

IYPT 2010

# №13 SHRIEKING ROD

**GEORGIAN Team**

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**A metal rod is held between two fingers and hit. Investigate how the sound produced depends on the position of holding and hitting the rod?**

# Presentation Plan

## a) Wave types in rod

- ❖ Longitudinal Waves
- ❖ Bending Waves
- ❖ Torsion waves

## b) Longitudinal waves

- ❖ Wave equation
- ❖ Solution of Wave equation
- ❖ Frequency modes ; Node positions

## c) Experiments with Longitudinal waves

## d) Bending waves

- ❖ Wave equation
- ❖ Solution of Wave equation
- ❖ Frequency modes ; Node positions

## e) Experiments with Bending waves

## f) Conclusion

# Wave types in rod

## (A) Quasi-longitudinal **Compression waves** in a thin rod

Quasi longitudinal due to transverse strains– as rod stretches, it grows thinner



## (B) **Bending waves** in a thin rod

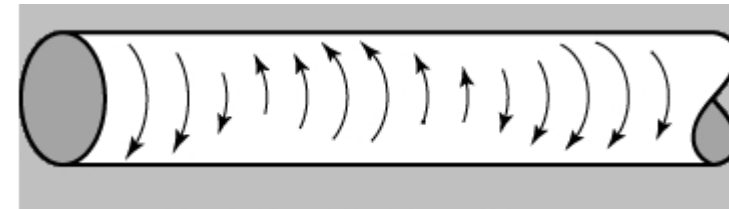
Bending waves involve both compression and shear strains. Their velocity depends on frequency - they are **DISPERSIVE**



## (C) Transverse **Torsion waves** in a thin rod

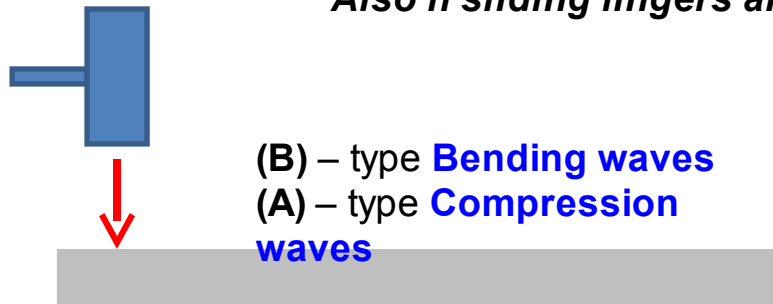
A lateral displacement  $\chi$  which varies with  $x$  gives rise to a **shear strain**.

In a thin rod torsion shear waves travel at a speed which is always little less than longitudinal wave speed.

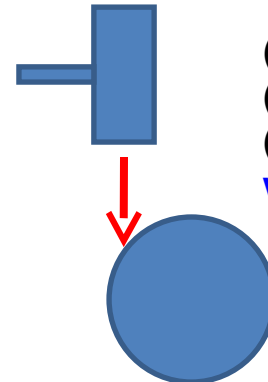


(A) – type **Compression waves** (*in ideal case*)

*Also if sliding fingers along the rod*



(B) – type **Bending waves**  
(A) – type **Compression waves**

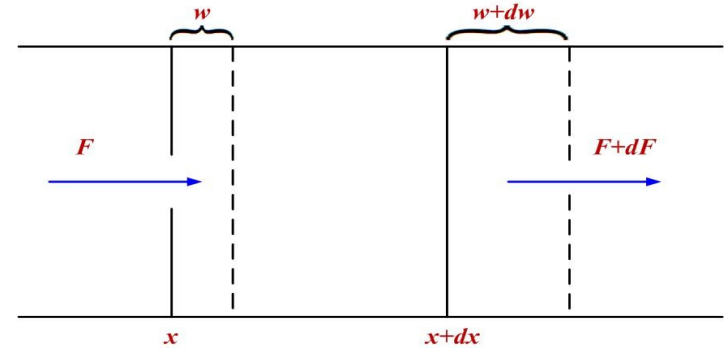


(C) – type **Torsion waves**  
(B) – type **Bending waves**  
(A) – type **Compression waves**

# Longitudinal Waves

## Wave equation

- Short segment of length  $dx$
- Cross-section area  $S$
- Force  $F(x)$
- The plane at  $x$  moves a distance  $w$  to the right
- The **Stress**  $F/S$
- The **Strain** (change in length per unit of original length)  $\partial w / \partial x$
- The Young's module  $E$



The **Hooke's law**:

$$\frac{F}{S} = E \frac{\partial w}{\partial x} \quad (1)$$

The net force:

$$dF_{net} = F(x + dx) - F(x) = \frac{\partial F}{\partial x} dx = SE \frac{\partial^2 w}{\partial x^2} dx$$

The **Newton's second law**:

$$dF_{net} = a \cdot dm = \rho S dx \frac{\partial^2 w}{\partial t^2} = SE \frac{\partial^2 w}{\partial x^2} dx$$

or

$$\frac{\partial^2 w}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 w}{\partial x^2} = c_L^2 \frac{\partial^2 w}{\partial x^2} \quad (2)$$

- one-dimensional wave equation for waves with a **velocity**

$$c_L = \sqrt{E/\rho} \quad (3)$$

## Solution of Wave equation

Let us search the **harmonic standing wave modes** in the form:  $w = f(x)\text{Cos}(\omega t - \varphi)$

Substituting in (2) gives: 
$$\frac{d^2 f(x)}{dx^2} = -\frac{\omega^2}{c_L^2} f(x)$$

This yields: 
$$f(x) = A\text{Cos}(kx) + B\text{sin}(kx)$$

Where the wave number 
$$k = \omega/c_L = 2\pi/\lambda \quad . \quad (4)$$

**Boundary conditions** for both ( $x=0$  ;  $x=l$ ) ends free:  $F = 0 \rightarrow \frac{\partial w}{\partial x} = 0$

This gives :  $B = 0$  ;  $\text{Sin}(kl) = 0 \rightarrow k = \frac{\pi n}{l} \rightarrow \omega = \frac{\pi n c_L}{l}$  .

i.e.

**Frequency modes:** 
$$v_n = \frac{\omega}{2\pi} = n \frac{c_L}{2l} \quad ; n = 1, 2, 3, \dots \quad (5)$$

**Wave lengths:** 
$$\lambda_n = \frac{c_L}{v_n} = \frac{2l}{n} \quad ; n = 1, 2, 3, \dots \quad (6)$$

**Wave solution:** 
$$w = A \text{Cos}\left(\frac{\pi n}{l} x\right) \cdot \text{Cos}\left(\frac{\pi n c_L}{l} t - \varphi\right) \quad ; \quad c_L = \sqrt{E/\rho}$$

# Standing wave modes

## **The First Harmonic:**

Wave length:  $\lambda_1 = 2l$

Frequency:  $\nu_1 = \frac{\sqrt{E/\rho}}{2l}$

$C_L = \sqrt{E/\rho}$

Nod positions:  $l/2$

## **The Second Harmonic:**

Wave length:  $\lambda_2 = l$

Frequency:  $\nu_2 = \frac{\sqrt{E/\rho}}{l} = 2\nu_1$

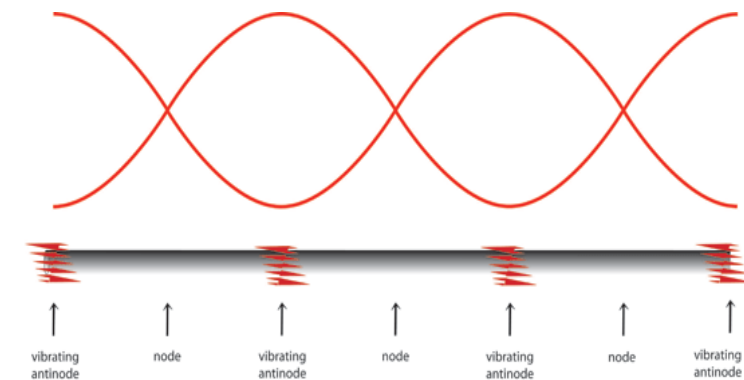
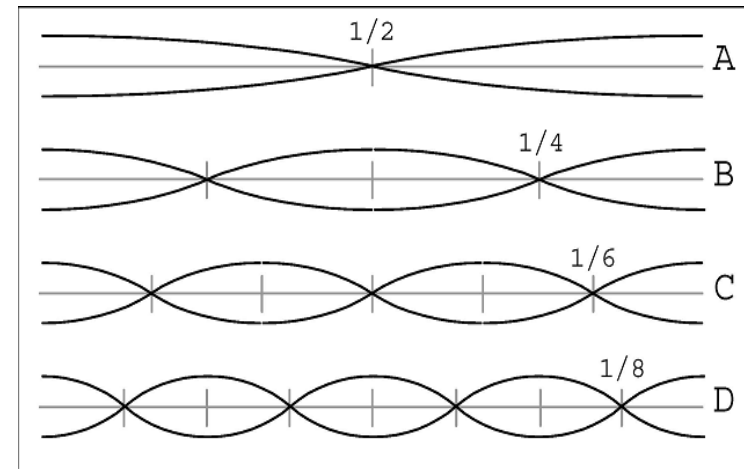
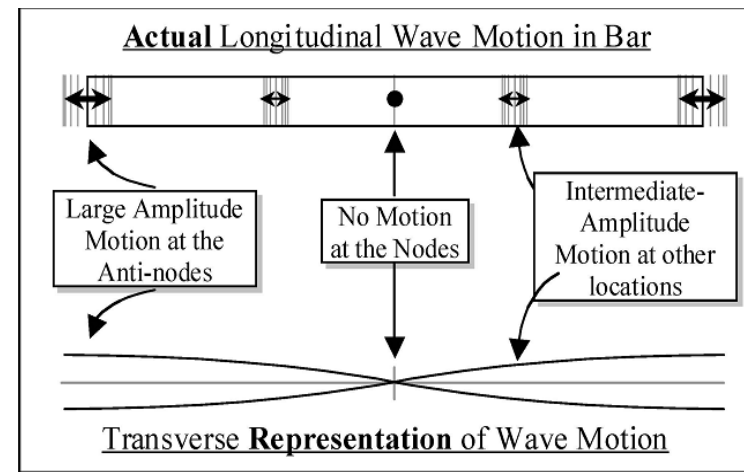
Nod positions:  $l/4 ; 3l/4$

## **The Third Harmonic:**

Wave length:  $\lambda_3 = \frac{2l}{3}$

Frequency:  $\nu_3 = \frac{3\sqrt{E/\rho}}{2l} = 3\nu_1$

Nod positions:  $l/6 ; l/2 ; 5l/6$



**The modal frequencies, wavelengths, and the locations of nodes and anti-nodes for the first nine harmonics associated with a vibrating rod of length,  $L$**

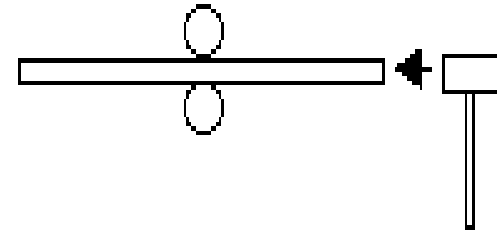
Harmonic Mode #, $n$	Frequency $f$ (Hz)	Wavelength $\lambda$ (m)	Node Locations, (m)	Anti-Node Locations, (m)
1	$f_1$	$\lambda_1 = 2L$	0	$\pm^{1/2}L$
2	$f_2 = 2f_1$	$\lambda_2 = \frac{1}{2}\lambda_1 = L$	$\pm^{1/4}L$	$0, \pm^{2/4}L = \pm^{1/2}L$
3	$f_3 = 3f_1$	$\lambda_3 = \frac{1}{3}\lambda_1 = \frac{2}{3}L$	$0, \pm^{2/6}L = \pm^{1/3}L$	$\pm^{1/6}L, \pm^{3/6}L = \pm^{1/2}L$
4	$f_4 = 4f_1$	$\lambda_4 = \frac{1}{4}\lambda_1 = \frac{1}{2}L$	$\pm^{1/8}L, \pm^{3/8}L$	$0, \pm^{2/8}L = \pm^{1/4}L, \pm^{4/8}L = \pm^{1/2}L$
5	$f_5 = 5f_1$	$\lambda_5 = \frac{1}{5}\lambda_1 = \frac{2}{5}L$	$0, \pm^{2/10}L = \pm^{1/5}L, \pm^{4/10}L = \pm^{2/5}L$	$\pm^{1/10}L, \pm^{3/10}L, \pm^{5/10}L = \pm^{1/2}L$
6	$f_6 = 6f_1$	$\lambda_6 = \frac{1}{6}\lambda_1 = \frac{1}{3}L$	$\pm^{1/12}L, \pm^{3/12}L = \pm^{1/4}L, \pm^{5/12}L$	$0, \pm^{2/12}L = \pm^{1/6}L, \pm^{4/12}L = \pm^{1/3}L, \pm^{6/12}L = \pm^{1/2}L$
7	$f_7 = 7f_1$	$\lambda_7 = \frac{1}{7}\lambda_1 = \frac{2}{7}L$	$0, \pm^{2/14}L = \pm^{1/7}L, \pm^{4/14}L = \pm^{2/7}L, \pm^{6/14}L = \pm^{3/7}L$	$\pm^{1/14}L, \pm^{3/14}L, \pm^{5/14}L, \pm^{7/14}L = \pm^{1/2}L$
8	$f_8 = 8f_1$	$\lambda_8 = \frac{1}{8}\lambda_1 = \frac{1}{4}L$	$\pm^{1/16}L, \pm^{3/16}L, \pm^{5/16}L, \pm^{7/16}L$	$0, \pm^{2/16}L = \pm^{1/8}L, \pm^{4/16}L = \pm^{1/4}L, \pm^{6/16}L = \pm^{3/8}L$
9	$f_9 = 9f_1$	$\lambda_9 = \frac{1}{9}\lambda_1 = \frac{2}{9}L$	$0, \pm^{2/18}L = \pm^{1/9}L, \pm^{4/18}L = \pm^{2/9}L, \pm^{6/18}L = \pm^{1/3}L, \pm^{8/18}L = \pm^{4/9}L$	$\pm^{1/18}L, \pm^{3/18}L, \pm^{5/18}L, \pm^{7/18}L, \pm^{9/18}L = \pm^{1/2}L$

All modes ( $n = 0, 1, 2, 3, 4, 5, \dots$ ) of vibration of the rod all have the same longitudinal speed of propagation of sound in the rod  $C_L = \sqrt{E/\rho} = v_i \lambda_i$

# Experiments with Longitudinal Waves

To make rod sound **clearer and louder**, we shall:

3. Hit the rod **as fast as possible**
4. Hit it **not very hard**, because harmonicity of waves will be violated.
5. Hit the rod with thing, that **doesn't produce good sound** (e.g. Ebonite Rod), not to interrupt Main one.
6. Try to hit it **vertically or horizontally**, and **not intermediate**.



- Also **sliding of hand** down the length of the rod.
- Addition of **Rosin** makes **fingers more sticky**.
- The **pitch** of the sound can be **varied** by changing **holding places of the Rod** or by changing the **length of the rod** itself.

# Ringling The Rod



Experiment

# Experiments with Longitudinal Waves

1. **Aluminium rod** ( $\rho_{AL} = 2,7 \cdot 10^3 \text{ kg/m}^3$  ;  $E_{AL} = 70 \times 10^9 \text{ N/m}^2$  ;  $C_{L(AL)} = 5082.4 \text{ m/s}$  )

Length of the rod 1:  $l = 1,2 \text{ m}$   $v_1 = \frac{\sqrt{E/\rho}}{2l}$   $C_L = \sqrt{E/\rho}$   $\lambda_n = \frac{2l}{n}$   $v_n = n \frac{C_L}{2l}$

The first harmonic:  $\lambda_1 = 2.4 \text{ m}$  ;  $v_1 = 2117 \text{ Hz}$  ; nodes:  $0.6 \text{ m}$

The second harmonic:  $\lambda_2 = 1.2 \text{ m}$  ;  $v_2 = 4234 \text{ Hz}$  ; nodes:  $0.3 \text{ m}$  ,  $0.9 \text{ m}$

The third harmonic:  $\lambda_3 = 0.8 \text{ m}$  ;  $v_3 = 6351 \text{ Hz}$  ; nodes:  $0.2 \text{ m}$  ,  $0.6 \text{ m}$  ,  $1.0 \text{ m}$

If hold the rod in the place where nodes of several modes are placed all these modes will occur.

Touching the rod at the ends will stop the sound .

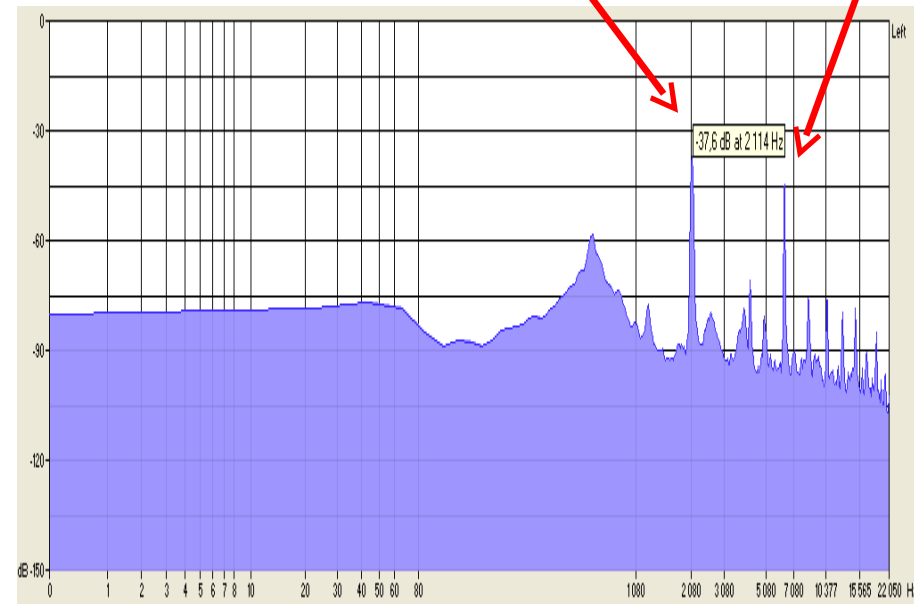
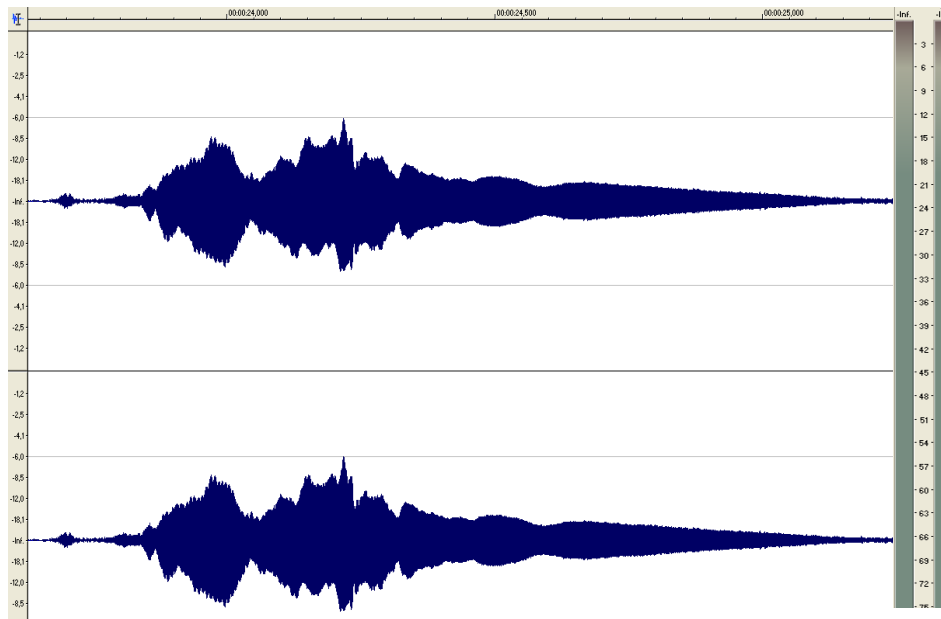


????? ??????????

holding in center

$v_1$

$v_3$



# Experiments with Longitudinal Waves

2. **Aluminium rod** ( $\rho_{AL} = 2,7 \cdot 10^3 \text{ kg/m}^3$  ;  $E_{AL} = 70 \times 10^9 \text{ N/m}^2$  ;  $C_{L(AL)} = 5082.4 \text{ m/s}$  )

Length of the rod 2:  $l = 0.75 \text{ m}$   $v_1 = \frac{\sqrt{E/\rho}}{2l}$   $C_L = \sqrt{E/\rho}$   $\lambda_n = \frac{2l}{n}$   $v_n = n \frac{C_L}{2l}$

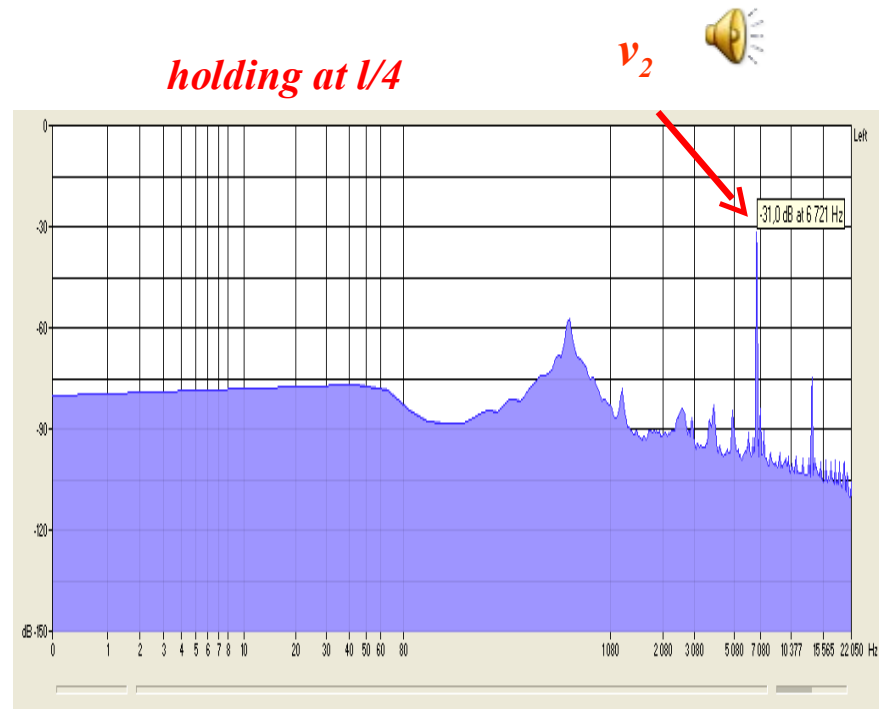
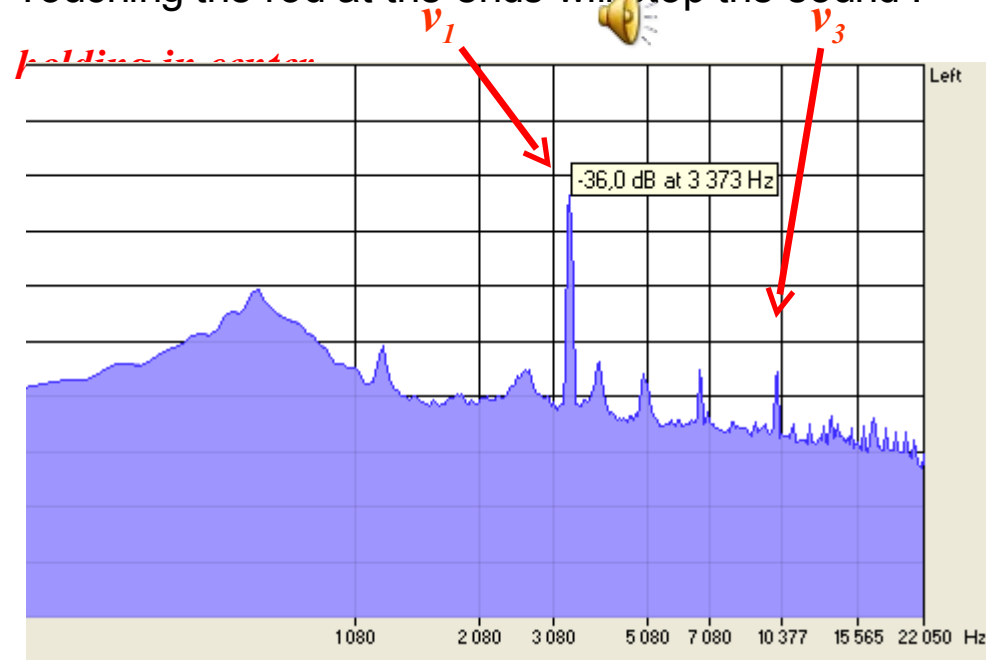
The first harmonic:  $\lambda_1 = 1.5 \text{ m}$  ;  $v_1 = 3388 \text{ Hz}$  ; nodes:  $0.37 \text{ m}$

The second harmonic:  $\lambda_2 = 0.75 \text{ m}$  ;  $v_2 = 6776 \text{ Hz}$  ; nodes:  $0.19 \text{ m}$  ,  $0.56 \text{ m}$

The third harmonic:  $\lambda_3 = 0.5 \text{ m}$  ;  $v_3 = 10\ 164 \text{ Hz}$  ; nodes:  $0.12 \text{ m}$  ,  $0.37 \text{ m}$  ,  $0.63 \text{ m}$

If hold the rod in the place where nodes of several modes are placed all these modes will occur.

Touching the rod at the ends will stop the sound .



# Experiments with Longitudinal Waves

3. **BRASS rod** ( $\rho_{Br} = 8,5 \cdot 10^3 \text{ kg/m}^3$  ;  $E_{Br} = 95 \times 10^9 \text{ N/m}^2$  ;  $C_{L(Br)} = 3480 \text{ m/s}$  )

Length of the rod 3:  $l = 0.4 \text{ m}$     $v_1 = \frac{\sqrt{E/\rho}}{2l}$     $C_L = \sqrt{E/\rho}$     $\lambda_n = \frac{2l}{n}$     $v_n = n \frac{C_L}{2l}$

The first harmonic:  $\lambda_1 = 0.8 \text{ m}$  ;  $v_1 = 4350 \text{ Hz}$  ; nodes:  $0.2 \text{ m}$

The second harmonic:  $\lambda_2 = 0.4 \text{ m}$  ;  $v_2 = 8700 \text{ Hz}$  ; nodes:  $0.1 \text{ m}$  ,  $0.3 \text{ m}$

The third harmonic:  $\lambda_3 = 0.27 \text{ m}$  ;  $v_3 = 13\ 050 \text{ Hz}$  ; nodes:  $0.07 \text{ m}$  ,  $0.2 \text{ m}$  ,  $0.33 \text{ m}$

If hold the rod in the place where nodes of several modes are placed all these modes will occur.

Touching the rod at the ends will stop the sound .

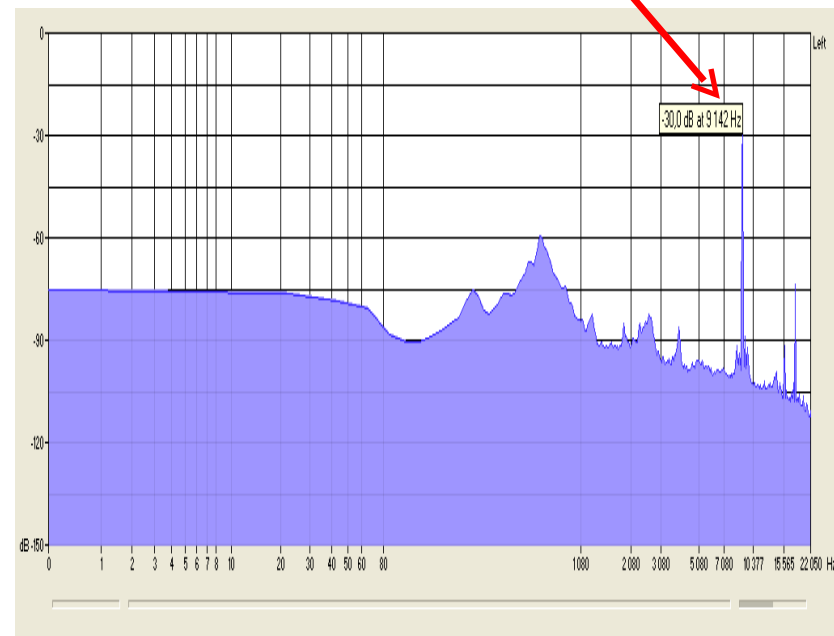
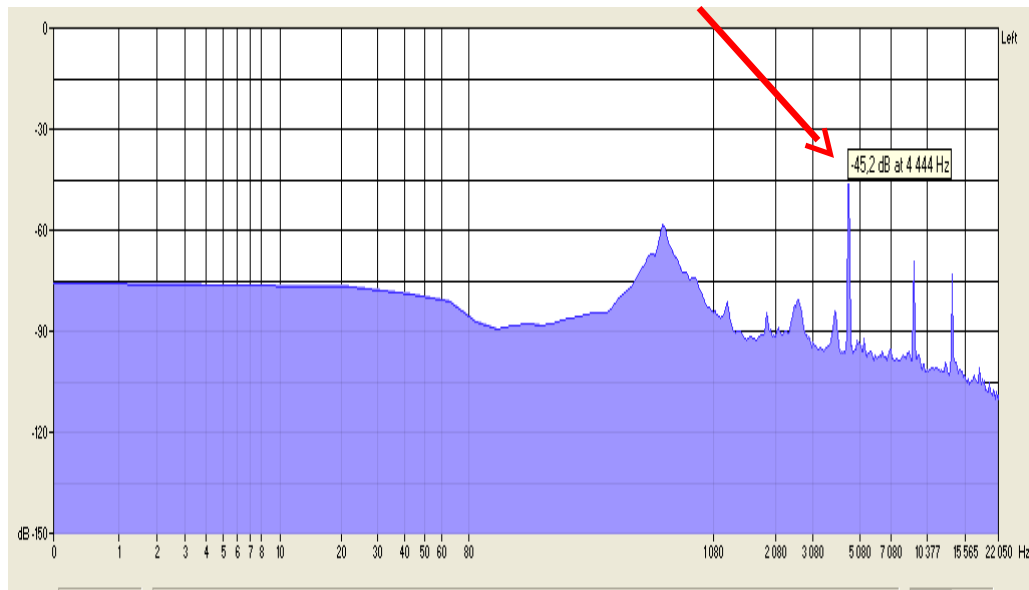
*holding in center*

$v_1$



*holding at 1/4*

$v_2$



# Experiments with Longitudinal Waves

4. **STEEL rod** ( $\rho_{St} = 7,8 \cdot 10^3 \text{ kg/m}^3$  ;  $E_{St} = 200 \times 10^9 \text{ N/m}^2$  ;  $C_{L(St)} = 5150 \text{ m/s}$  )

Length of the **rod 4**:  $l = 0.6 \text{ m}$      $v_1 = \frac{\sqrt{E/\rho}}{2l}$      $C_L = \sqrt{E/\rho}$      $\lambda_n = \frac{2l}{n}$      $v_n = n \frac{C_L}{2l}$

The first harmonic:  $\lambda_1 = 1.2 \text{ m}$  ;  $v_1 = 4291 \text{ Hz}$  ; **nodes: 0.3 m**

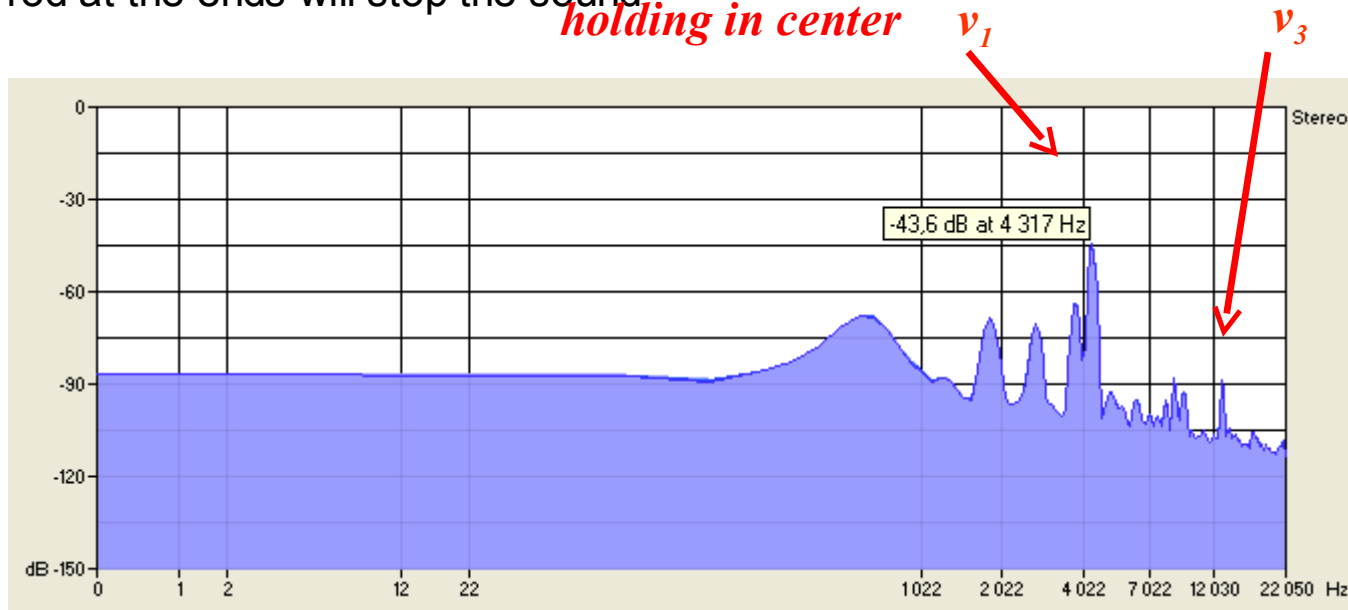
The second harmonic:  $\lambda_2 = 0.6 \text{ m}$  ;  $v_2 = 8582 \text{ Hz}$  ; **nodes: 0.15 m , 0.45 m**

The third harmonic:  $\lambda_3 = 0.4 \text{ m}$  ;  $v_3 = 12\ 873 \text{ Hz}$  ; **nodes: 0.1 m , 0.3 m , 0.5 m**

If hold the rod in the place where nodes of several modes are placed all these modes will occur.

Touching the rod at the ends will stop the sound

*holding in center*



# BENDING Waves

## Wave equation

Euler-Bernoulli beam theory equation of motion

$$\frac{\partial^2 y}{\partial t^2} = - \frac{EK^2}{\rho} \cdot \frac{\partial^4 y}{\partial x^4}$$

Where  $y$  is displacement normal to rod axis.

"Radius of Gyration": 
$$K^2 = \frac{1}{S} \int z^2 dS$$

$z$  is distance from central axis of rod.

Here shear deformations and rotary inertia are neglected.

Harmonic solutions are of the following form:

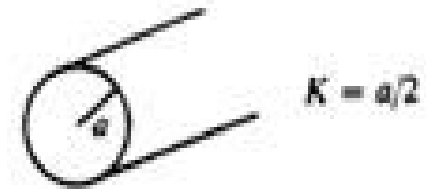
$$y(x, t) \equiv Y(x)e^{i\omega t}$$

$$Y(x) = \cos(\omega t + \varphi) [A \cosh(kx) + B \sinh(kx) + C \cos(kx) + D \sin(kx)]$$

$k = \omega/v$  is the wave (propagation) number.

Velocity is dependent on frequency. So here is **dispersion**.

$$v = \sqrt{2\pi\nu Kc_L}$$



$$K = \frac{\sqrt{(\text{inner radius})^2 + (\text{outer radius})^2}}{2}$$

# Frequency Modes of Standing Bending Waves



Frequency modes depend on the end conditions

For our task we consider FREE end - no torque and no shearing force

They give restrictions on standing wave frequencies:

$$\frac{\omega l}{2v} = \frac{\pi}{4} \cdot [3.011 ; 5 ; 7 ; \dots ; (2n + 1) \dots]$$

$$v = \sqrt{2\pi\nu K c_L}$$

$$\nu_n = \frac{\omega_n}{2\pi} = \frac{\pi K}{8l^2} \sqrt{\frac{E}{\rho}} \cdot [3.011^2 ; 5^2 ; 7^2 ; \dots ; (2n + 1)^2 \dots]$$

And wave lengths:

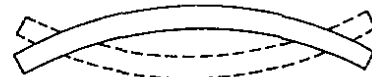
$$\lambda_n = \frac{4l}{[3.011 ; 5 ; 7 ; \dots ; (2n + 1) \dots]}$$

Table I. Characteristics of transverse vibrations in a bar with free ends.

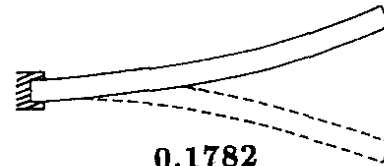
Frequency (Hz)	Wavelength (m)	Nodal positions (m from end of 1-m bar)
$f_1 = 3.5607 (K/L^2)\sqrt{E/\rho}$	1.330L	0.224, 0.776
$2.756 f_1$	0.800L	0.132, 0.500, 0.868
$5.404 f_1$	0.572L	0.094, 0.356, 0.644, 0.906
$8.933 f_1$	0.445L	0.073, 0.277, 0.500, 0.723, 0.927

To obtain actual frequencies multiply

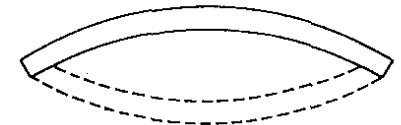
$$\frac{\pi K}{l^2} \sqrt{\frac{E}{\rho}}$$



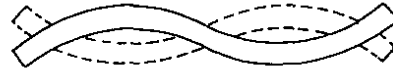
1.133



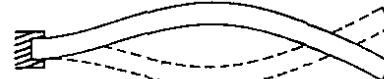
0.1782



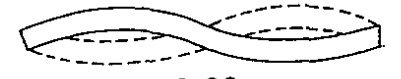
0.50



3.125



1.116



2.00



6.125



3.125



4.50



10.125



6.125



8.00

(a)

(b)

(c)

**Note:** Frequencies for **bending waves** are sufficiently **lower than for the the longitudinal waves** due to extra coefficient  $\frac{K}{l}$ .

Higher modes are vanishing very rapidly.

**Waves are damped for supported or clamped ends**

# Experiments with BENDING Waves

1. **Aluminium rod** ( $\rho_{AL} = 2,7 \cdot 10^3 \text{ kg/m}^3$  ;  $E_{AL} = 70 \times 10^9 \text{ N/m}^2$  ;  $C_{L(AL)} = 5082.4 \text{ m/s}$  )

**FREE Ends**

$$v_n = \sqrt{2\pi v_n K C_L} \quad ; \quad K = \frac{1}{2} \sqrt{a^2 + b^2} \quad ; \quad v_n = \frac{\pi K}{8l^2} \sqrt{\frac{E}{\rho}} \cdot [3.011^2; 5^2; 7^2; \dots; (2n+1)^2 \dots]$$

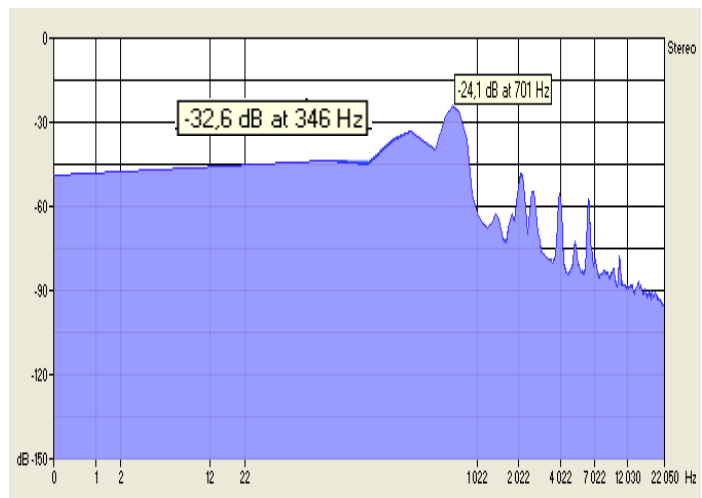
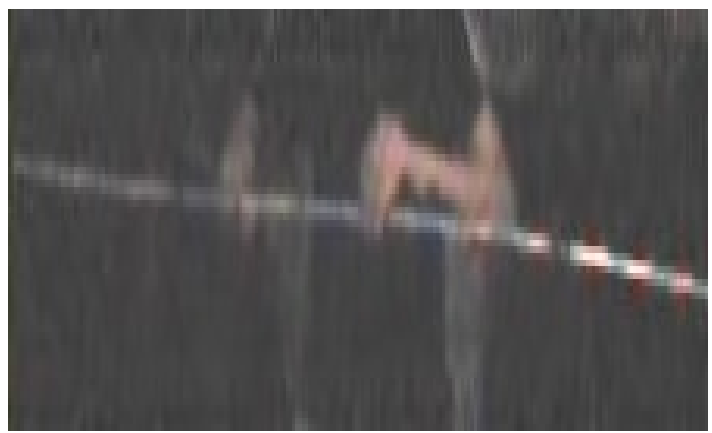
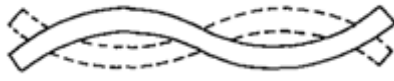
$$\lambda_n = \frac{4l}{[3.011; 5; 7; \dots; (2n+1) \dots]}$$

Length of the **rod 1**:  $l = 1,2\text{m}$  ; Radii:  $a = 0.005\text{m}$ ,  $b = 0.004\text{m}$  ;  $K = 0.0032\text{m}$

The first harmonic:  $\lambda_{B1} = 1,596\text{m}$  ;  $v_1 = 40 \text{ Hz}$  ; nodes:  $0.27\text{m}$  ,  $0,93\text{m}$

The second harmonic:  $\lambda_{B2} = 0,96\text{m}$  ;  $v_2 = 110 \text{ Hz}$  ; nodes:  $0.16\text{m}$  ,  $0.6\text{m}$  ,  $1.04\text{m}$

The third harmonic:  $\lambda_{B3} = 0.69\text{m}$  ;  $v_3 = 215 \text{ Hz}$  ; nodes:  $0.11\text{m}$ ,  $0.43\text{m}$  ,  $0.77\text{m}$



*holding in center*



# Experiments with BENDING Waves

4. **STEEL rod** ( $\rho_{St} = 7,8 \cdot 10^3 \text{ kg/m}^3$  ;  $E_{St} = 200 \times 10^9 \text{ N/m}^2$  ;  $C_{L(St)} = 5150 \text{ m/s}$  )

**FREE Ends**

$$v_n = \sqrt{2\pi v_n K c_L} \quad ; \quad K = a/2 \quad ; \quad v_n = \frac{\pi K}{8l^2} \sqrt{\frac{E}{\rho}} \cdot [3.011^2; 5^2; 7^2; \dots; (2n+1)^2 \dots]$$

$$\lambda_n = \frac{4l}{[3.011; 5; 7; \dots; (2n+1) \dots]}$$

Length of the **rod 4**:  $l = 0.6 \text{ m}$  ; **Radius**  $a = 0.007 \text{ m}$  ;  $K = 0.0035 \text{ m}$

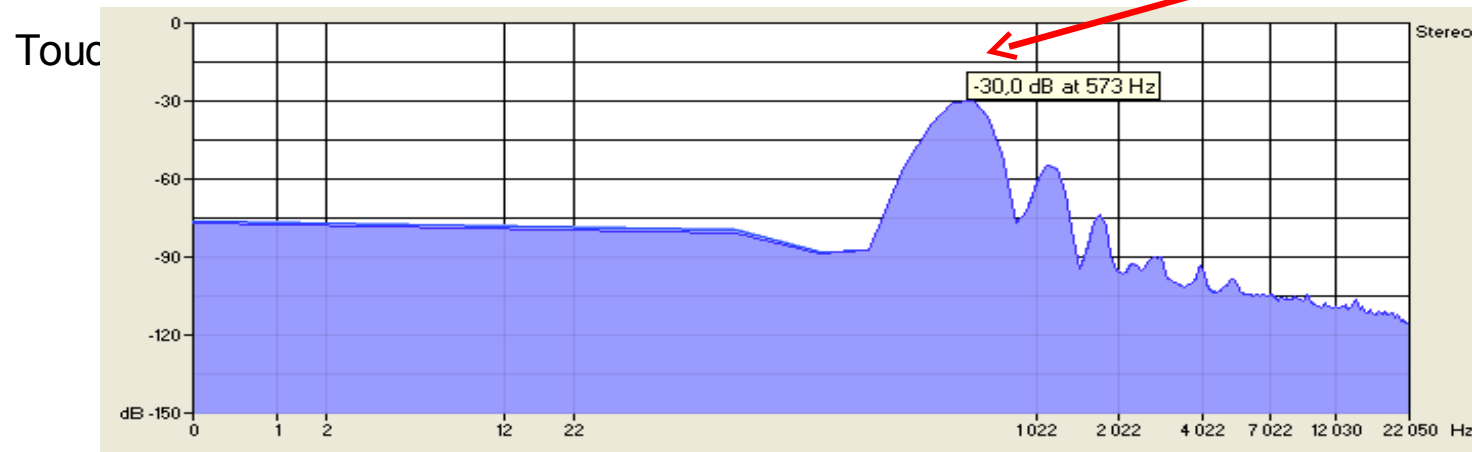
The first harmonic:  $\lambda_{B1} = 0.8 \text{ m}$  ;  $v_1 = 190 \text{ Hz}$  ; **nodes**:  $0.13 \text{ m}$  ,  $0.47 \text{ m}$

The second harmonic:  $\lambda_{B2} = 0.48 \text{ m}$  ;  $v_2 = 525 \text{ Hz}$  ; **nodes**:  $0.08 \text{ m}$  ,  $0.3 \text{ m}$  ,  $0.52 \text{ m}$

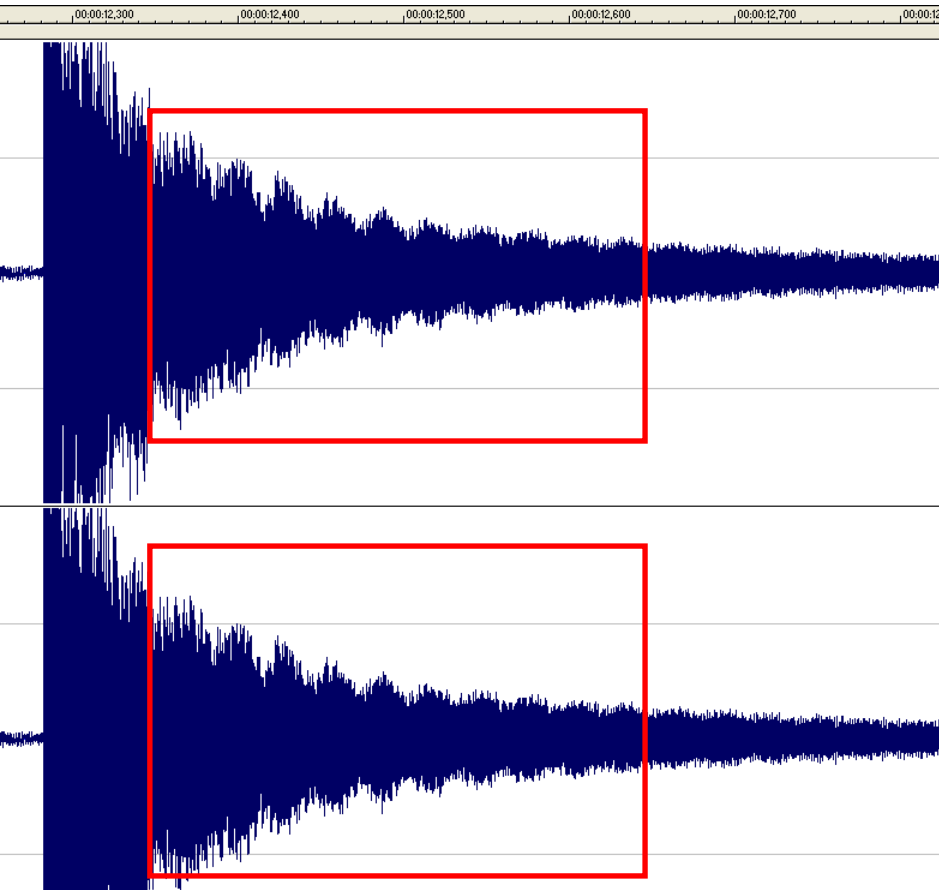
The third harmonic:  $\lambda_{B3} = 0.34 \text{ m}$  ;  $v_3 = 1029 \text{ Hz}$  ; **nodes**:  $0.056 \text{ m}$  ,  $0.21 \text{ m}$  ,  $0.39 \text{ m}$  ,  $0.544 \text{ m}$

If hold the rod in the place where nodes of several modes are placed ~~at~~ all these modes will occur.

**holding at 0.3m**



# Beats



Sound in Experiments sometimes became stronger, sometimes weaker.

This was because of Beats.

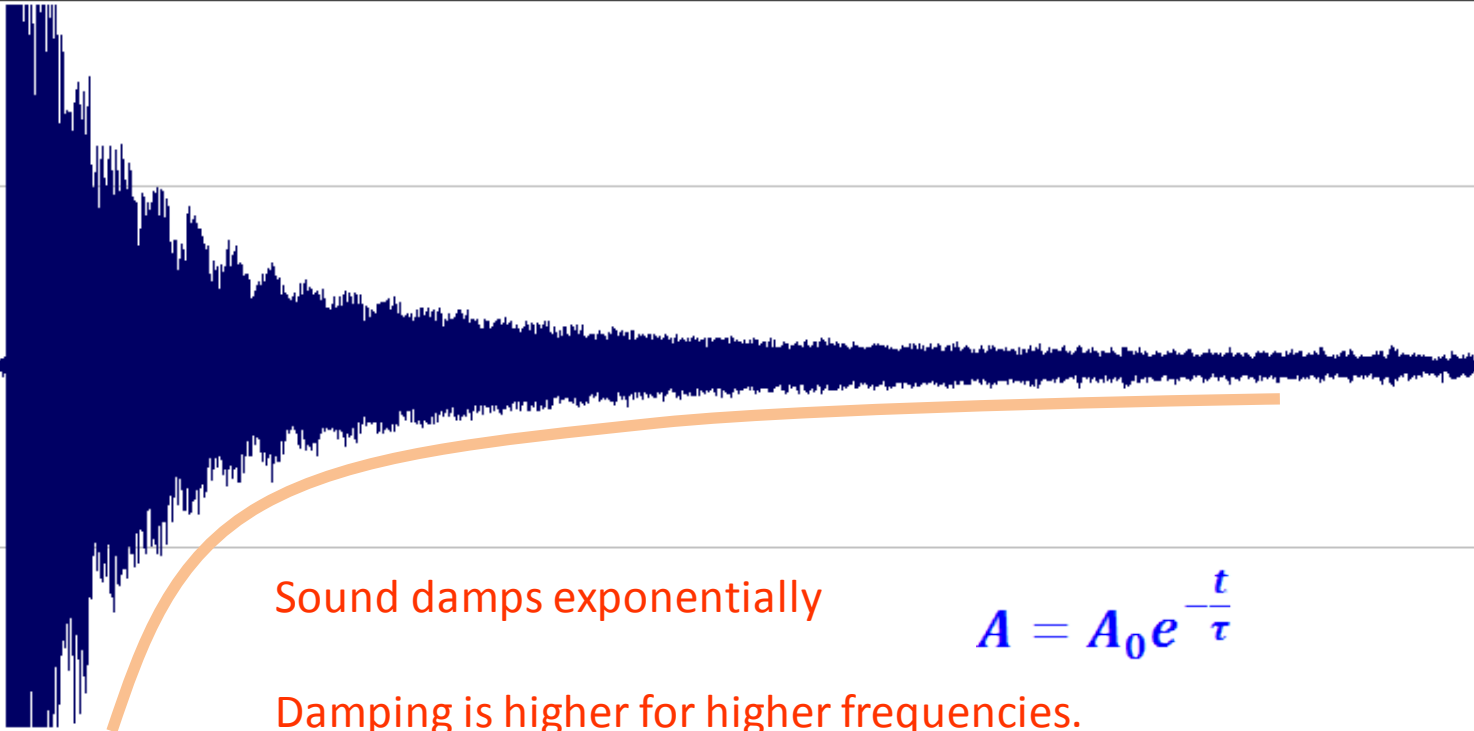
A **beat** is an interference between two sounds of slightly different frequencies.

Our experimental Beats

# Sound Damping

00:00:12,500

00:00:13,000



Sound damps exponentially

$$A = A_0 e^{-\frac{t}{\tau}}$$

Damping is higher for higher frequencies.

We saw it comparing damping for transverse and longitudinal waves.

- Internal Damping

The "decay time"

$$\tau_{int} \sim \frac{1}{\pi v}$$

- Air damping.

The "decay time"

$$\tau_a \sim \rho r / \sqrt{v}$$

- Transfer of energy to other systems (e.g. supports)

$$\tau_{sup} \sim \frac{1}{lGv^2} \quad (G- \text{"energy conductance"})$$

# Conclusion

- There are different types of waves in the rod
- The type of wave depends on how do we hit the rod.
- The frequency of standing wave depends on where we hold the rod.
- The frequencies of bending waves are lower than of compression waves
- Damping of the waves depends on frequency

**Thank you for your attention!**