger, sometimes weaker.

This was because of Beats.

A **beat** is an interference between two sounds of slightly different frequencies.

Sound Damping



Sound damps exponentially

$$A = A_0 e^{-\frac{t}{\tau}}$$

Damping is higher for higher frequencies.
We saw it comparing damping for transverse and longitudinal waves.

- Internal Damping The "decay time" $\tau_{int} \sim \dfrac{1}{\pi\nu}$

- Air damping. The "decay time" $\tau_a \sim \rho r/\sqrt{\nu}$

- Transfer of energy to other systems (e.g. supports) $\tau_{sup} \sim \dfrac{1}{lG\nu^2}$ *(G- "energy conductance")*

Rossing, Fletcher "Physics of Musical Instruments"

Conclusion

- There are different types of waves in the rod
- The type of wave depends on how do we hit the rod.
- The frequency of standing wave depends on where we hold the rod.
- The frequencies of bending waves are lower than of compression waves
- Damping of the waves depends on frequency

Thank you for your attention!