IYPT 2009
Nankai University, Tianjin

2. Coupled Compasses

Republic of Korea
Lee, Chan

Flowchart

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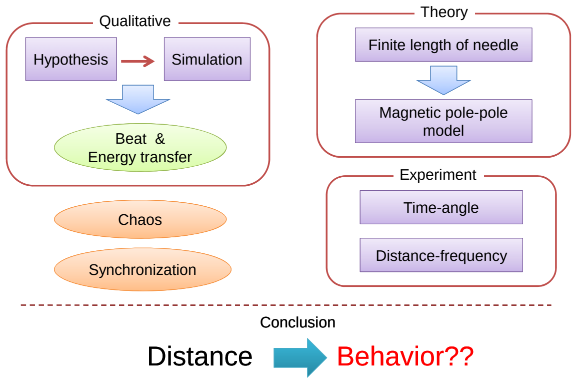

PROBLEM

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Place a **compass** on a table. Place a **similar compass** next to the first one and **shake it gently** to make the needle start oscillating. The original compass' needle will start oscillating.

Coupled oscillator

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"More than two oscillating systems connected together so as to interact with each other"

- *"A Dictionary of Physics,"* Oxford (2001)

Mechanical oscillators, biochemical phenomena, electron behavior, …

- S.Strogatz, *"Sync: the emerging science of spontaneous order," Sanders co.* (1998)

Beat, Chaos, Synchronization

PROBLEM

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Observe and explain the behavior of these coupled oscillators.

Qualitative hypothesis

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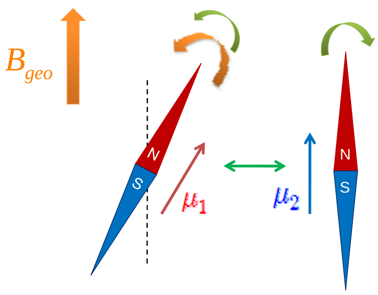
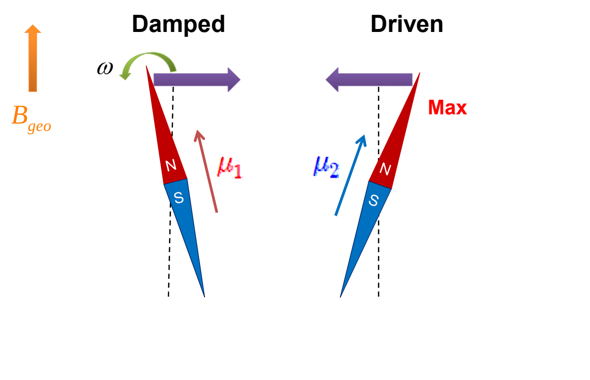
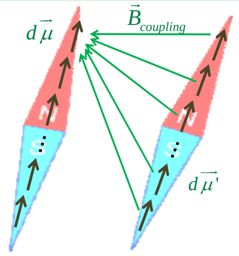


Qualitative hypothesis



Simulation model



Divided into infinitesimal point dipoles

Numerically calculate the force applied on each dipole

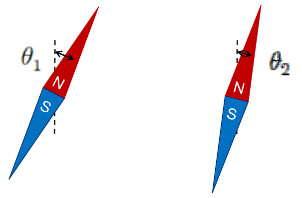
➡ **Obtain torque for each time step**

$$\vec{B}_{coupling} = \int \frac{\mu_0}{4\pi} \frac{3(d\vec{\mu}' \cdot \hat{r})\hat{r} - d\vec{\mu}'}{r^3}$$

Griffith, *"Introduction to electrodynamics"* (1991)

$$I\ddot{\theta}_1\hat{n} = -\int d\vec{\mu} \times (\vec{B}_{geo} + \vec{B}_{coupling}) - c\dot{\theta}_1\hat{n}$$

Qualitative hypothesis



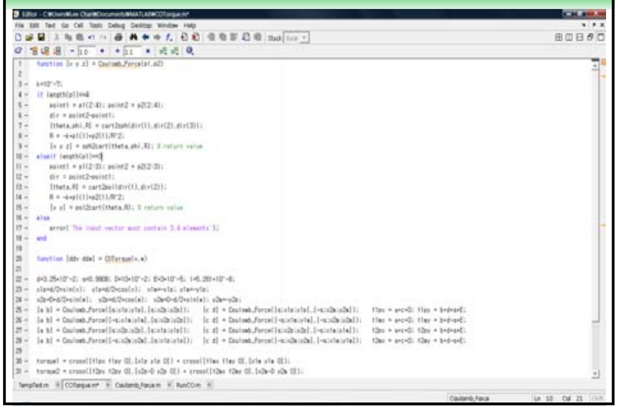
$\theta_{2(max)}$ would keep increasing until $\theta_{1(max)} \to 0$

➡ **Relative phase shifts**

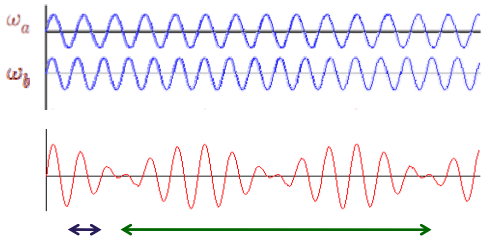
➡ **$\theta_{1(max)}$ would keep increasing until $\theta_{2(max)} \to 0$**

Energy transfer, Beat

Simulation code (MATLAB)



Beat



Carrier frequency

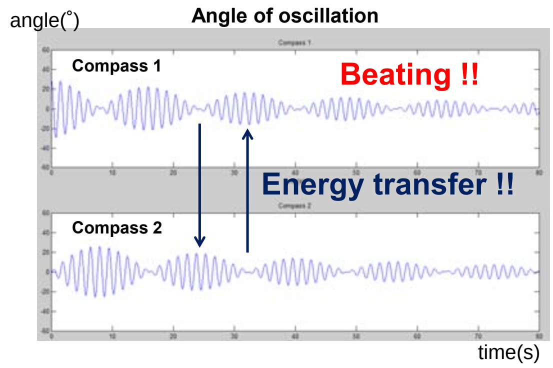
$$\frac{\omega_a + \omega_b}{2}$$

Beat frequency

$$\frac{\omega_a - \omega_b}{2}$$

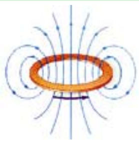
Halliday, Resnick, Walker, *"Fundamentals of Physics"* (2006)

Simulation results



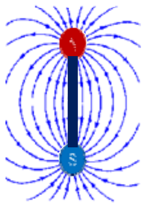
Theory

$\mu \equiv iA$



Does not consider length!!

Finite length



Magnetic pole-pole model

$$I(\ddot{\theta}_1+\ddot{\theta}_2) = -\mu B(\theta_1+\theta_2) - c(\dot{\theta}_1+\dot{\theta}_2)$$

$$\omega_a = \frac{\sqrt{4I\mu B - c^2}}{2I}$$

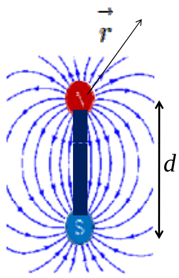
$$I(\ddot{\theta}_2-\ddot{\theta}_1) = -\mu B(\theta_2-\theta_1) + \frac{\mu_0\mu^2}{2\pi}\left(\frac{1}{D^3-2(D^2+d^2)^{\frac{3}{2}}}\right)(\theta_2-\theta_1) - c(\dot{\theta}_2-\dot{\theta}_1)$$

$$\omega_b = \frac{\sqrt{4I(\mu B-2\gamma)-c^2}}{2I}$$

$$\gamma \equiv \frac{\mu_0}{4\pi}\left(\frac{1}{D^3} - \frac{1}{2(D^2+d^2)^{\frac{3}{2}}}\right)$$

Coupling coefficient

Theory



$\mu = q_m d$ **(magnetic pole-pole model)**

$q_m \equiv \frac{\Phi}{\mu_0}$ **(magnetic pole strength)**

- D. Kim, *"Basic theory and application on magnetism"* (2003)

$$q_m = \frac{\vec{B}\cdot\vec{A}}{\mu_0} = \frac{B(4\pi r^2)}{\mu_0}$$

$$B = \frac{\mu_0}{4\pi}\frac{q_m}{r^2} \Leftrightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q_e}{r^2}$$

Theory

When $\theta_1(0) = \theta_0,\ \theta_2(0) = 0$

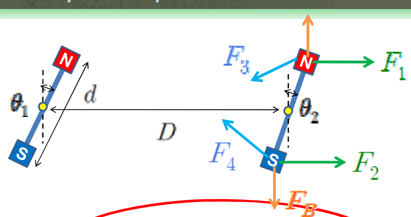
$$\theta_1 = \theta_0 \cos\left(\frac{\omega_a+\omega_b}{2}t\right)\cos\left(\frac{\omega_a-\omega_b}{2}t\right)e^{-\frac{c}{2I}t}$$

$$\theta_2 = -\theta_0 \sin\left(\frac{\omega_a+\omega_b}{2}t\right)\sin\left(\frac{\omega_a-\omega_b}{2}t\right)e^{-\frac{c}{2I}t}$$

Carrier frequency   Beat frequency

1. **Beat**
2. **Energy transfer**

Magnetic pole-pole model

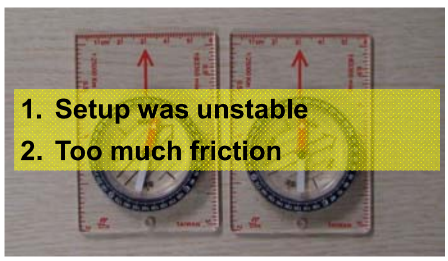


$$I\ddot{\theta}_1 = -\mu B\theta_1 - \frac{\mu_0\mu^2}{4\pi}\left(\frac{1}{D^3 \quad 2(D^2+d^2)^{\frac{3}{2}}}\right)(\theta_2-\theta_1) - c\dot{\theta}_1$$

$$I\ddot{\theta}_2 = -\mu B\theta_2 + \frac{\mu_0\mu^2}{4\pi}\left(\frac{1}{D^3-2(D^2+d^2)^{\frac{3}{2}}}\right)(\theta_2-\theta_1) - c\dot{\theta}_2$$

Geomagnetic field   **Coupling**   **Damping**

Initial setup



Need stable observation, controlled experiment

Experimental Setup

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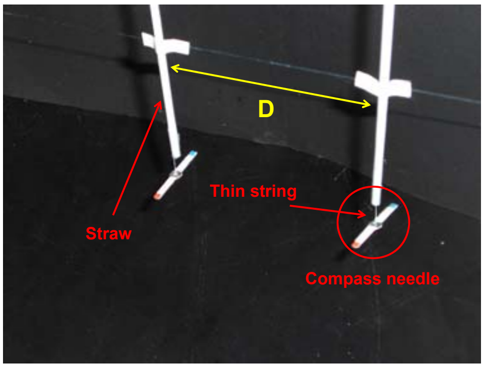

Image processing (Java)

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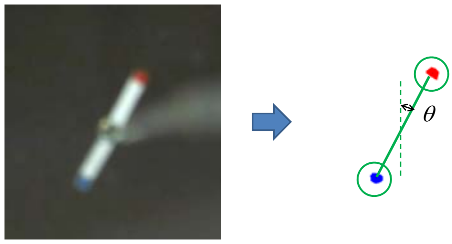

30 frame/s

Experiment

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Controlled variables

Length of needle (d) = 3.25 cm

Inertia of needle (I) = 5.28×10^{-8} kg·m^2

Magnetic moment of needle (μ) = 0.016 N·m/T

Geomagnetic field (B) = 0.49 G

Independent variables

Distance between needle (D) = 5 ~ 20 cm

Dependent variables

Frequency components ← Time-angle plot

Time-angle graph

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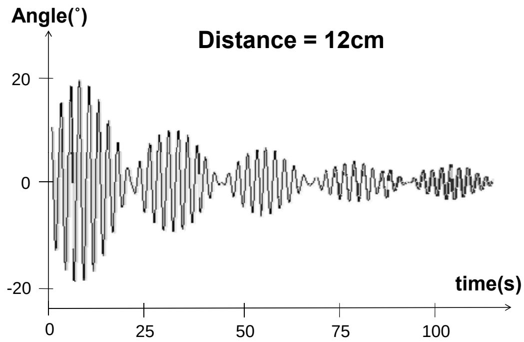

Video

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Tendency of oscillation

Time-angle graph

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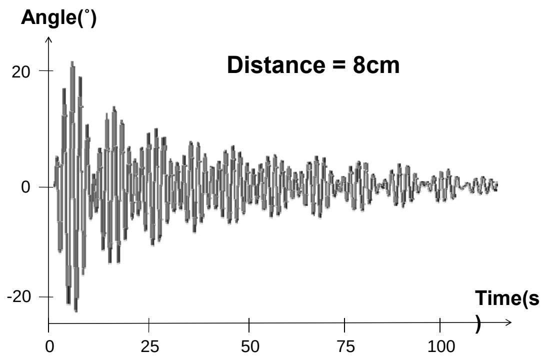
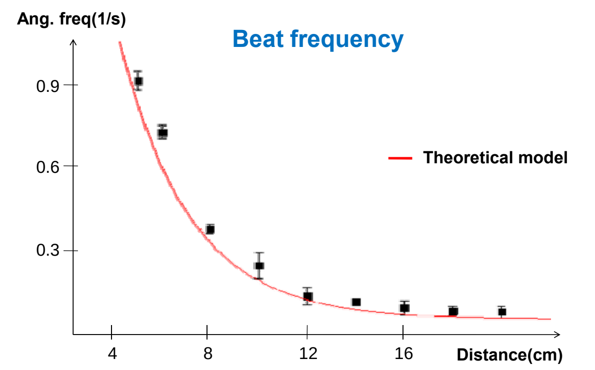


Distance–frequency relation



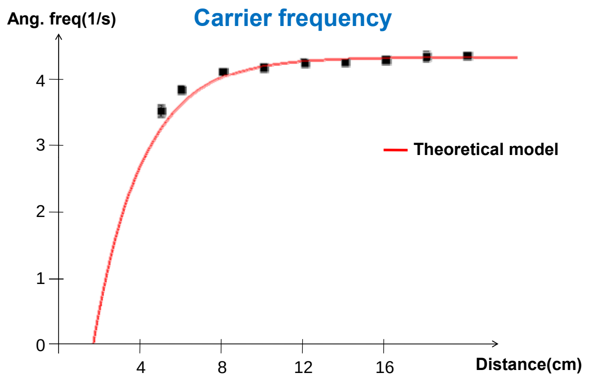
New state of equilibrium

Distance ↓↓ (3.8~4.6 cm)

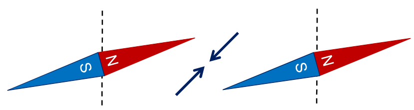
Coupling > Geomagnetic field

Synchronization/chaos becomes observable

Distance–frequency relation



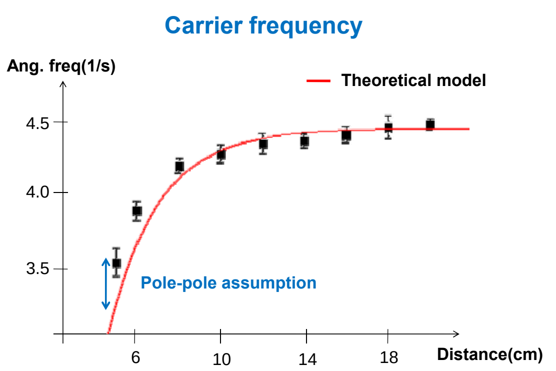
Synchronization



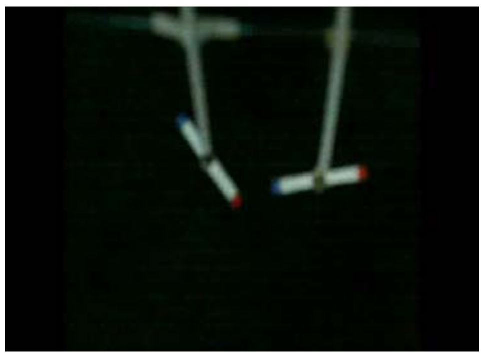
- **Strong attraction between nearby poles**
- **Nearby poles oppose movement in other direction**

➡ **Synchronized in opposite phase**

Distance–frequency relation



Synchronization



Chaos

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- **Governed by deterministic physical laws**
- **Extremely sensitive to initial condition**
  - ➡ **Unpredictable, random**

- **Non-linear term becomes dominant**

  Distance ↓, Initial angle ↑

-S.Strogatz, *"Sync: the emerging science of spontaneous order," Sanders co.* (1998)

Chaos

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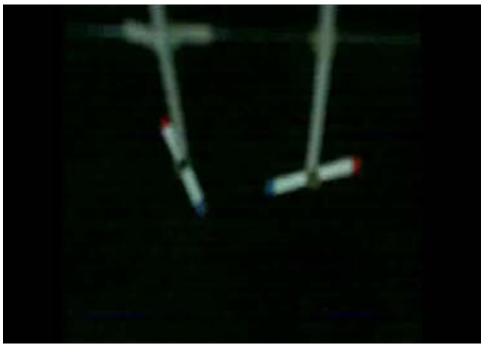

Conclusion

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1. Coupled compasses mainly exhibit beat and energy transfer

2. Beat becomes more frequent as distance decreases

3. Sync/chaos occurs in extreme conditions

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THANK YOU !

Republic of Korea
Lee, Chan