№12. LEVITATING SPINNER

GEORGIA

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A toy (called "Levitron") consists of a magnetic spinning top and a plate containing magnets The top may levitate above the magnetic plate.

Under what conditions can one observe the phenomenon?

Presentation Plan

1. Description of LEVITRON

- Levitron components;
- Experiment; Description of Levitron action (Phenomenology);
- Dependence on environment .

2. Physics of Levitron

- Qualitative analysis;
- Quantitative analysis; Basic assumptions;
- System Energy analysis - Equilibrium condition , Stability condition;
- Top's spin upper limit.

3. Equilibrium area

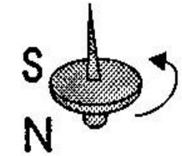
- Theoretical calculation of base magnetic field;
- Measured field;
- Area of equilibrium.

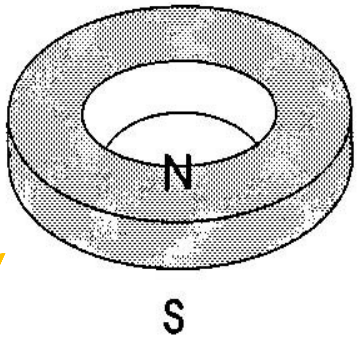
4. Conclusion

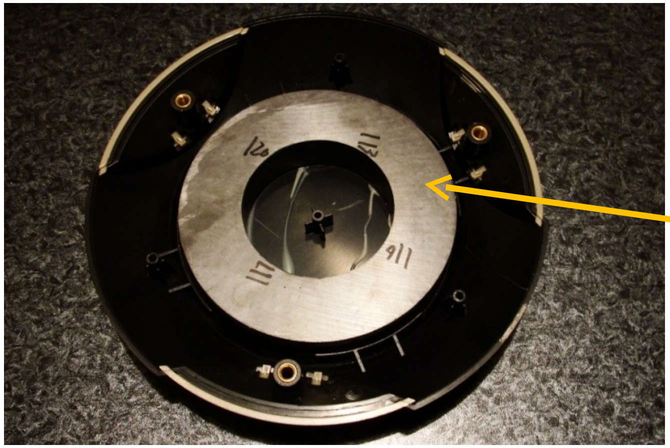
Description of Levitron

1. **Spinner (Top)** – nonmagnetic spindle inserted in a flat, toroidal, permanent magnet (Ceramic)
2. **Base** – relatively large toroidal ceramic permanent magnet.
3. **Plastic lifter plate**

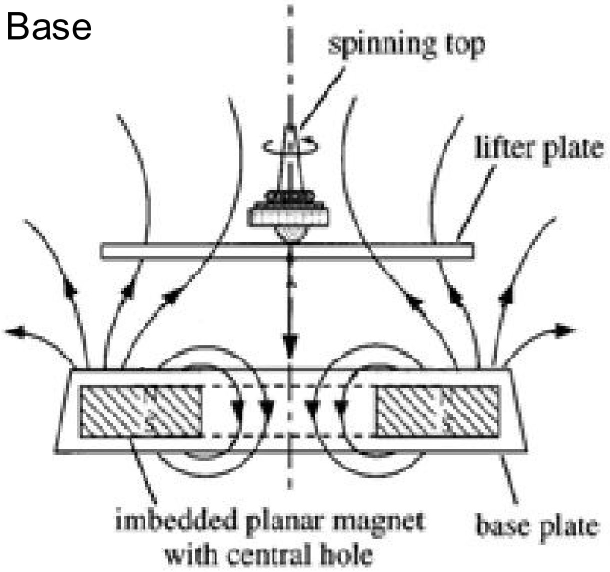
spinning magnetic top







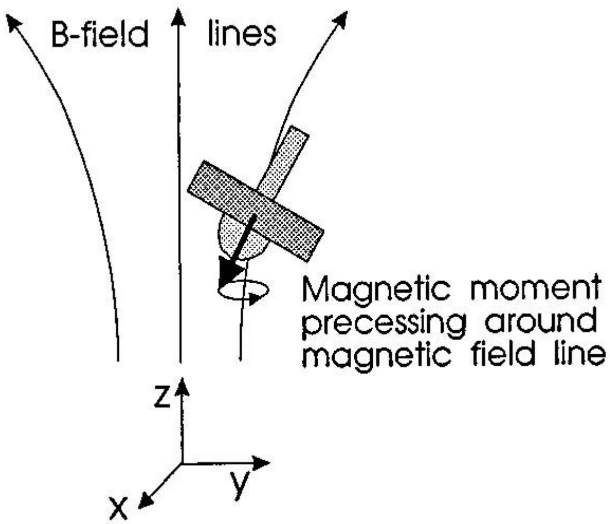
Magnetic Base

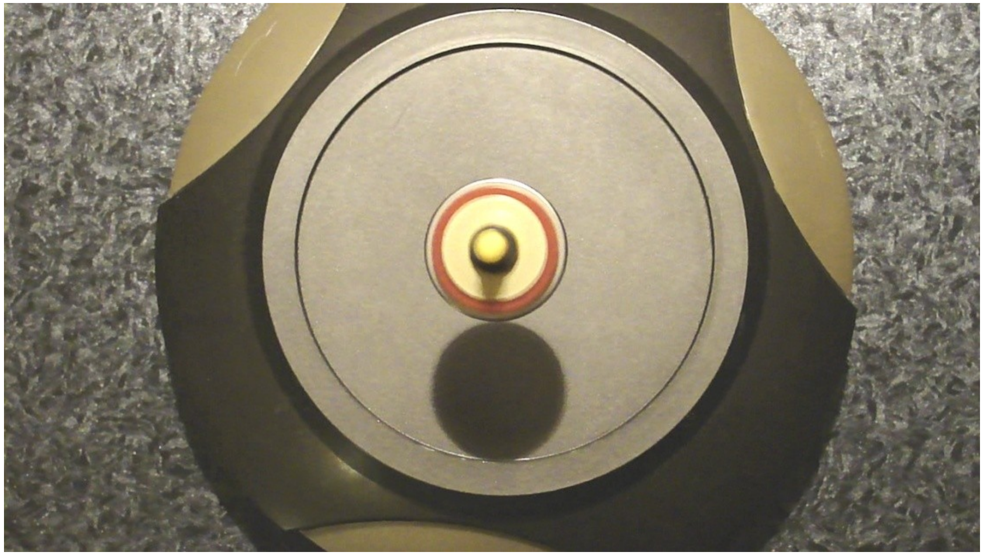


Between Base and Top acts **dipole-dipole repulsion** force

Magnetic Top levitates above the Base due to **gradient** of base magnetic field.

Description of Levitron action (Phenomenology)





Video ; video from above ;
Slow Motion

- Top rotation speed: ($\Omega_{spin} \cong 35Hz$)
- Top precession speed : ($\Omega_{precession} \cong 5Hz$)
- Top "oscillation" frequency: ($\Omega_{lateral} \cong 1Hz$)

$$\Omega_{spin} \gg \Omega_{precession} \gg \Omega_{lateral}$$

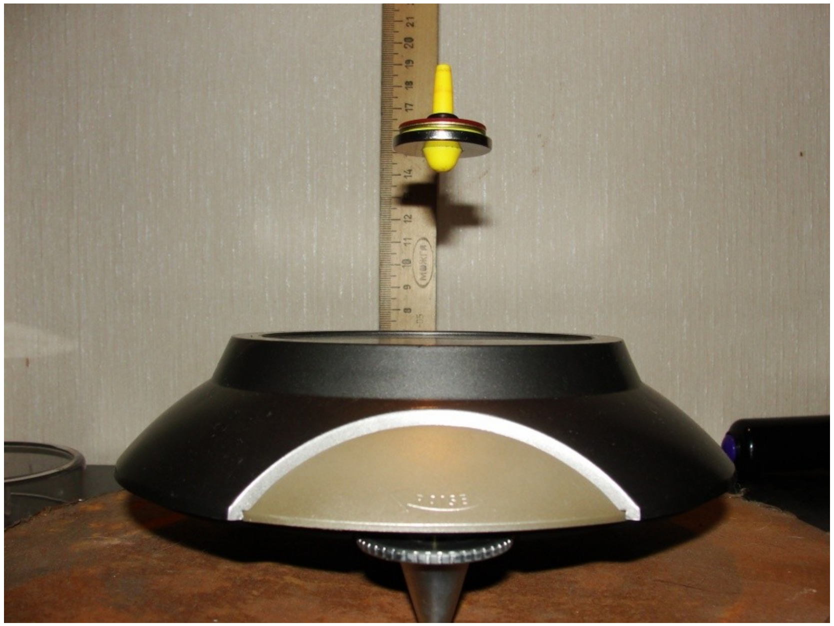
- Spin precesses around the local direction of the field and Magnetic moment of Top effectively on the average points antiparallel to the local magnetic field lines.
- Base magnetic field lines are not vertically directed. Their direction is varying.

The magnetic field lines

Sensitivity to environment

Top Levitation is influenced by:

- **External magnets or magnetized bodies**, video



- **Iron bodies;**

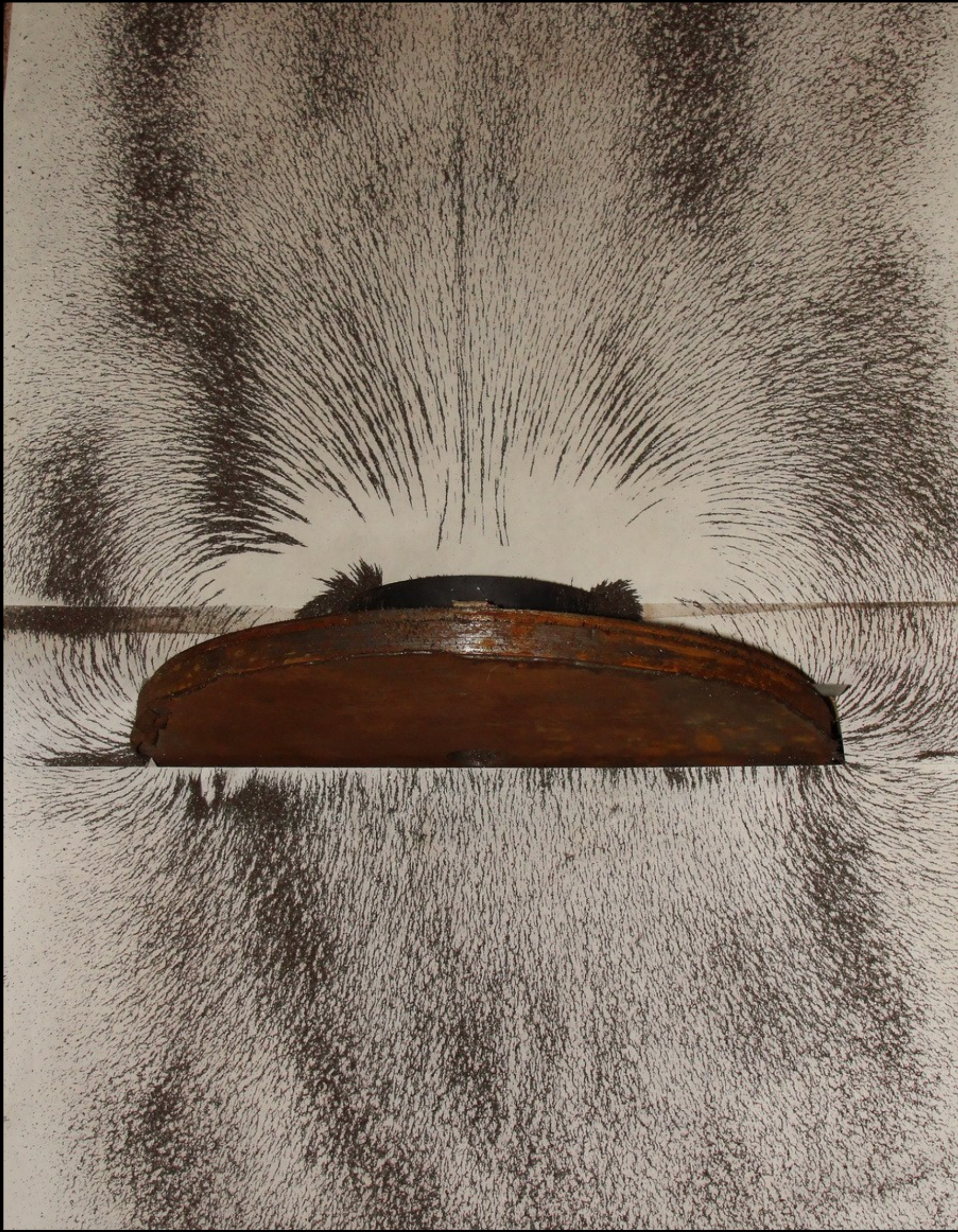
Placing the base on **thick iron plate** changes the shape and value of magnetic field of base.

Equilibrium point is lower while top's mass must be larger.

- **Temperature fluctuations**

One needs to change top's mass when the temperature of environment changes.

The magnetic field lines

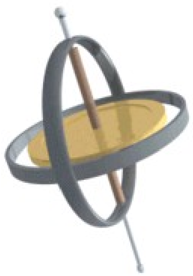


Physics of Levitron

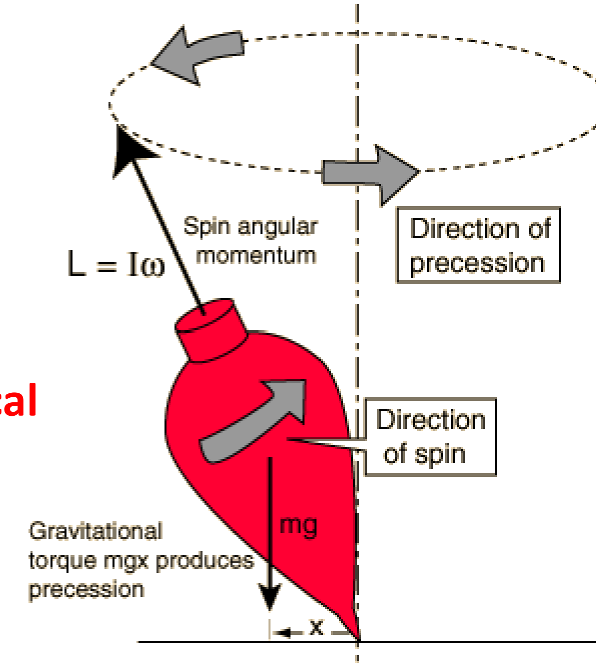
Qualitative analysis

For the stable levitation two conditions must be satisfied:

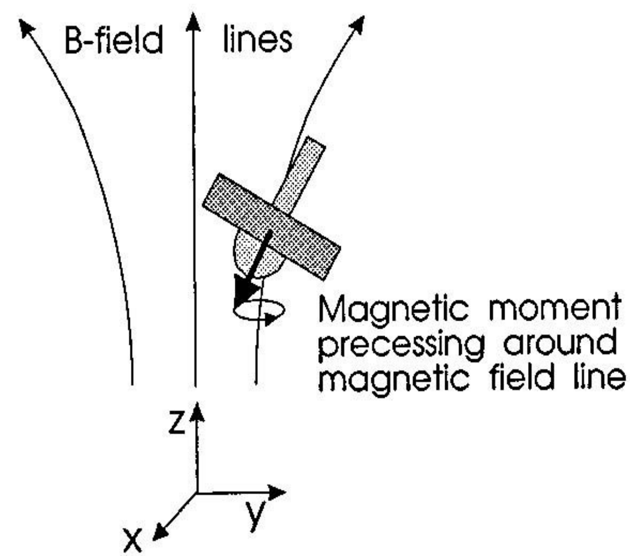
a) Top's weight must be <u>balanced</u> by magnetic Repulsion force
b) In the levitation point potential <u>energy</u> must have <u>minimum</u>.



<u>Ordinary</u> spinner precesses around **vertical gravitational** field lines.



<u>Magnetic</u> top precesses around **local magnetic** field lines (which **in different points have different directions**).



Due to gyroscopic effect top's precession axis always tries to be aligned on average to the direction of local magnetic field line.

At radial excursion top's precession axis tries to <u>"reorient"</u> to magnetic field local line direction.

<u>Reorientation energy</u> of precession axis creates the <u>potential well</u>.

Quantitative analysis; Basic Assumptions

For our further investigation let us make such **assumptions**:

a)

In the magnetic field of base there are no currents and magnetic charges, i.e. Maxwell equations take the form:

- **Gauss law** for magnetic field: $\vec{\nabla} \cdot \vec{B} = 0$ – magnetic charge does not exist;
- **Amper's law**: $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \cdot \frac{\partial \vec{E}}{\partial t}$ - electrical current and electric field variation create magnetic field. Since on our case there are no electric currents and electric field

$$\vec{\nabla} \times \vec{B} = 0$$

b)

- Top is **Magnetic Dipole and its magnetic dipole centre** coincides with **Mass centre** ;
- Top is **„Fast“, precession angle is small -** the **angular momentum is along the spin axis** of the top which **also coincides with the magnetic moment axis**;
- Top's precession frequency is much larger than lateral oscillations frequency

$$\Omega_{precession} \gg \Omega_{lateral}$$

- Top's magnetic moment $\vec{\mathfrak{M}}$ has constant value

$$\mathfrak{M} = \left|\vec{\mathfrak{M}}\right| = const$$

Quantitative analysis; Potential Energy of Magnetic Spinner

The force exerted by the magnetic field $\vec{B}$ on a dipole of moment $\vec{\mathfrak{M}}$ in air or vacuum depends on the *directed gradient* of the field (Сивухин. т.3, стр.243):

$$\overrightarrow{F_{\mathfrak{M}}} = (\vec{\mathfrak{M}} \cdot \vec{\nabla}) \cdot \vec{B} \qquad (1)$$

The potential energy of the top (since in average $\vec{\mathfrak{M}} \parallel \vec{B}$)

$$U = -(\vec{\mathfrak{M}} \cdot \vec{B}) + mgz \approx \mathfrak{M}B + mgz \qquad \textbf{(2)}$$

Representing $\vec{B}$ as a **Taylor series** in the vicinity of levitating point and taking into account Maxwell equations we get:

$$B_z = B_0\left[1 + \alpha_1 z + \alpha_2\left(z^2 - \frac{1}{2}r^2\right) + O((r,z)^3)\right]$$

$$B_r = B_0\left[-\alpha_1\frac{r}{2} - \alpha_2 zr + O((r,z)^3)\right] \qquad (3)$$

Where

$$\alpha_1(z_0) \equiv \left[\frac{1}{B_z}\frac{\partial B_z}{\partial z}\right]_{z=z_0=0;\, r=0} \quad ; \quad \alpha_2(z_0) \equiv \frac{1}{2}\left[\frac{1}{B_z}\frac{\partial^2 B_z}{\partial z^2}\right]_{z=z_0=0;\, r=0}$$

In this approximation the potential energy of top is: [1,2]

$$U \approx \left[B_0 + \left\{\frac{mg}{\mathfrak{M}} + B_0\alpha_1\right\}z + B_0\alpha_2 z^2 + \frac{1}{2}B_0\alpha_2\left\{\frac{1}{4}\frac{\alpha_1^2}{\alpha_2} - 1\right\}r^2 + \ \ldots\right] \qquad \textbf{(4)}$$

Let us investigate this expression:

$$U \approx \left[B_0 + \left\{\frac{mg}{\mathfrak{M}} + B_0\alpha_1\right\} z + B_0\alpha_2 z^2 + \frac{1}{2} B_0 \alpha_2 \left\{\frac{1}{4}\frac{\alpha_1^2}{\alpha_2} - 1\right\} r^2 + \ \dots\right]$$

(4)

a) In the levitating point must be energy extremum: $\frac{\partial U}{\partial z} = 0$.

Gradient of base magnetic field balances gravitational force (Equilibrium condition):

$$\alpha_1 = \left[\frac{1}{B_0}\frac{\partial B_z}{\partial z}\right]_{z=z_0=0;\ r=0} = -\frac{mg}{\mathfrak{M}B_0}$$

(5a)

b) For stable equilibrium **energy** must have **minimum**: $\frac{\partial^2 U}{\partial z^2} > 0\ ;\ \frac{\partial^2 U}{\partial r^2} > 0$.

i.e. coefficients of z^2 and r^2 must be positive $\alpha_2 > 0$ (Stability condition):

$$\frac{1}{4}\frac{\alpha_1^2}{\alpha_2} - 1 > 0 \quad or \quad \alpha_1^2 > 4\alpha_2$$

(5b)

(5c) $\frac{1}{4}\frac{\alpha_1^2}{\alpha_2}$

Top Spin Frequency Upper Limit

Trapping conditions set a restriction on the top spin frequency upper limit: [1]

$$\omega \leq \frac{1}{r_{eff}^2 \cdot g}\left(\frac{\mathfrak{M}}{m} \cdot B_0\right)^{3/2}$$

The top spin frequency must be less than this value, to have the levitation.

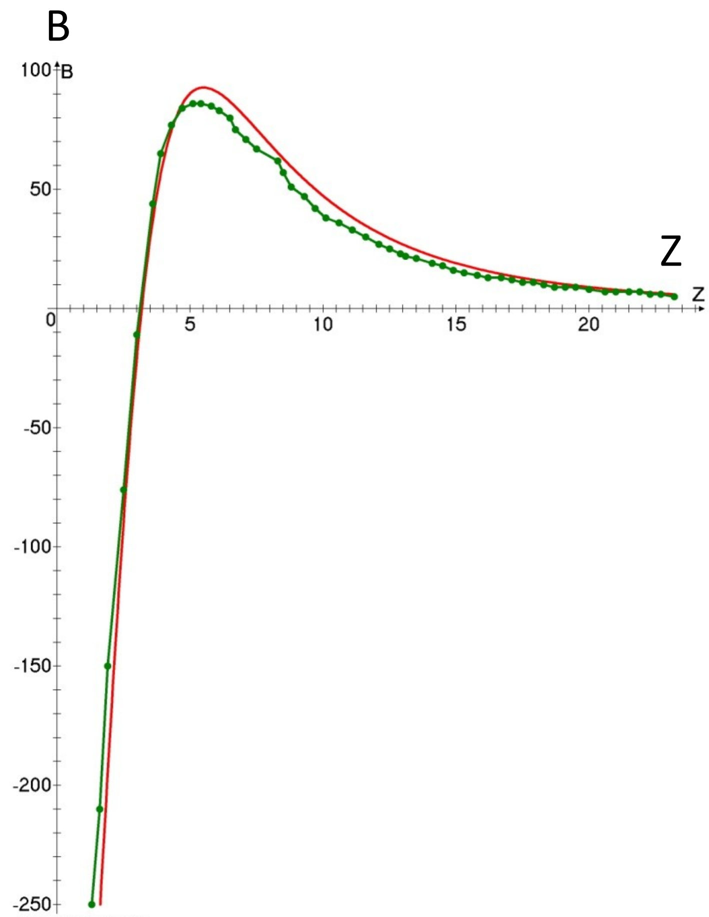
If the **top** is "**too fast"**, the **precession** frequency will be **too slow** to allow the top to reorient to the local field direction as the top makes its radial excursion.

Equilibrium area

Assume base as a consequence of vertically oriented magnetic dipoles stacked as a horizontal ring.

Similarly to electrostatic problem the magnetic field along the ring axis will be: [2]

$$B_z = \frac{\mu_0 \mathfrak{M}_{ring}}{4\pi R^3} \cdot \left\{ \frac{2\left(\frac{z}{R}\right)^2 - 1}{\left[\left(\frac{z}{R}\right)^2 + 1\right]^{5/2}} \right\} \qquad (8)$$



$\mathfrak{M}_{ring}$ – ring dipole moment

R – Radius of base magnet (4,5 cm)

z – Distance from the base along axis.

When $z \simeq 0$ $z = R/\sqrt{2}$, $B_z < 0$;

When $z = R\sqrt{3/2}$, $B_z = 0$;

When , $B_z = max$;

When $z \Rightarrow \infty$, $B_z \Rightarrow 0$

Here B_z is given in ($\times 10^{-4}$ Tesla);

z – in cm;

Magnetic constant $\mu_0 = 1{,}257 \cdot 10^{-6}$ V·Sec/(A·m);

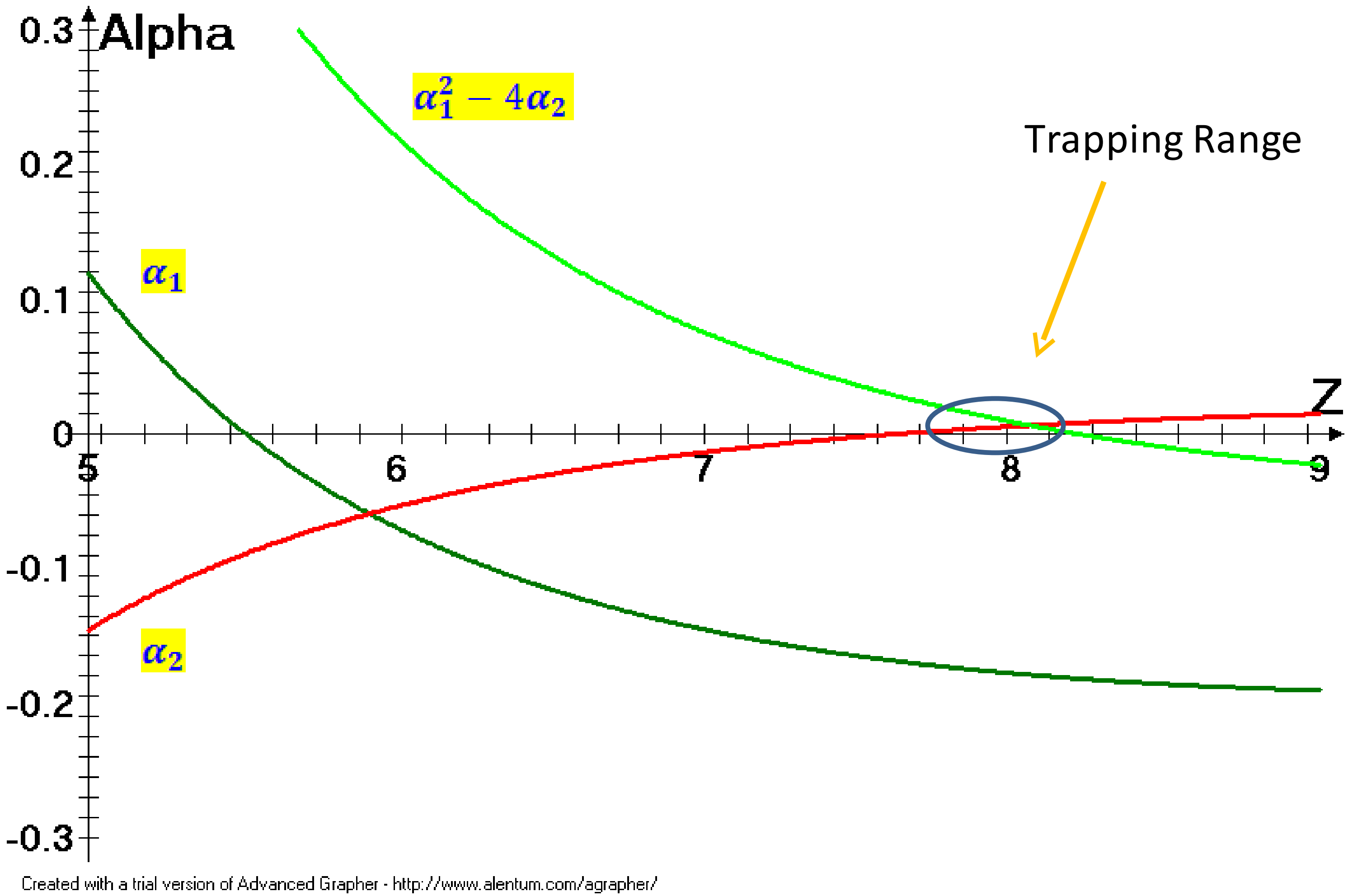
Comparing (8) with measurements we get

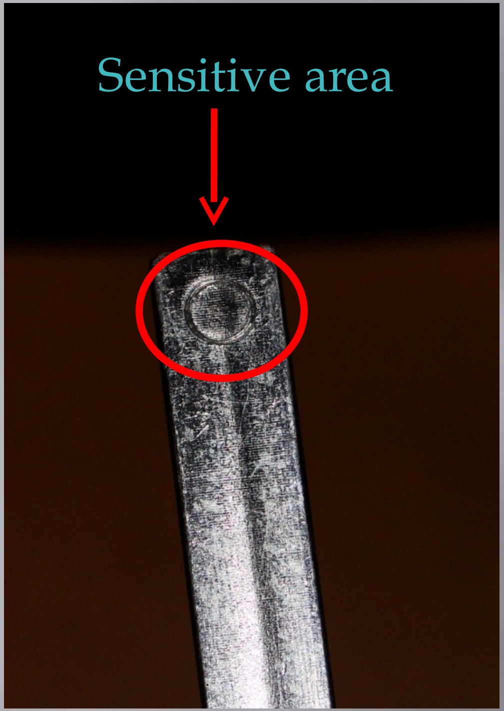
$\mathfrak{M}_{ring}$ = 41,7 A·m²

Calculated Equilibrium area

From (8) calculating relevant derivatives and using "Trapping Conditions" (5) we found the "Top Trapping Range".

For our case it is around 8 cm from the Base magnet.




Sensitive area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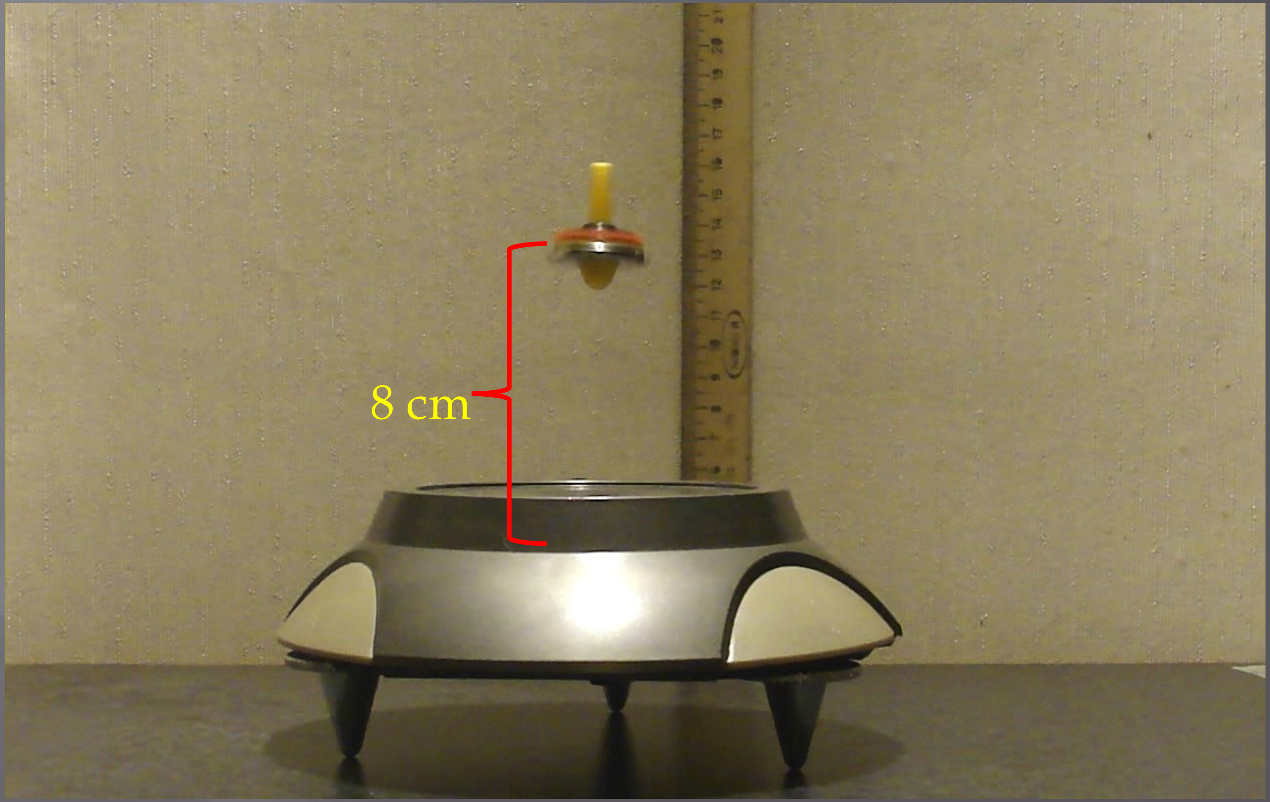
8 cm

Thank you for attention!

References:
•M.Simon, L.Heflinger, S.Ridgway. Am. J. Phys. 65 (4), April 1997. P.286.
•T.B.Jones, M.Washizu, R.Gans. J. Appl. Phys. 82 (2), 15 July 1997. P.883

Top Spin Frequency Upper Limit

Top motion equations (the *torque* and *force* equations) :

torque $\frac{d\vec{L}}{dt} = \vec{M}$ => [since angular momentum $\vec{L} = I\omega\left(\frac{\vec{\mathfrak{M}}}{\mathfrak{M}}\right)$ =>

$$\frac{d\vec{\mathfrak{M}}}{dt} = \frac{\mathfrak{M}}{I\omega}\vec{\mathfrak{M}} \times \vec{B} \qquad \textbf{(6a)}$$

force

$$m\frac{d^2\vec{r}}{dt^2} = \left(\vec{\mathfrak{M}} \cdot \vec{\nabla}\right) \cdot \vec{B} - mg\hat{z} \qquad \textbf{(6b)}$$

$I = mr_{eff}^2$ - rotational Inertia of the Top.

From (6a)

$$\omega_{precession} = \frac{\mathfrak{M}B}{I\omega}$$

the precession frequency is inversely proportional to the spin frequency.

Substituting in (6) $\vec{B}$ in **Taylor** series and imposing condition (5c) one can get [Simon et all. AJP 65 (4) 1997)]

$$\omega \le \frac{1}{r_{eff}^2 \cdot g}\left(\frac{\mathfrak{M}}{m} \cdot B_0\right)^{3/2} \qquad \textbf{(7)}$$

If the **top** is "**too fast**", the **precession** frequency will be **too slow** to allow the top to reorient to the local field direction as the top makes its radial excursion.

This is *the origin of the upper spin limit*.

$\vec{\mathfrak{M}}$ of very fast top is oriented always vertically. Thus:

(a) – we can not use approximation (2).

(b) – in the force (1), (6) the radial component of magnetic field will not act on the top.

Earnshaw Theorem

Two conditions for stable levitation:

a) Balance of magnetic and gravitational forces;
b) Potential energy must have minimum in the levitation point.

Problem! **„Earnshaw" theorem:**
If the interaction energy satisfies the Laplace equation:

$$\nabla^2 U(\vec{r}) = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0$$

It can not have the local minimum. It has only saddle points.

This concerns $\frac{1}{r}$ - type potentials. Really, if $f(r) = \frac{f_0}{r}$, (and magnetic interaction is such) then in spherical coordinates

$$\Delta f \equiv \nabla^2 f \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(-r^2 \frac{f_0}{r^2}\right) = 0$$

Though it is for static case. We have DYNAMICAL stable equilibrium.