2. Coupled Compasses

Republic of Korea
Lee, Chan

PROBLEM
Place a compass on a table. Place a similar compass next to the first one and shake it gently to make the needle start oscillating. The original compass' needle will start oscillating.

Coupled oscillator
“More than two oscillating systems connected together so as to interact with each other”

Mechanical oscillators, biochemical phenomena, electron behavior, …

Beat, Chaos, Synchronization

PROBLEM
Observe and explain the behavior of these coupled oscillators.

Qualitative hypothesis

Flowchart

- Qualitative
  - Hypothesis
  - Simulation
  - Beat & Energy transfer
- Theory
  - Finite length of needle
  - Magnetic pole-pole model
- Experiment
  - Time-angle
  - Distance-frequency
- Conclusion
  - Distance ➔ Behavior??

Beat, & Energy transfer

Synchronization

N
S

B_{geo}

\mu_1

\mu_2
Qualitative hypothesis

\[ \theta_{2(\text{max})} \text{ would keep increasing until } \theta_{1(\text{max})} \rightarrow 0 \]

Relative phase shifts

\[ \theta_{1(\text{max})} \text{ would keep increasing until } \theta_{2(\text{max})} \rightarrow 0 \]

Energy transfer, Beat

Simulation model

Divided into infinitesimal point dipoles

Numerically calculate the force applied on each dipole

Obtain torque for each time step

\[ \vec{B}_{\text{coupling}} = \frac{1}{4\pi} \int \frac{3(d\vec{\mu} \cdot \vec{r})\vec{r} - d\vec{\mu}}{r^3} \, dr \]

Griffith, "Introduction to Electrodynamics" (1991)

\[ I\ddot{\vec{n}} = -\int d\vec{\mu} \times (\vec{B}_{\text{geo}} + \vec{B}_{\text{coupling}}) - c\vec{\mu} \]

Simulation code (MATLAB)

Simulation results

Beat

Angle of oscillation

Compass 1

Beating !!

Energy transfer !!

Compass 2

Carrier frequency

Beat frequency

Theory

\[ \mu = iA \]

Does not consider length!!

Finite length

Magnetic pole–pole model

\[ I(\dot{q} + \dot{\theta}) = -\mu B(\dot{q} + \dot{\theta}) - c(\dot{\theta} + \dot{\theta}) \]

\[ \omega_k = \frac{\sqrt{4I \mu B - c^2}}{2I} \]

\[ I(\dot{\theta}_2 - \dot{\theta}_1) = -\mu B(\dot{\theta}_2 - \dot{\theta}_1) + \frac{\mu J^2}{2\pi} \frac{1}{D' - 2(D + d)^2} (\dot{\theta}_2 - \dot{\theta}_1 - c(\dot{\theta}_2 - \dot{\theta}_1)) \]

\[ \omega_k = \frac{\sqrt{4I \mu B - 2\gamma c^2} - c^2}{2I} \]

\[ \gamma = \frac{\mu J}{4\pi} \left( \frac{1}{D'} - \frac{1}{2(D + d)^2} \right) \]

Coupling coefficient

Theory

\[ \mu = q_m d \] (magnetic pole–pole model)

\[ q_m = \frac{\Phi}{\mu_0} \] (magnetic pole strength)

- D. Kim, “Basic theory and application on magnetism” (2003)

\[ q_m = \frac{\bar{B} \cdot \bar{A}}{\mu_0} = \frac{B(4\pi r^2)}{\mu_0} \]

\[ B = \frac{\mu_0 q_m}{4\pi r^2} \]

\[ E = \frac{1}{4\pi\mu_0} \frac{q_m}{r^2} \]

Theory

When \( \theta_1(0) = \theta_{01}, \theta_2(0) = 0 \)

\[ \theta_1 = \theta e^{\cos \left( \frac{\omega_a + \omega_b}{2} t \right)} \cos \left( \frac{\omega_a - \omega_b}{2} \right) e^{-\frac{c}{2I} t} \]

\[ \theta_2 = -\theta e^{\sin \left( \frac{\omega_a + \omega_b}{2} t \right)} \sin \left( \frac{\omega_a - \omega_b}{2} \right) e^{-\frac{c}{2I} t} \]

Carrier frequency

Beat frequency

1. Beat
2. Energy transfer

Magnetic pole–pole model

Initial setup

1. Setup was unstable
2. Too much friction

Need stable observation, controlled experiment
**Experimental Setup**

- Compass needle
- Straw
- Thin string

**Experiment**

**Controlled variables**
- Length of needle \( (d) = 3.25 \text{ cm} \)
- Inertia of needle \( (I) = 5.28 \times 10^{-8} \text{ kg·m}^2 \)
- Magnetic moment of needle \( (\mu) = 0.016 \text{ N·m/T} \)
- Geomagnetic field \( (B) = 0.49 \text{ G} \)

**Independent variables**
- Distance between needle \( (D) = 5 - 20 \text{ cm} \)

**Dependent variables**
- Frequency components
- Time-angle plot

**Video**

**Tendency of oscillation**

**Time-angle graph**

- Distance = 12 cm

- Distance = 8 cm
**Distance-frequency relation**

- **Beat frequency**
- **Carrier frequency**
- **Pole-pole assumption**

**New state of equilibrium**

- Distance ↓↓ (3.8~4.6 cm)
  - Coupling > Geomagnetic field
  - Synchronization/chaos becomes observable

**Synchronization**

- Strong attraction between nearby poles
- Nearby poles oppose movement in other direction
  - Synchronized in opposite phase

**Distance** ↓↓ (3.8~4.6 cm)

**Carrier frequency**

- Pole-pole assumption
Chaos

- Governed by deterministic physical laws
- Extremely sensitive to initial condition
  - Unpredictable, random
- Non-linear term becomes dominant
  - Distance ↓, Initial angle ↑


THANK YOU!
Republic of Korea
Lee, Chan

Conclusion

1. Coupled compasses mainly exhibit beat and energy transfer
2. Beat becomes more frequent as distance decreases
3. Sync/chaos occurs in extreme conditions