Task of 20. IYPT – example 2

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Filament

There is a significant current surge when a filament lamp is first switched on. Propose a theoretical model and investigate it experimentally.

Model assumptions

We consider the lamp filament to be a tungsten cylinder of constant diameter *d*, length and mass density $\bar{\varrho}$, attached to a tough DC voltage source $\mathcal{E}$, and placed inside an evacuated bulb. Let the environmental temperature T_0 be common room temperature about 293 K. Moreover, we consider the dominating mechanism of filament cooling to be electromagnetic radiation governed by the Planck's blackbody law

$$\ell = \sigma S (T_4 - T_0^4) , \qquad (1)$$

where ℓ is total electromagnetic luminosity, and *T* denote Stefan–Boltzmann radiation constant and the thermodynamic temperature of the filament, respectively, and

$$S = \pi \ell d , \qquad (2)$$

is the filament radiating surface.

The filament specific thermal capacity c_s is considered to be constant within whole the temperature interval up to the melting temperature T_{melt}, while the temperature dependence of the filament resistance is considered to be linear,

$$R(T) = R_0[1 + \alpha(T - T_0)] , \qquad (3)$$

where R_0 is filament resistance at T_0,

$$R_0 = \frac{1}{\gamma}\frac{\ell}{A} = \frac{1}{\gamma}\frac{4\ell}{\pi d^2}. \tag{4}$$

Here A = (π d^2) / 4 is the filament cross section area, denotes specific electric conductivity at temperature T0, and _ is linear temperature coefficient.

Model parameters

The parameters of the model must be fine tuned to ensure proper lamp functionality, e.g., to prevent the filament melting caused by exceeding the melting point T_{melt}. We derive the relationship between model parameters later in Section 4, temporarily giving the list of parameters in Table 1 on page 2 without explanation.

Mathematical model

The lamp filament undergoes heating up caused by the electric current passing through it, with heat production rate given by

$$\mathcal{P} = \frac{\mathcal{E}^2}{R(T)}. \tag{5}$$

The heat production rate is temperature-dependent due to formula (3). On the other hand, the filament radiates electromagnetic waves with rate described by the Planck's radiation law (1), which causes cooling of the filament. Thus, the thermal disbalance of the filament is given by the net heat production rate inside the filament

$$\mathcal{H} = \mathcal{P} - \mathcal{I}, \tag{6}$$

causing filament temperature change. Taking into account the definition of thermal capacity

$$C = \frac{\mathrm{d}Q}{\mathrm{d}T}, \tag{7}$$

we obtain for temperature change induced by supplying the amount of heat dQ

$$\mathrm{d}T = \frac{1}{C}\,\mathrm{d}Q. \tag{8}$$

Dividing both sides of (8) by the corresponding time interval dt and realizing that dQ/dt equals the net heat rate (6), we arrive to

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{1}{C}\left[\frac{\mathcal{E}^2}{R(T)} - \sigma S(T^4 - T_0^4)\right], \tag{9}$$

where the total thermal capacity of the filament C can be expressed as

$$C = mc_s = \varrho V c_s = \tfrac{1}{4}\pi\varrho d^2 \ell c_s, \tag{10}$$

(here m and V denote the filament mass and volume).

The first-order ordinary differential equation (9), together with relations (2), (3), (4), (10), and initial condition $T(0) = T_0$, (11) represents the mathematical model to be solved.

Table 1: Parameters used for filament lamp theoretical model building. Instead of the filament specific electric conductivity , the program uses its inverse value (specific electric resistivity) $1/\gamma = 0.05\,\Omega\,\mathrm{mm}^2\,\mathrm{m}^{-1}$. Taken from [1].

Quantity	Denoted by	Value
filament length	ℓ	22 mm
filament diameter	d	0.04 mm
filament density	ϱ	$19.30\,g/cm^3$
environment temperature	T_0	293.16 K
filament melting point	T_{melt}	3410 K
filament specific thermal capacity at T_0	c_s	0.134 J/g K
filament voltage	$\mathcal{E}$	12 V
filament resistance temperature coeff.	α	$0.0045\,K^{-1}$
filament specific electric conductivity at T_0	γ	$20.0\,m/\Omega\,mm^2$
Stefan–Boltzmann radiation constant	σ	$5.67032 \times 10^{-8}\,W\,m^{-2}\,K^{-4}$

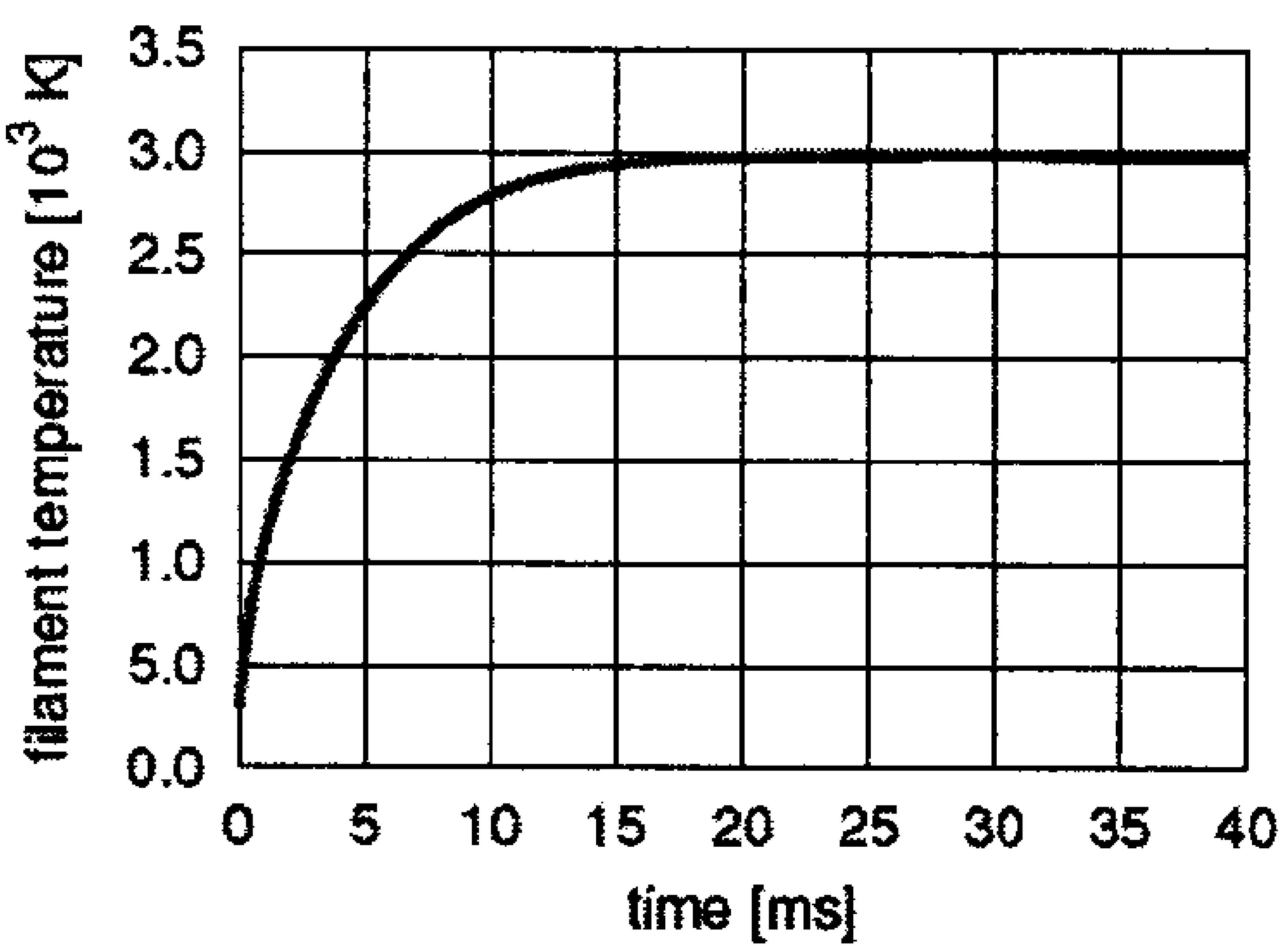

Figure 1: Filament temperature time dependence plot.

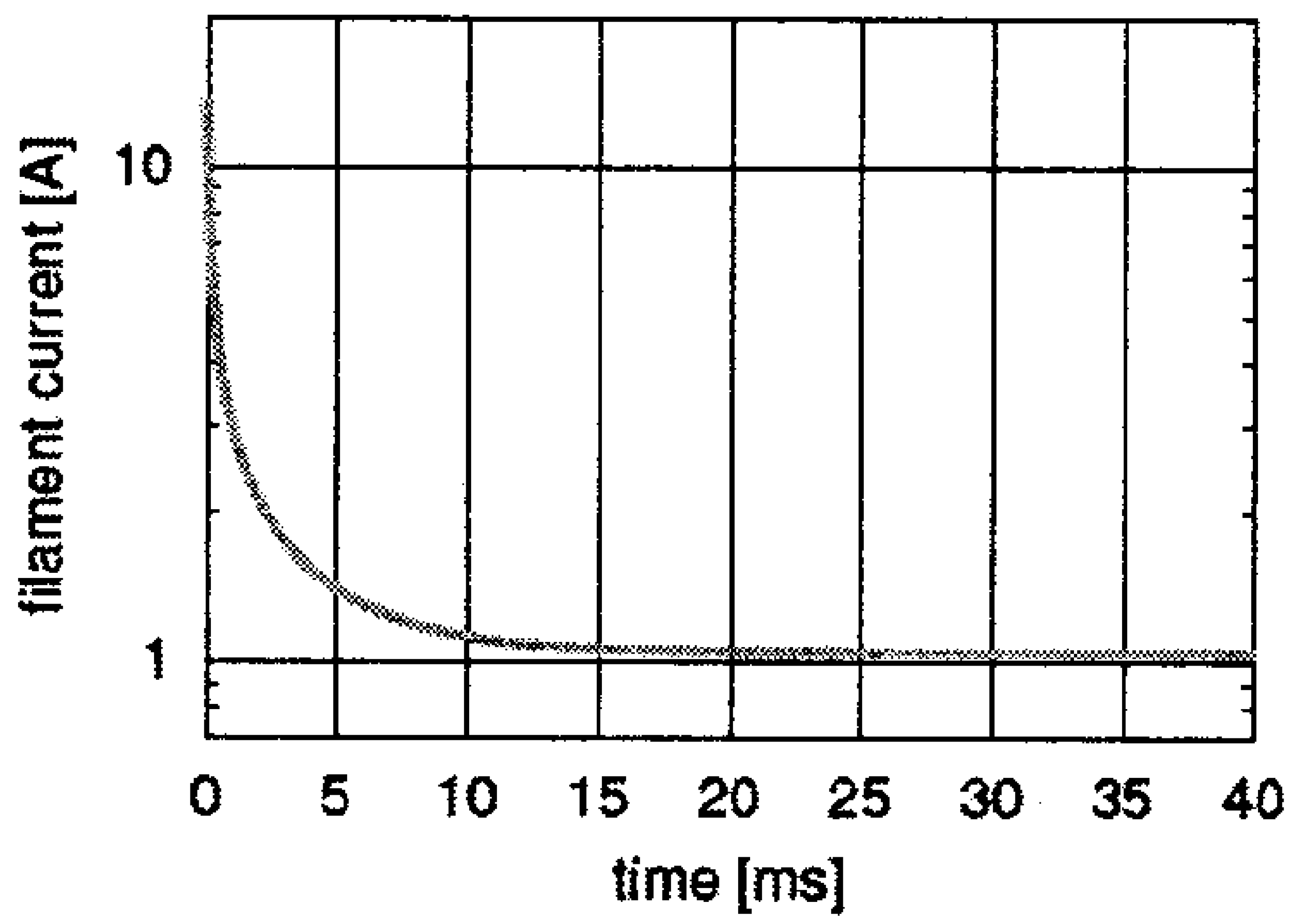

Figure 2: Filament electric current time dependence plot. Notice the logarithmic scale for current.