2009 IYPT

10. Optical Tube

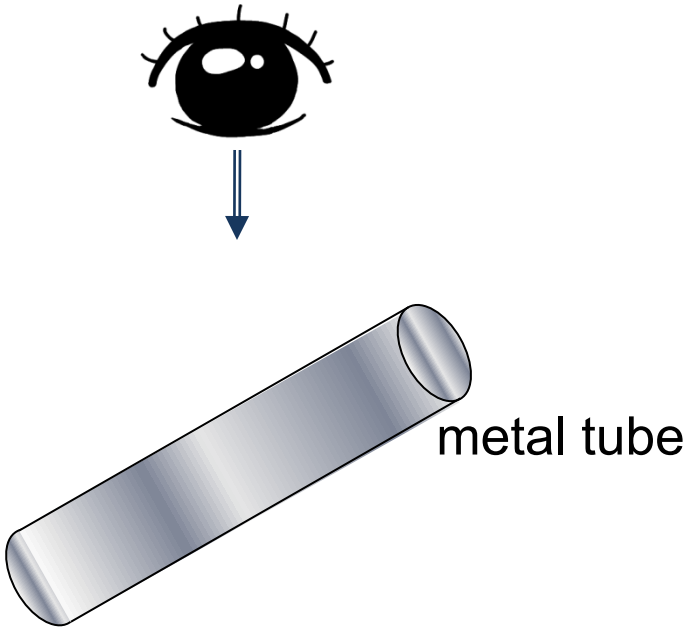
Republic of Korea

Minkyu Shim

INTRODUCTION

Republic of Korea

- **PROBLEM**
  - Look down a cylindrical metal tube which is shiny on the inside. You will notice dark and light bands. Investigate the phenomenon.



FLOWCHARTS

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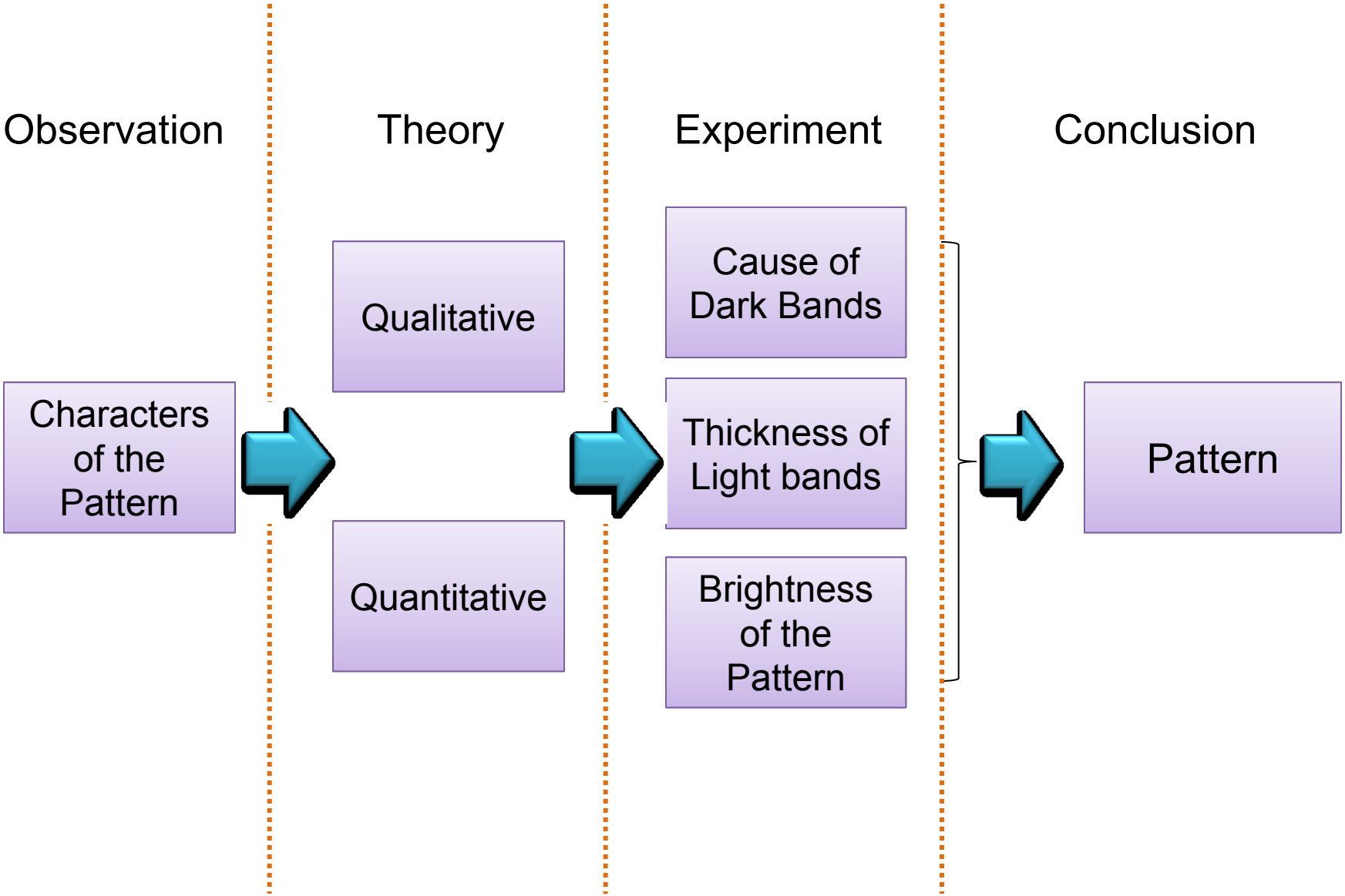

OBSERVATION

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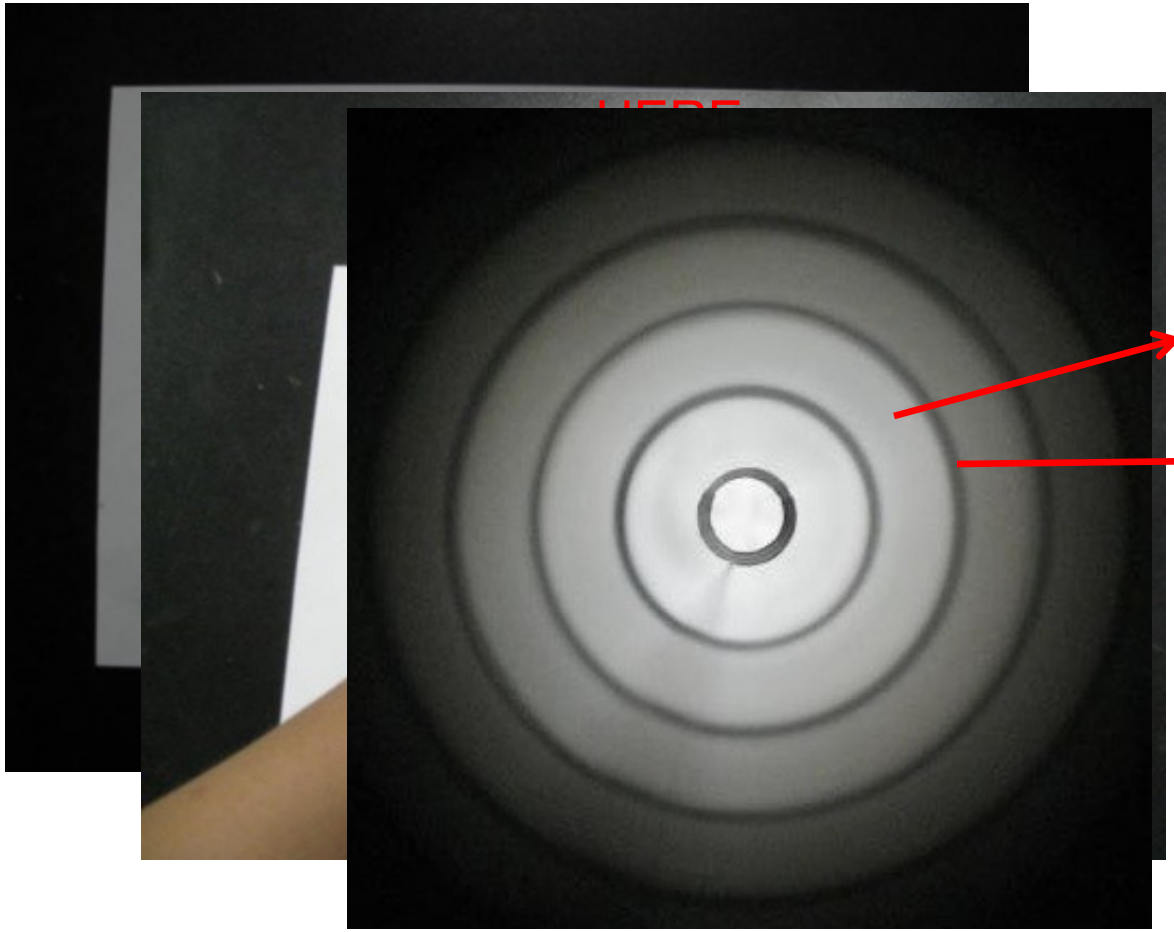
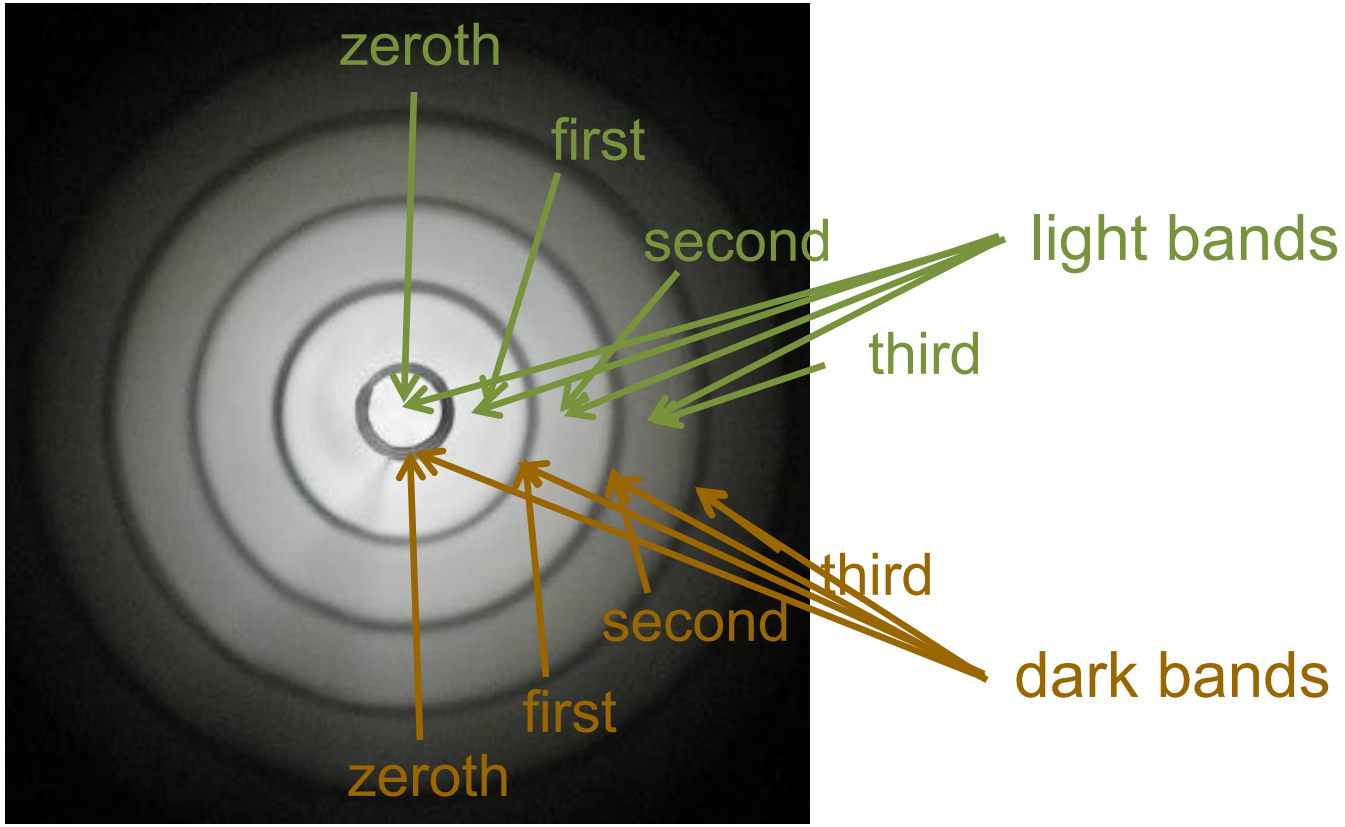


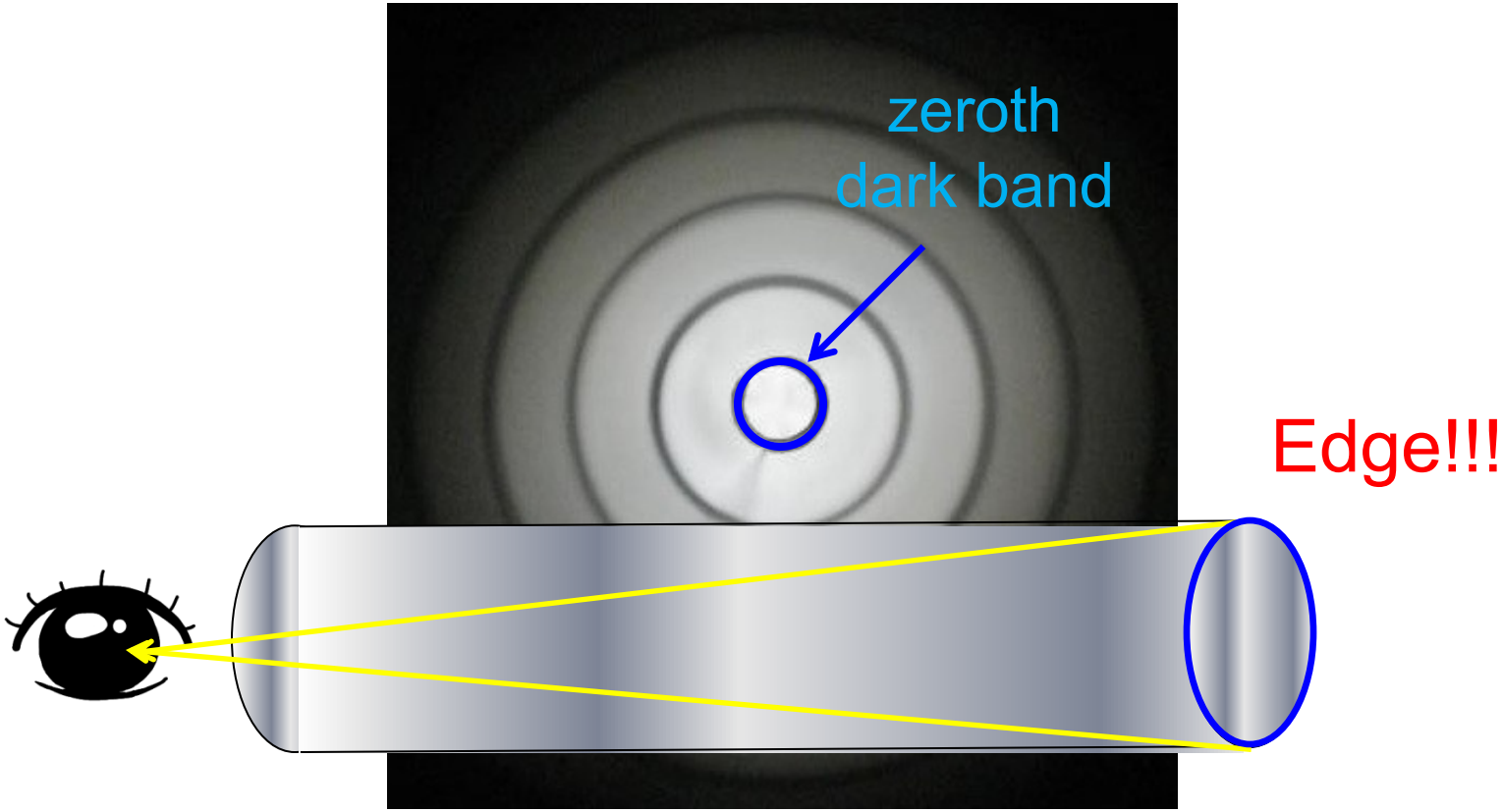
Thick Light Band

Thin Dark Band

- **Definitions**



- **Cause of Dark Bands**



- **Hypothesis on Dark Bands**

Edge of the metal tube causes the dark bands to be formed

EDGE

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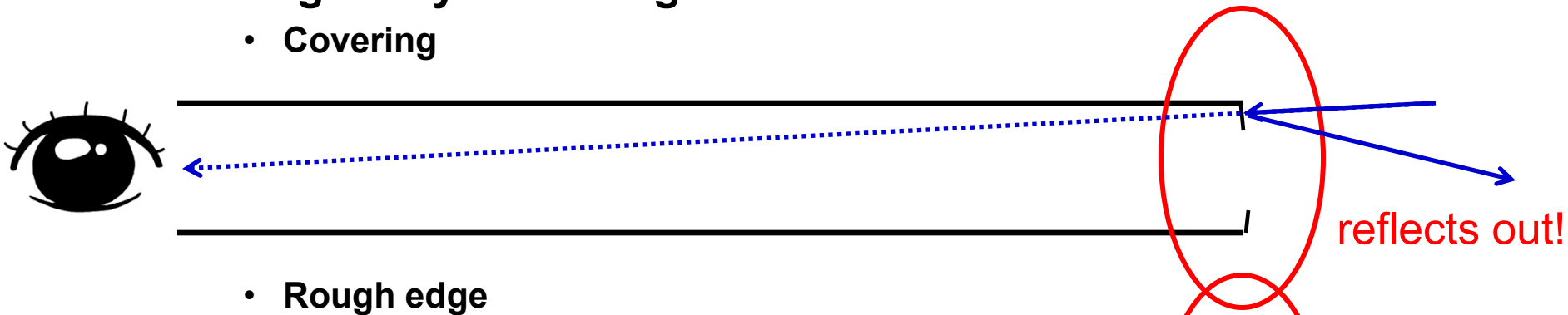
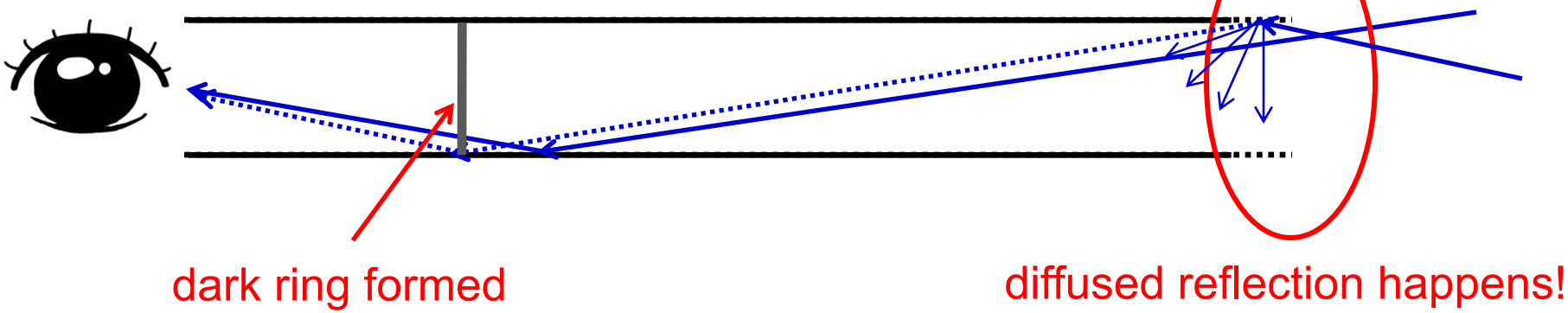
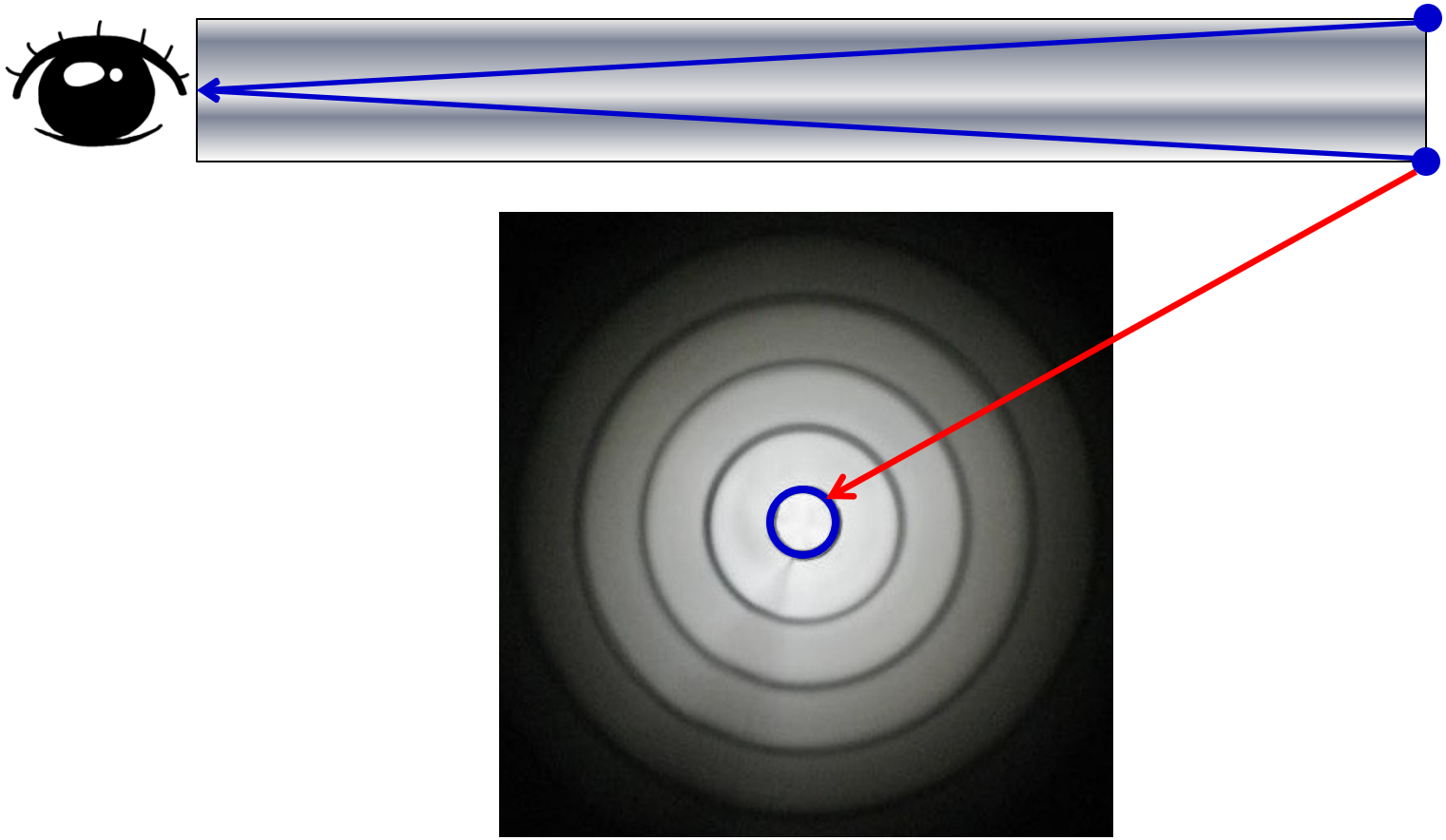
- **TWO REASONS**
  - **Irregularity of the edge**
    - **Covering**



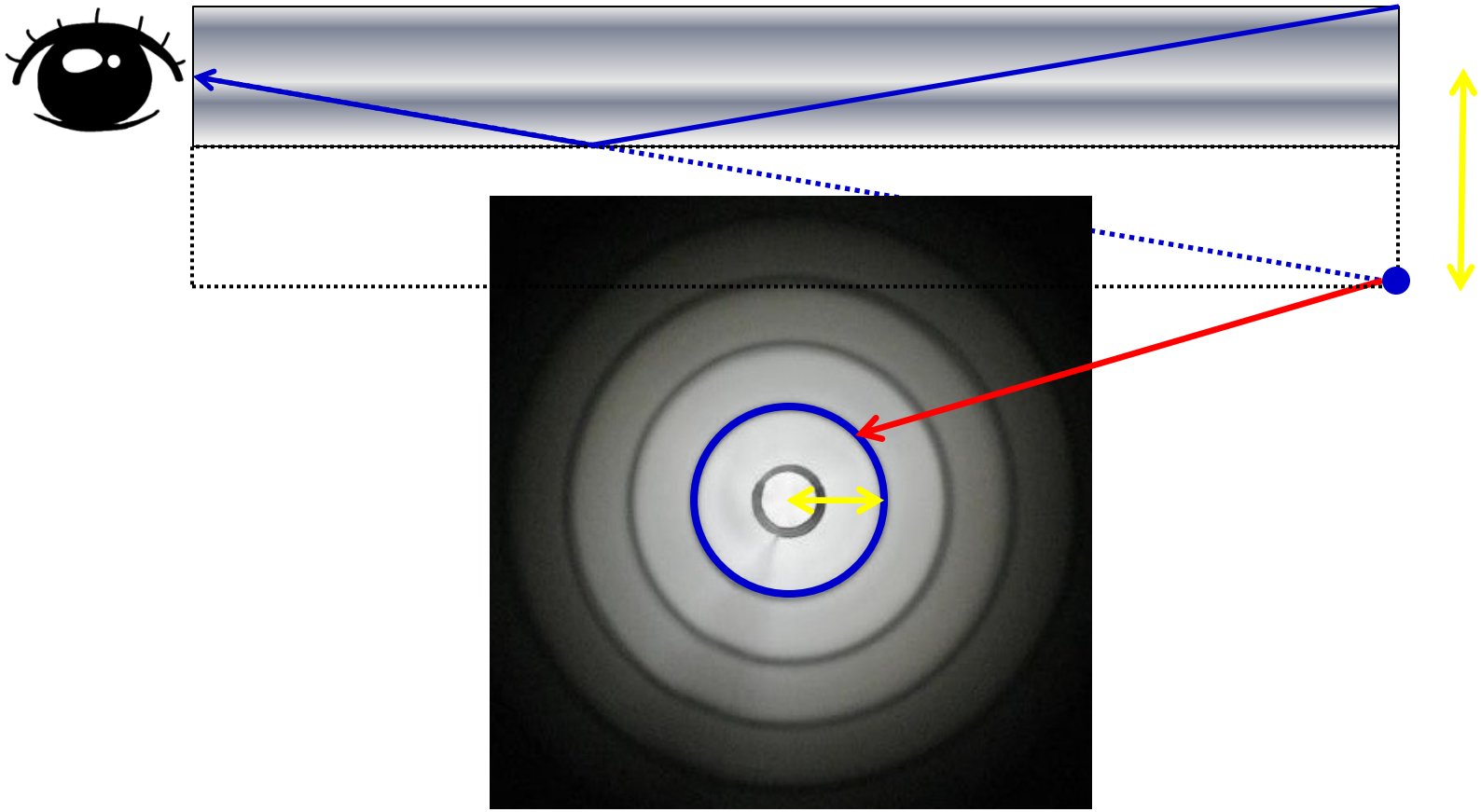
    - **Rough edge**



- **Ray Analysis on Edge – 0th dark band**



- **Ray Analysis on Edge – 1st dark band**



THEORY - QUALITATIVE

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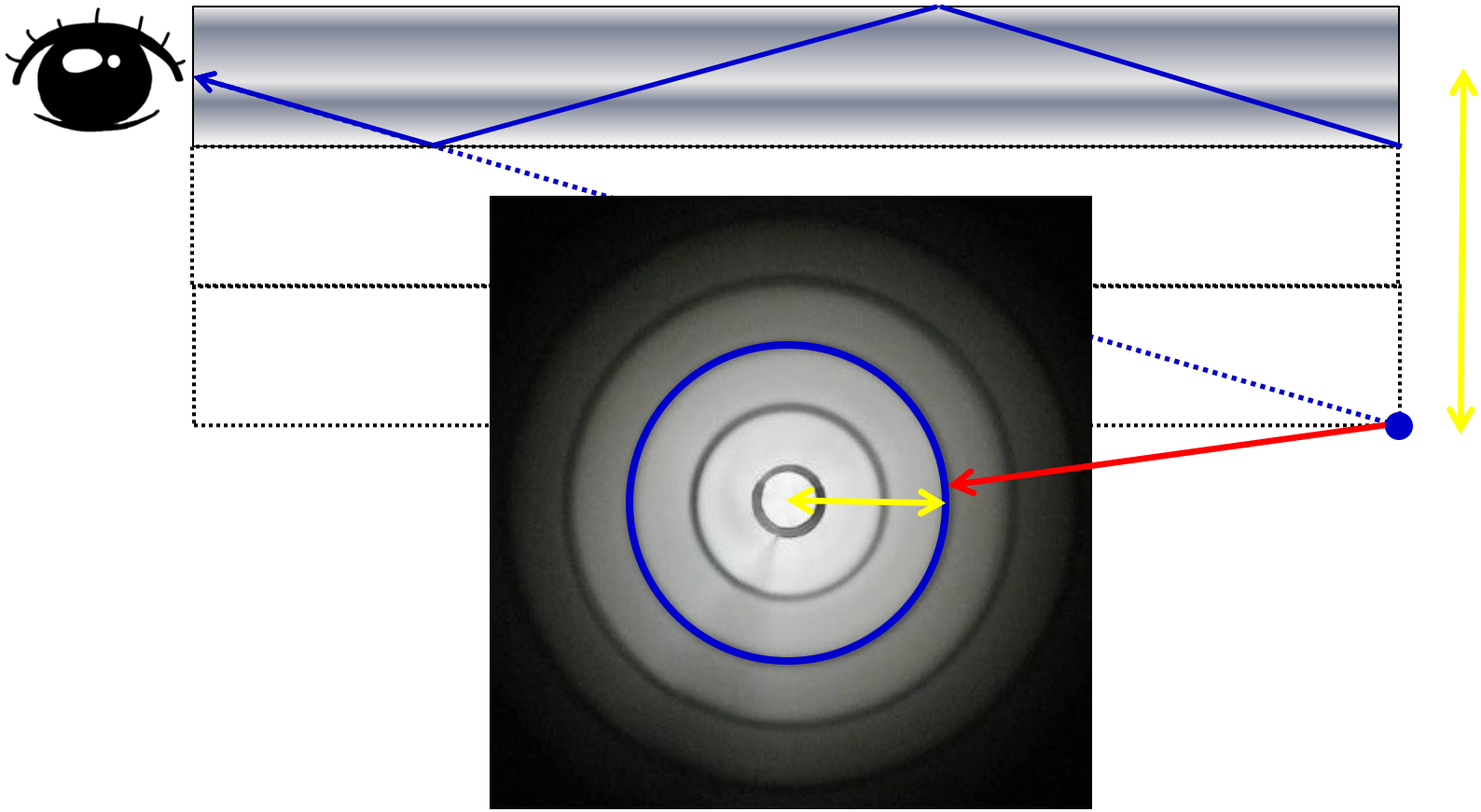
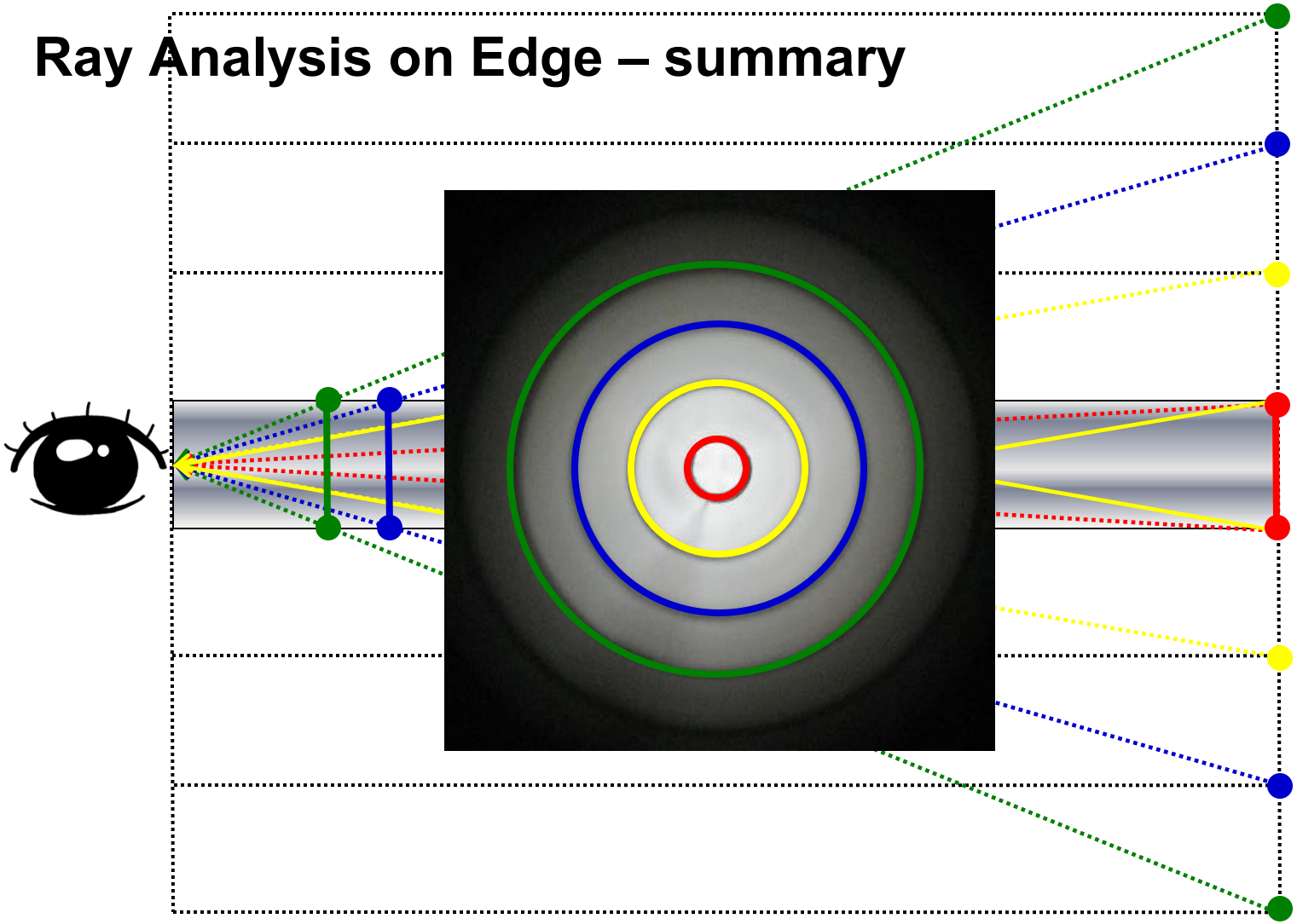
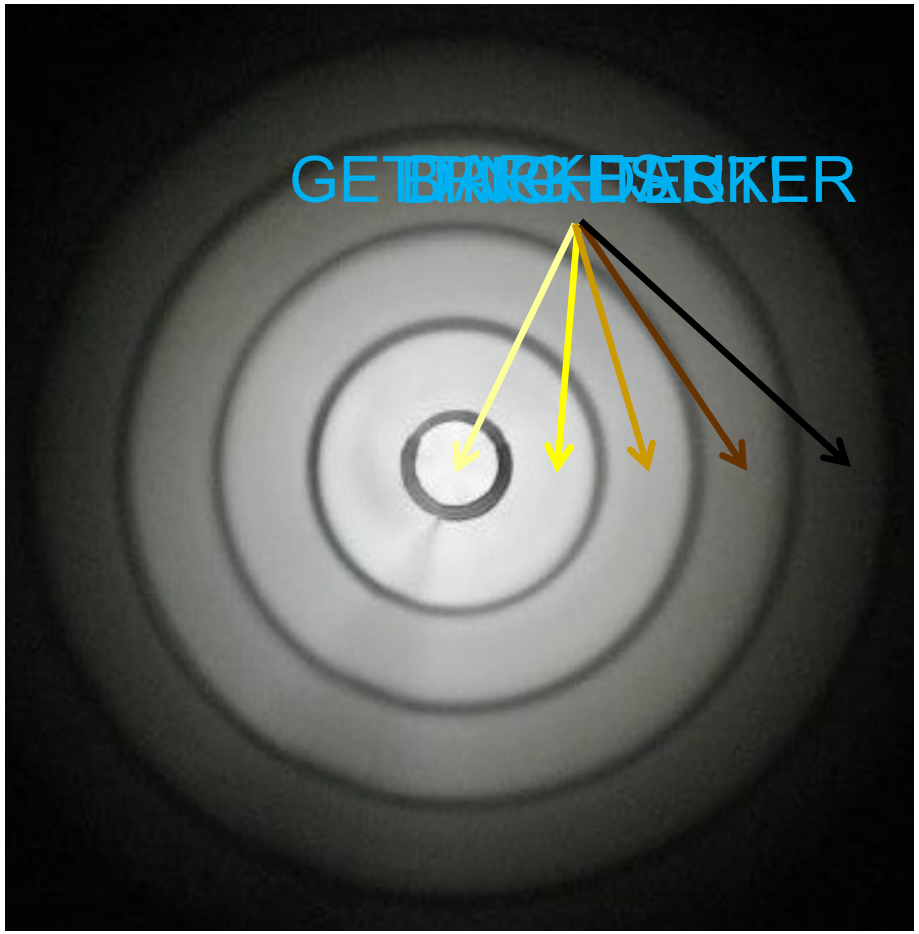
- **Ray Analysis on Edge – 2nd dark band**



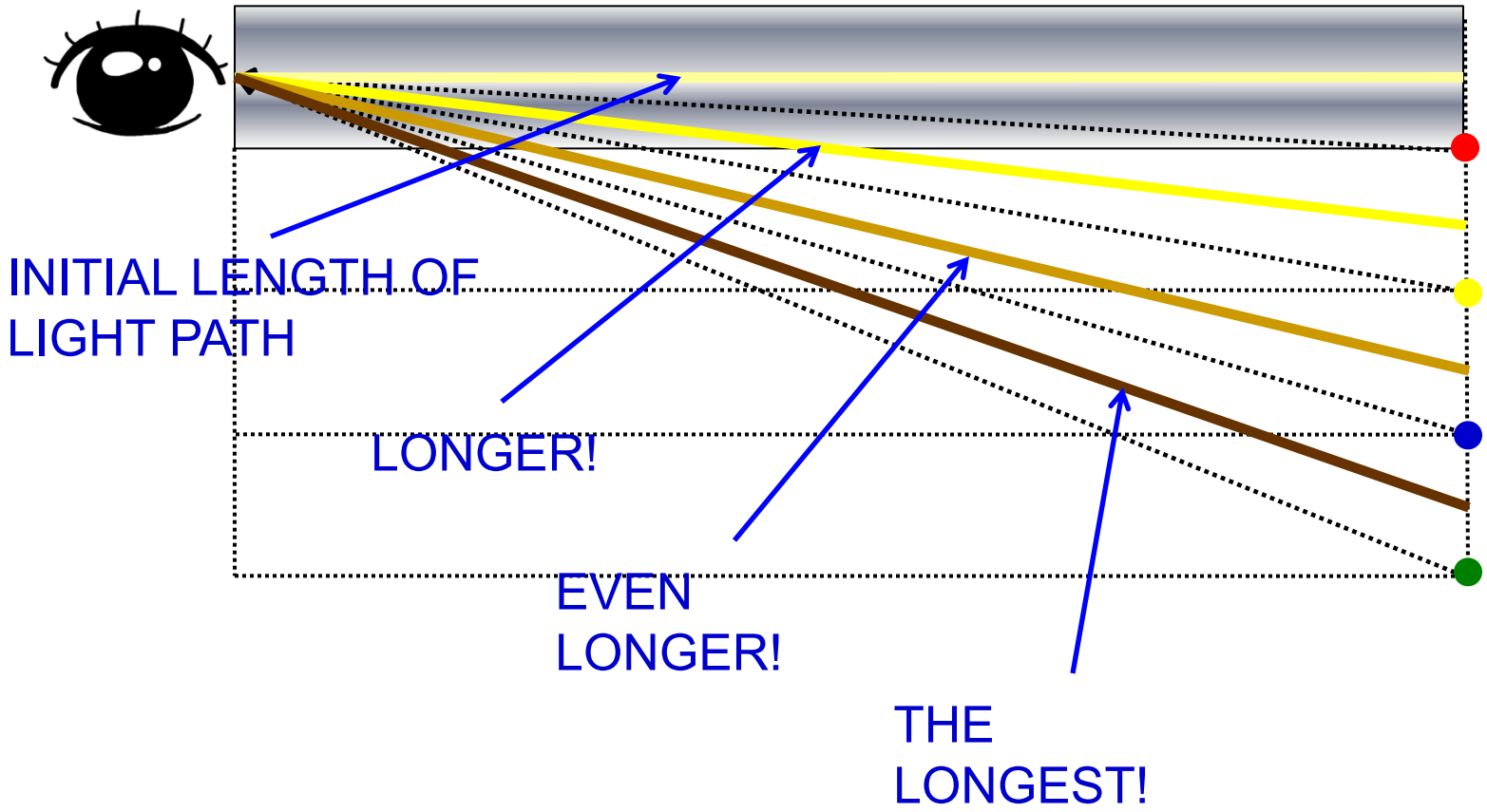
- **Ray Analysis on Edge – summary**



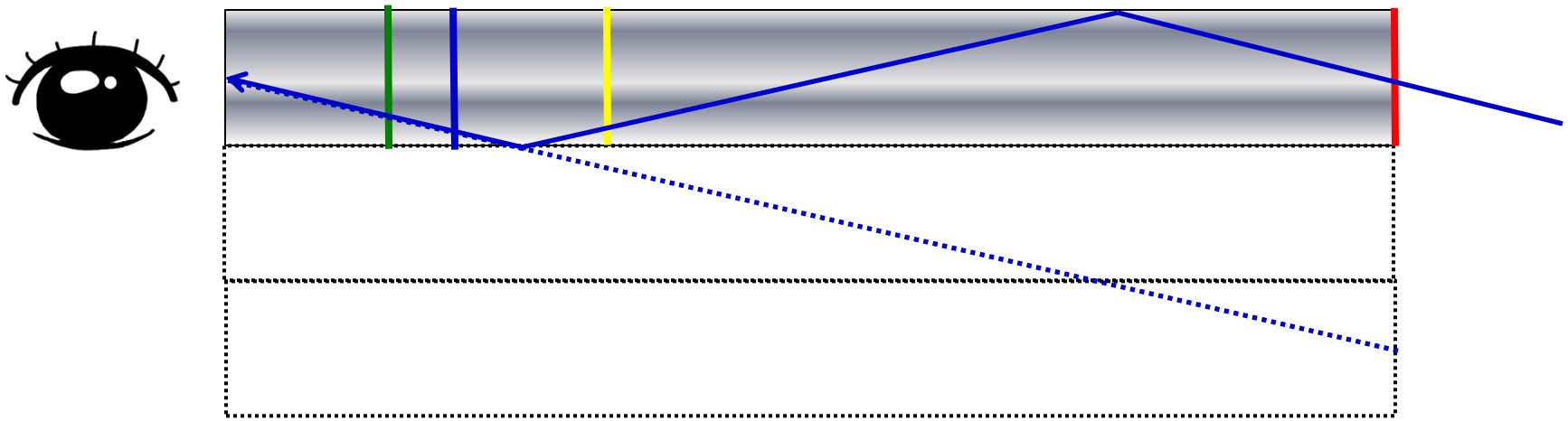
- **Ray Analysis for Brightness**



- **Decrease in Brightness 1 – longer path of light**



- **Decrease in Brightness 2 – loss in reflection**



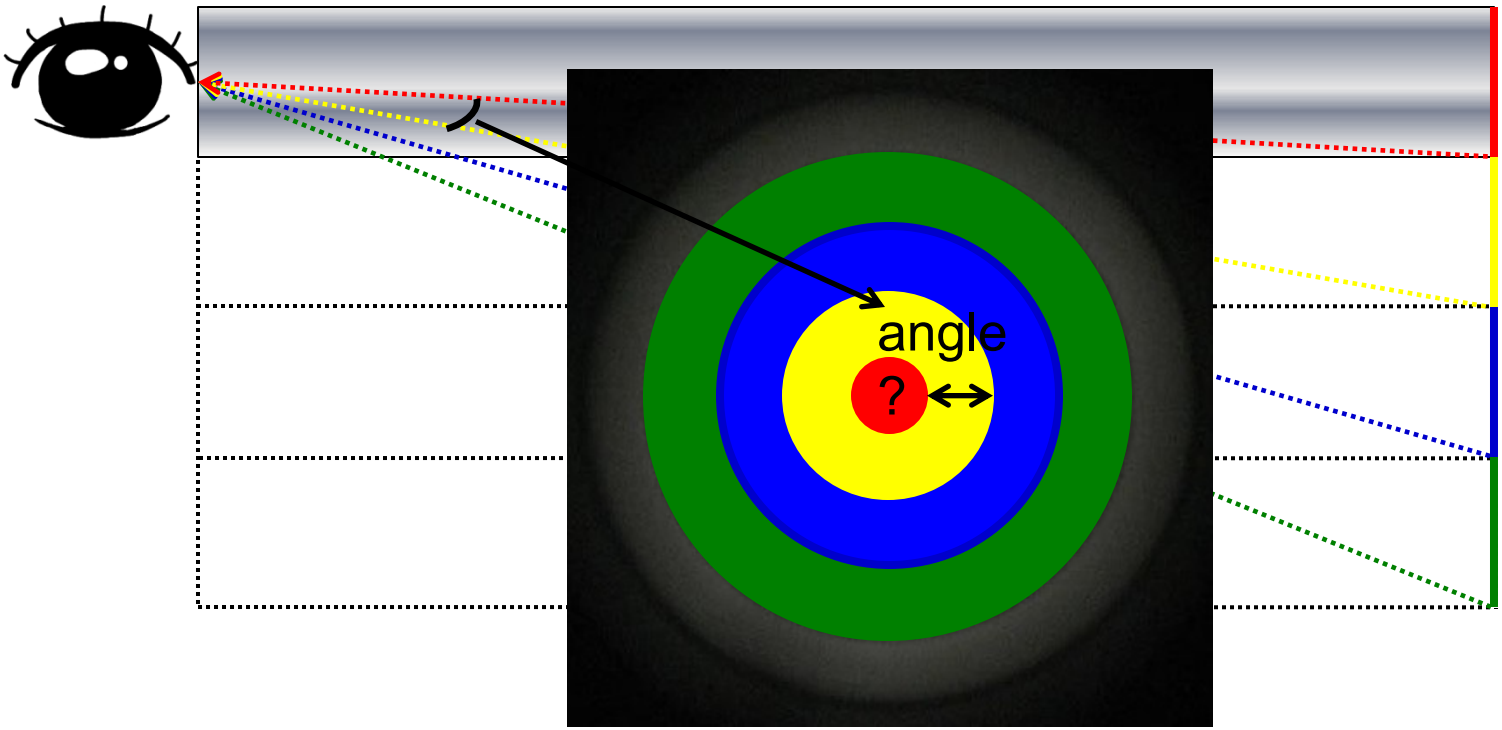
n-th light band reflects n times!

- **Summary of Qualitative Approach**

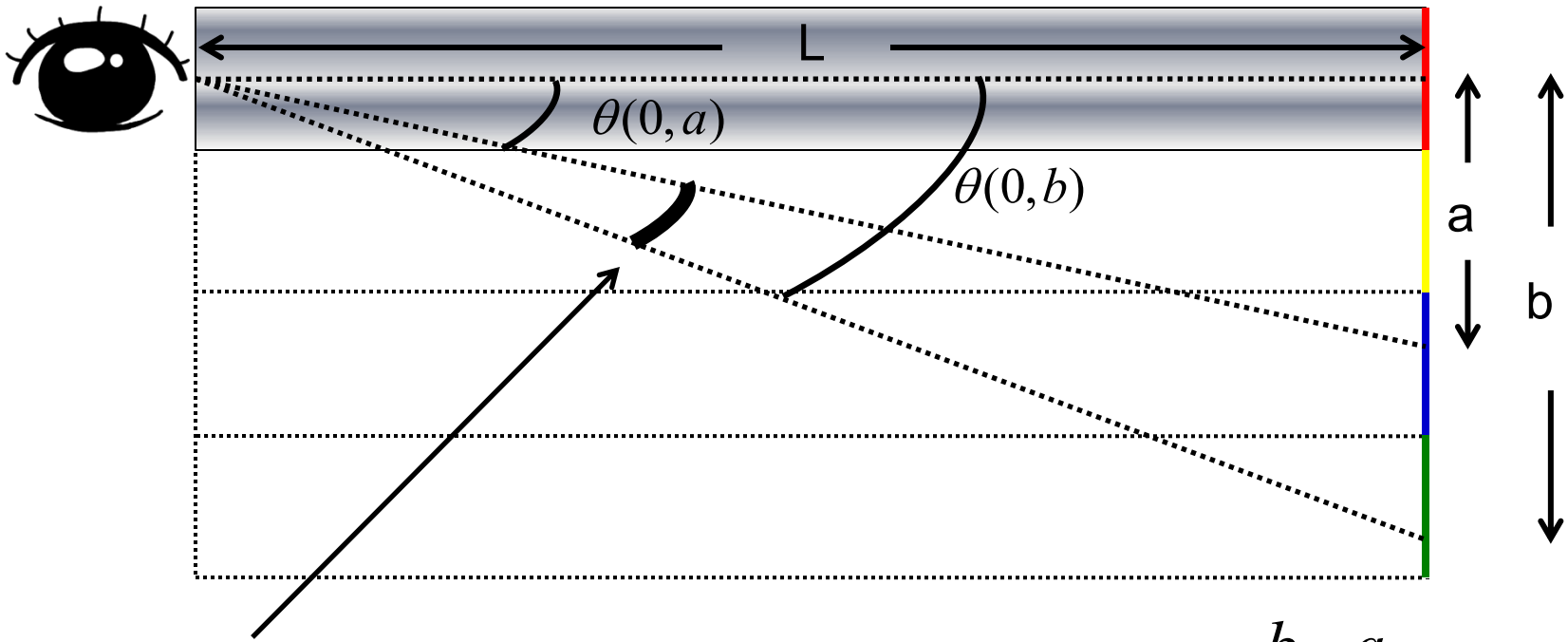
1. Edge of the metal tube causes the dark bands to be formed.

2. Longer path and reflection loss results in the decrease of brightness.

- **Thickness of Light Bands**

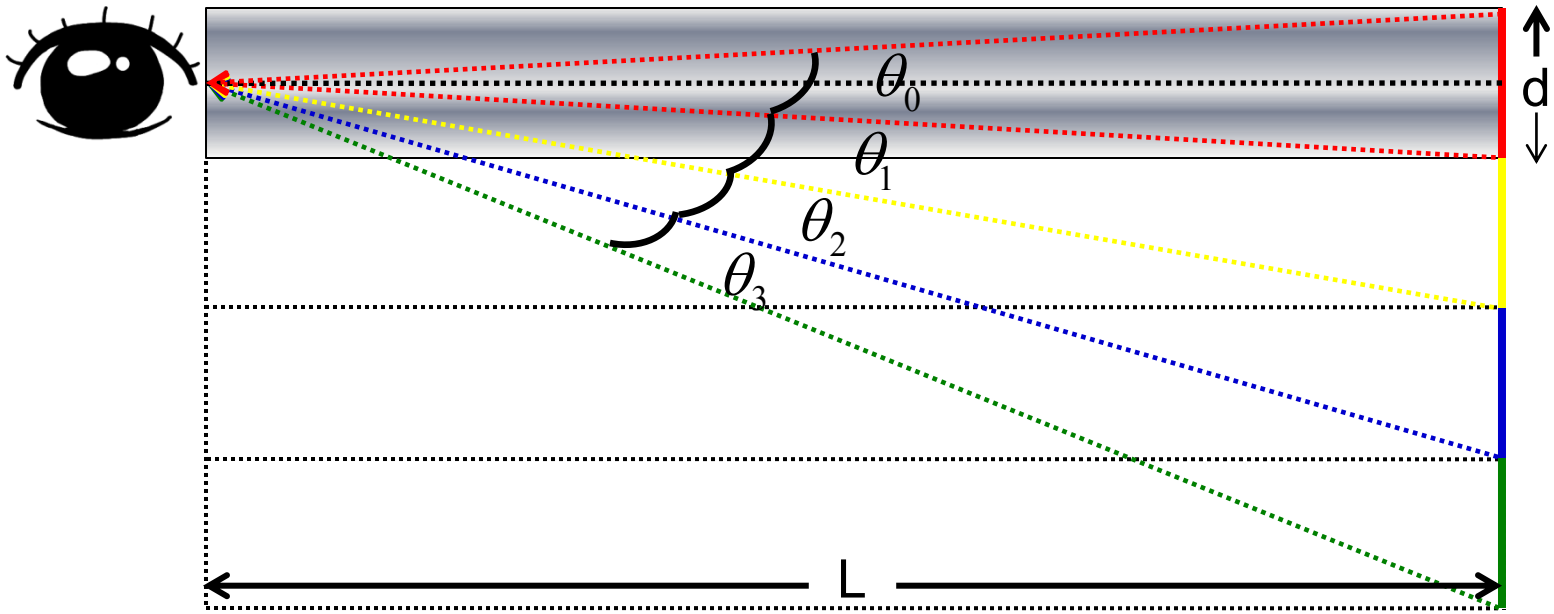


- **Thickness of Light Bands**



$$\theta(a,b) = \theta(0,b) - \theta(0,a) = \arctan(\frac{b}{L}) - \arctan(\frac{a}{L}) = \arctan(\frac{b-a}{L+\frac{ab}{L}})$$

- **Thickness of Light Bands**



Definition of θ_n : thickness of n-th light band in angle

Theoretically, $\theta_n = \arctan\left(\dfrac{1}{\dfrac{L}{d} + \dfrac{d}{L}\dfrac{(2n-1)(2n+1)}{4}}\right)$ (n=0,1,2...)

- **Thickness of Light Bands**

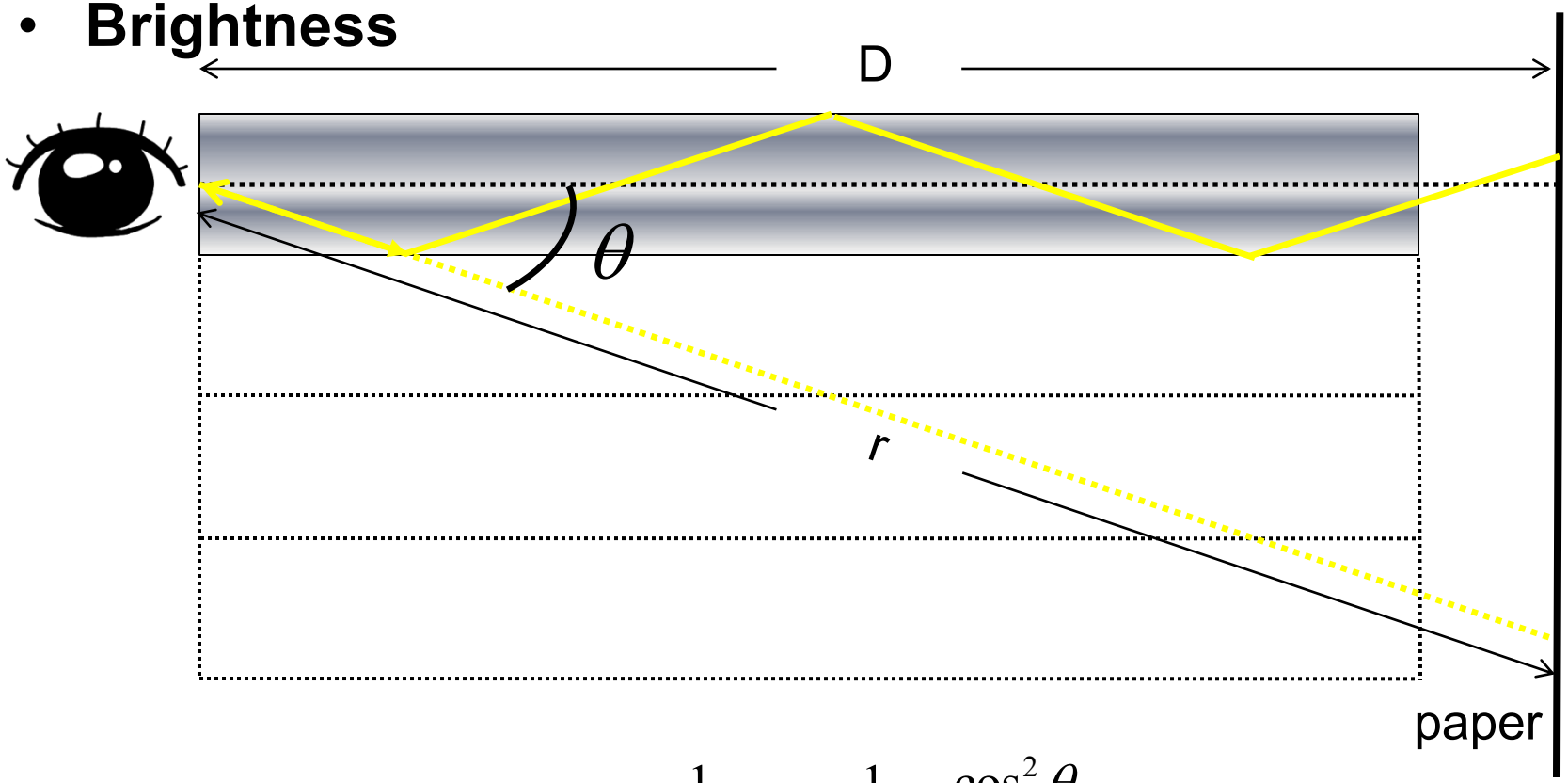
when $L >> d$,

$$\theta_n \approx \frac{d}{L} - \frac{d^3}{L^3}\frac{(2n-1)(2n+1)}{4}$$

If n is not sufficiently big, then

$$\theta_n \approx \frac{d}{L} \text{ (constant)}$$

- **Brightness**



$$\text{intensity} \propto \frac{1}{\text{distance}^2} = \frac{1}{r^2} = \frac{\cos^2\theta}{D^2} \propto \cos^2\theta$$

EXPERIMENT - OUTLINE

Republic of Korea

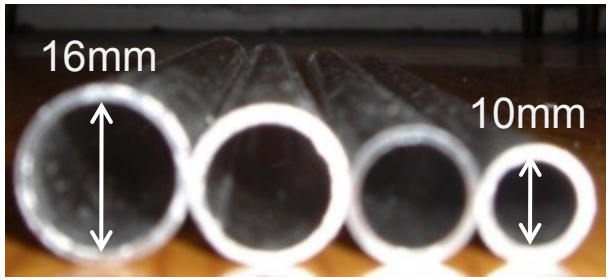
- **SUBJECTS FOR EXPERIMENT**
  - CAUSE OF DARK BANDS
  - THICKNESS OF LIGHT BANDS
  - BRIGHTNESS OF THE PATTERN

EXPERIMENT - OUTLINE

Republic of Korea

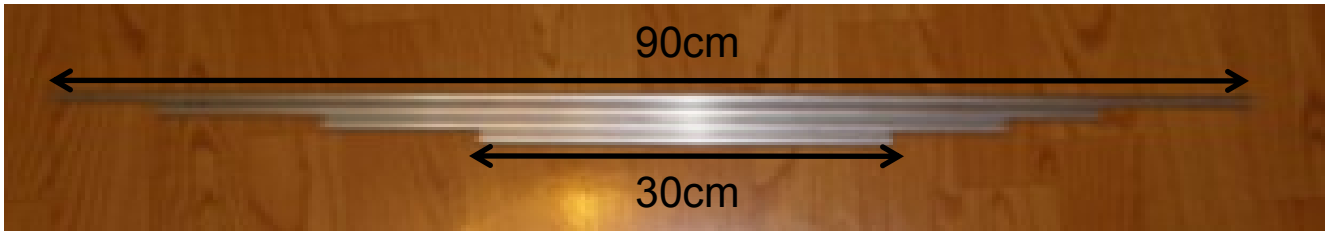
- **Tubes used (aluminum)**
  - Variable : Diameter

    Diameter 10, 12, 14, 16 mm Length 30 cm



  - Variable : Length

    Length 30, 50, 70, 90 cm Diameter 10 mm



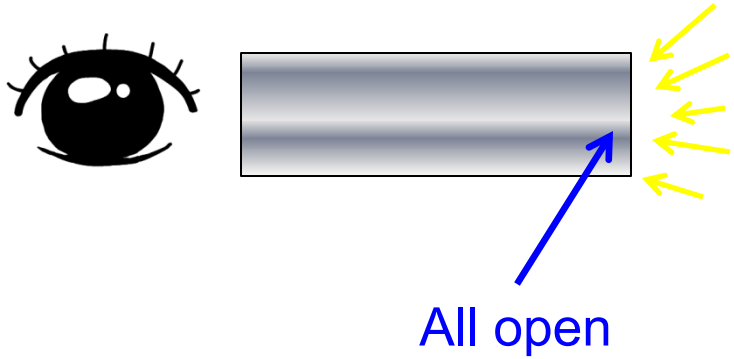
- **EXPERIMENT 1 – CAUSE OF DARK BANDS**

+

sand paper

=?

- **EXPERIMENT 1 – CAUSE OF DARK BANDS**



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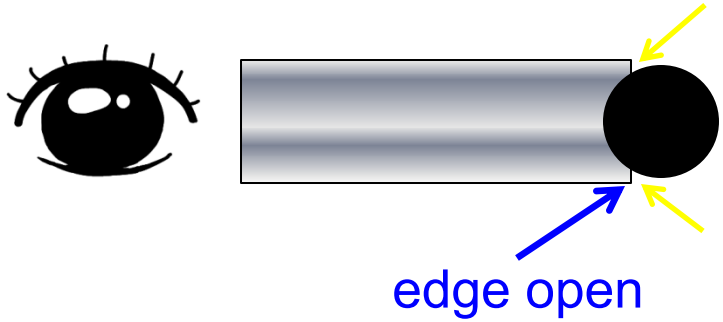

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EXPERIMENT

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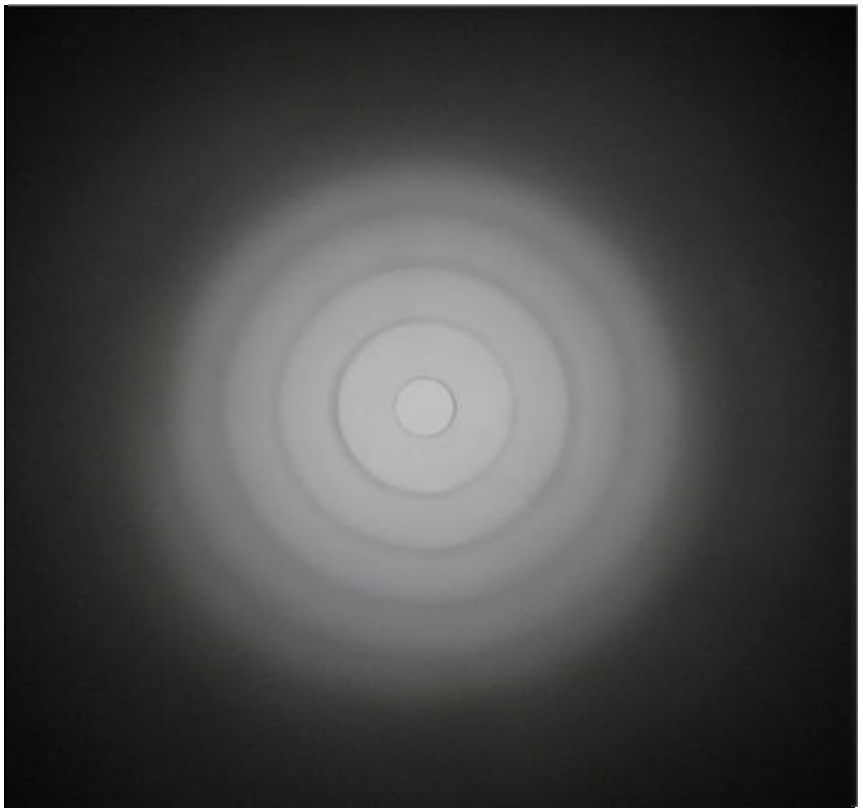
- **EXPERIMENT 1 – CAUSE OF DARK BANDS**



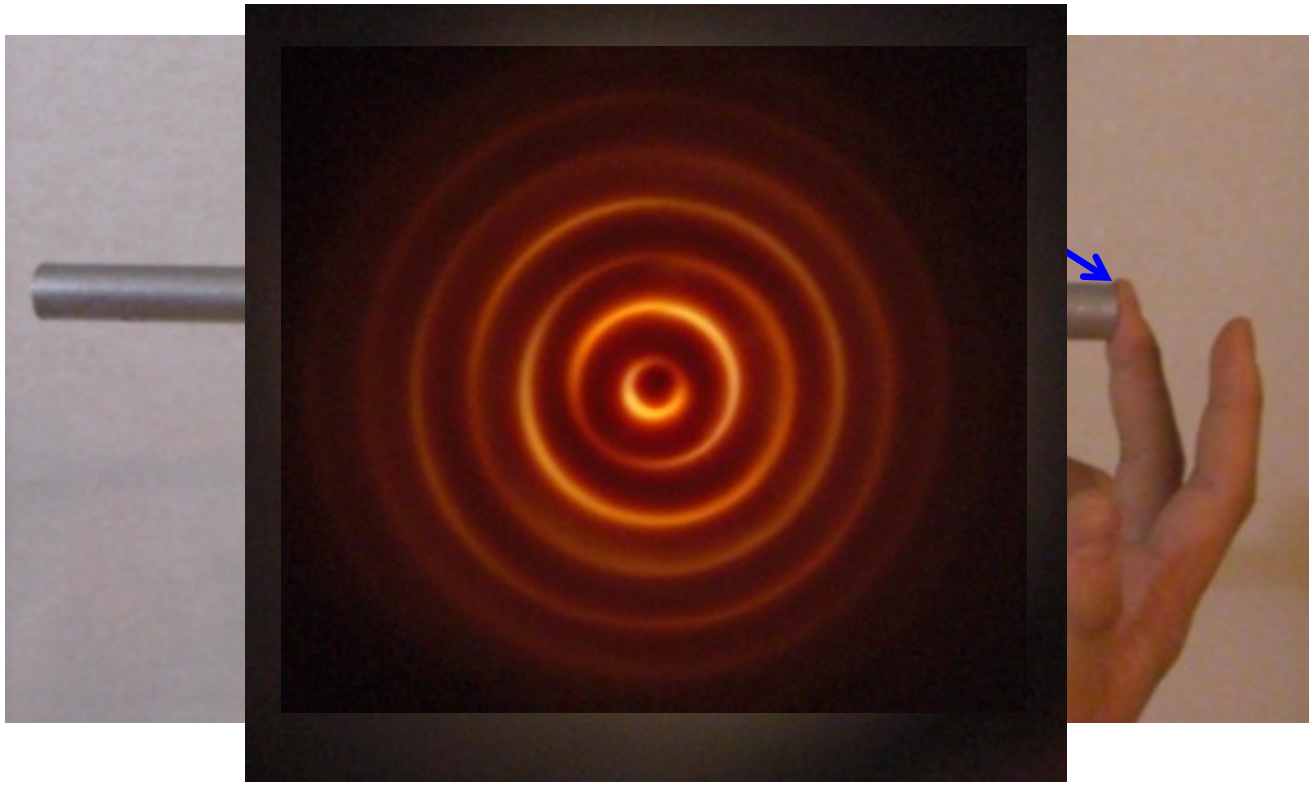
150 times of grinding

Dark bands
more distinct!

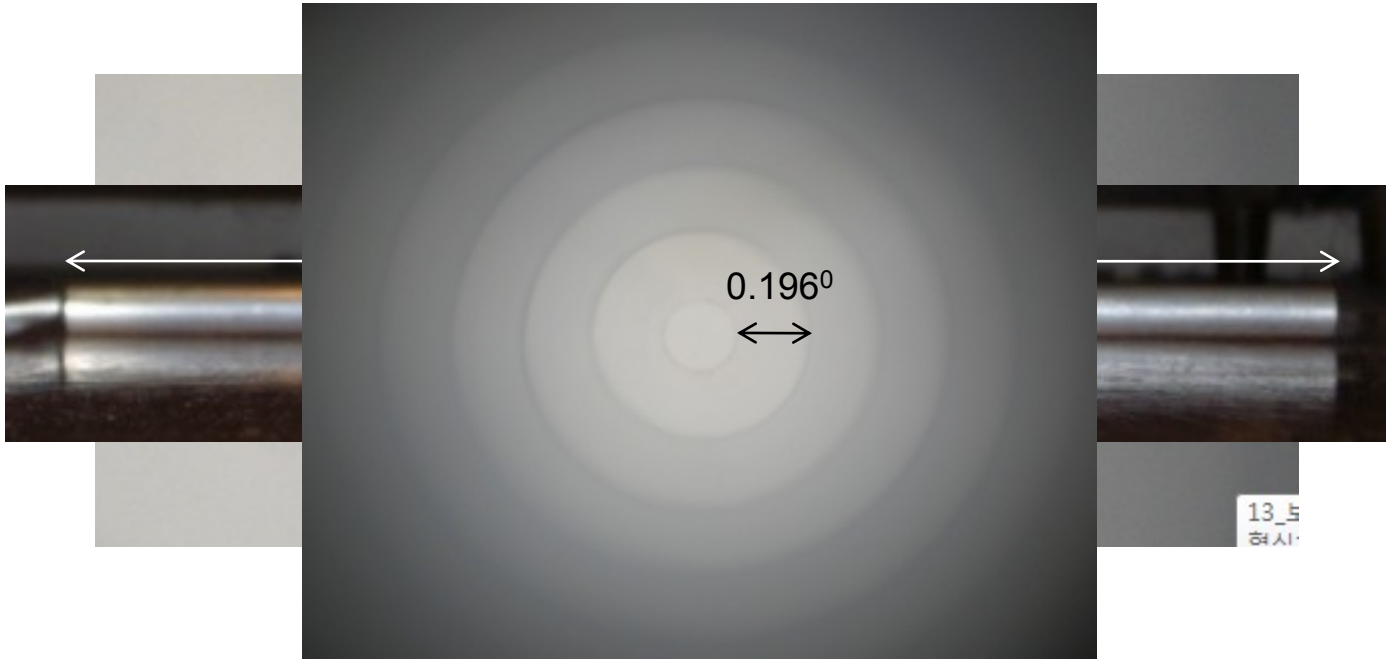
EXPERIMENT

Republic of Korea

- **EXPERIMENT 1 – CAUSE OF DARK BANDS**



- **EXPERIMENT 2 – THICKNESS OF LIGHT BAND**



- **EXPERIMENT 2 – THICKNESS OF LIGHT BAND**
  - **Variable : diameter . part1**

d = 8mm	n	Empirical theta n (degrees)	Theoretical theta n (degrees)	Error(%)
	0	0.02664	0.02666	0.075
	1	0.02654	0.02660	0.239
	2	0.02619	0.02638	0.717
	3	0.02595	0.02601	0.243
	4	0.02602	0.02552	1.968
d = 10mm				
	0	0.03344	0.03334	0.3
	1	0.03264	0.03321	1.717
	2	0.03221	0.03278	1.724
	3	0.03198	0.03207	0.295
	4	0.03073	0.03114	1.325

- **EXPERIMENT 2 – THICKNESS OF LIGHT BAND**
  - **Variable : diameter . part2**

d = 12mm	n	Empirical theta n (degrees)	Theoretical theta n (degrees)	Error(%)
	0	0.03814	0.04000	4.65
	1	0.03859	0.03979	3.011
	2	0.03710	0.03904	4.976
	3	0.03690	0.03786	2.537
	4	0.03697	0.03632	1.786
d = 14mm				
	0	0.04620	0.04666	0.493
	1	0.04467	0.04516	1.084
	2	0.04445	0.04325	2.782
	3	0.04245	0.04333	2.041
	4	0.04079	0.04101	0.545

- **EXPERIMENT 2 – THICKNESS OF LIGHT BAND**
  - **Variable : length . part1**

L = 30cm	n	Empirical theta n (degrees)	Theoretical theta n (degrees)	Error(%)
	0	0.016716	0.016665124	0.3036
	1	0.032638	0.033210376	1.7231
	2	0.032214	0.032775144	1.7133
	3	0.031979	0.032074558	0.2964
	4	0.030726	0.031142576	1.3376
L = 50cm				
	0	0.009826	0.009999667	1.7332
	1	0.019321	0.019973372	3.2678
	2	0.019782	0.019878097	0.4814
	3	0.019574	0.019721309	0.7479
	4	0.019151	0.019505914	1.8189

EXPERIMENT

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- **EXPERIMENT 2 – THICKNESS OF LIGHT BAND**
  - **Variable : length . part2**

L = 70cm	n	Empirical theta n (degrees)	Theoretical theta n (degrees)	Error(%)
	0	0.007091	0.007142736	0.726
	1	0.014288	0.014276003	0.0825
	2	0.014469	0.014241153	1.5968
	3	0.01388	0.014183446	2.136
	4	0.01349	0.014103437	4.3519
L = 90cm				
	0	0.005378	0.005555498	3.1987
	1	0.011757	0.011106541	5.8526
	2	0.010941	0.011090118	1.345
	3	0.011137	0.011062855	0.6661
	4	0.010802	0.011024912	2.0182

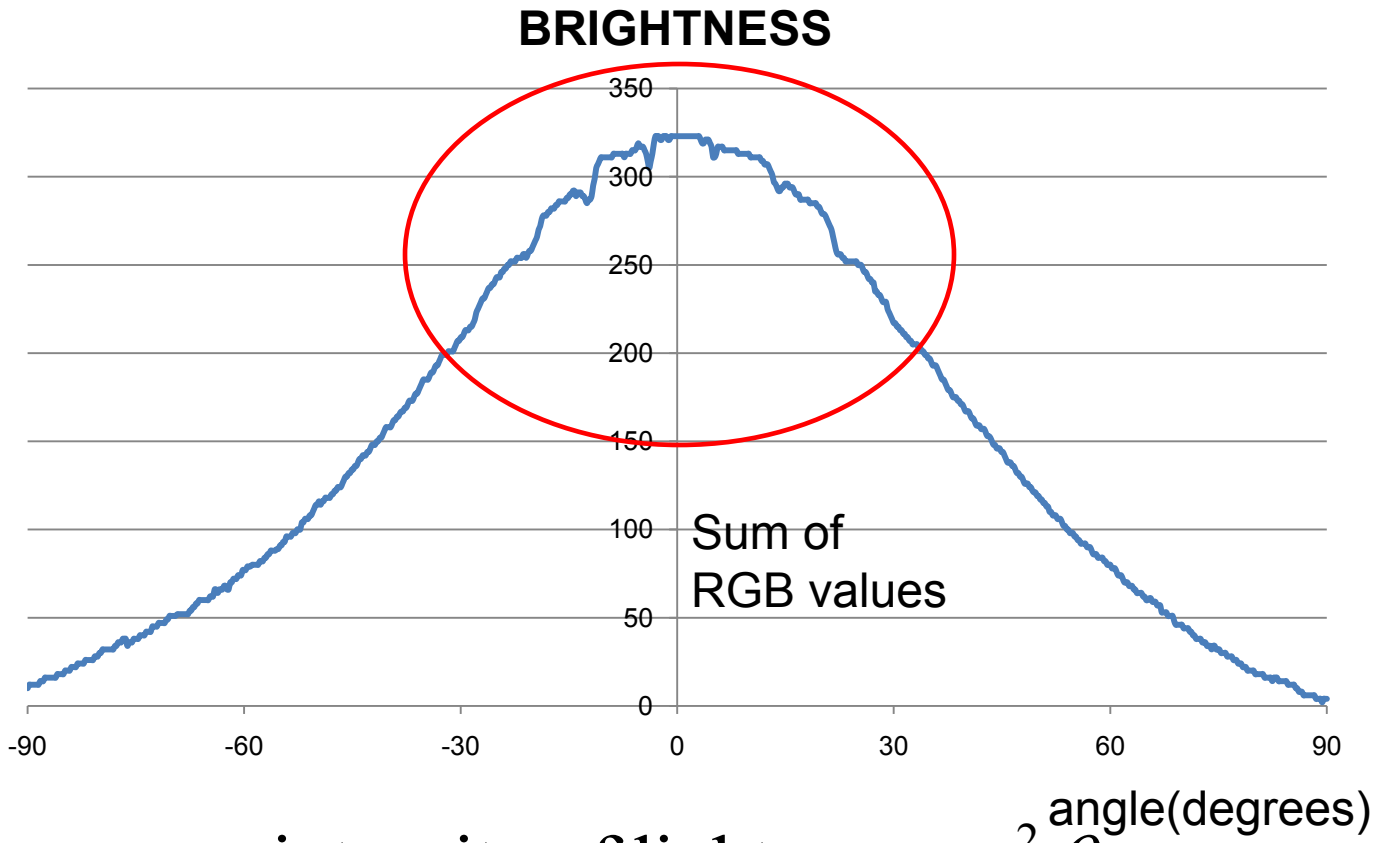
- **EXPERIMENT 3 – BRIGHTNESS OF THE PATTERN**

```java
import java.io.*;
import java.awt.*;
import javax.imageio.*;
import java.awt.event.*;
import java.awt.image.*;

public class Brightness
{
    BufferedImage img;
    private int width, height;
    private int[][] red, green, blue;
    private String filename;
    Color white = new Color(255,255,255);
    Color black = new Color(0,0,0);
    Color color1 = new Color(255,0,0);

    public Brightness(String name) {
        filename = name;
        try {
            img = ImageIO.read(new File(filename));
        } catch (IOException e) {
            System.out.println(e.getMessage());
        }

        BufferedImage copy = img;
        width = img.getWidth(); height = img.getHeight();
        setGrid();
    }
    private void setGrid() //brings the rgb values
    {
        red = new int[width][height];
        green = new int[width][height];
        blue = new int[width][height];

        for (int i = 0; i < width; i++)
        {
            for (int j = 0; j < height; j++)
            {
                Color color = new Color(img.getRGB(i, j));
```

Sum of RGB values

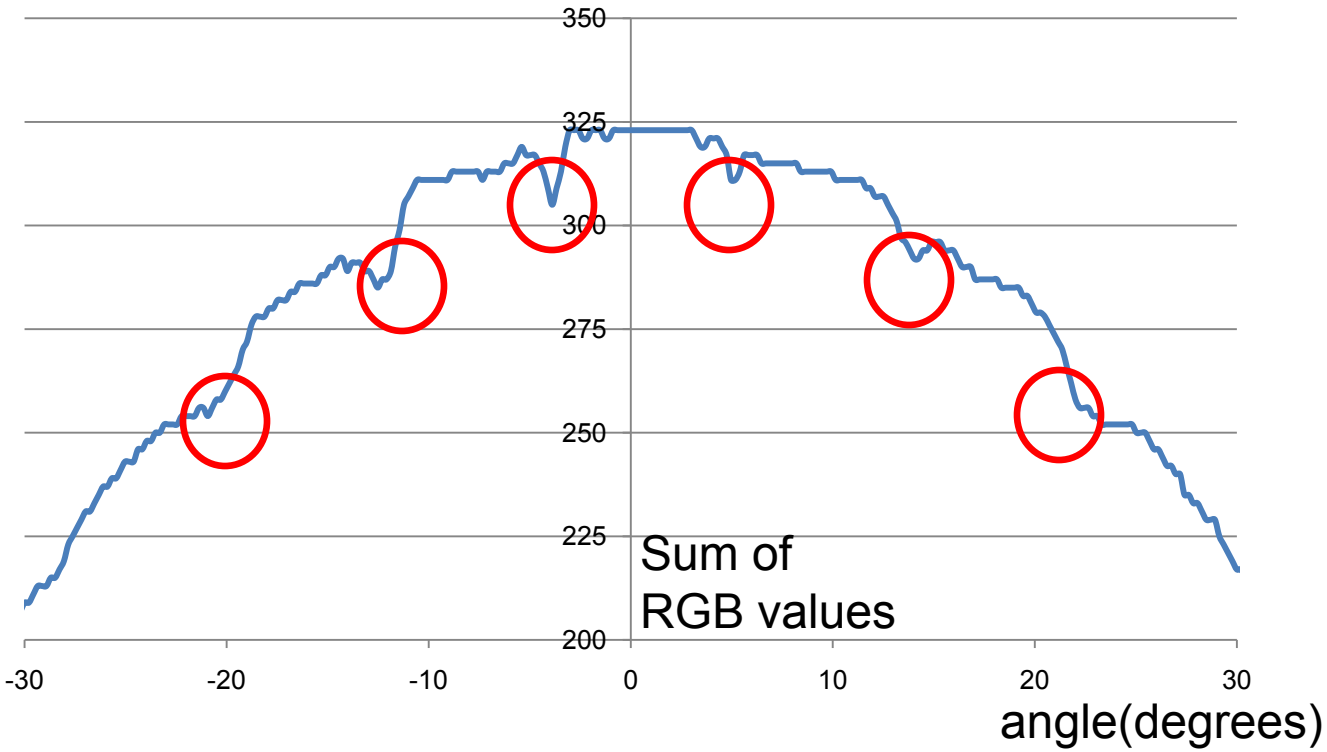
Brightness on center line

- **EXPERIMENT 3 – BRIGHTNESS OF THE PATTERN**



$$\text{intensity of light} \propto \cos^2\theta$$

- **EXPERIMENT 3 – BRIGHTNESS OF THE PATTERN**



DIFFERENT TUBES

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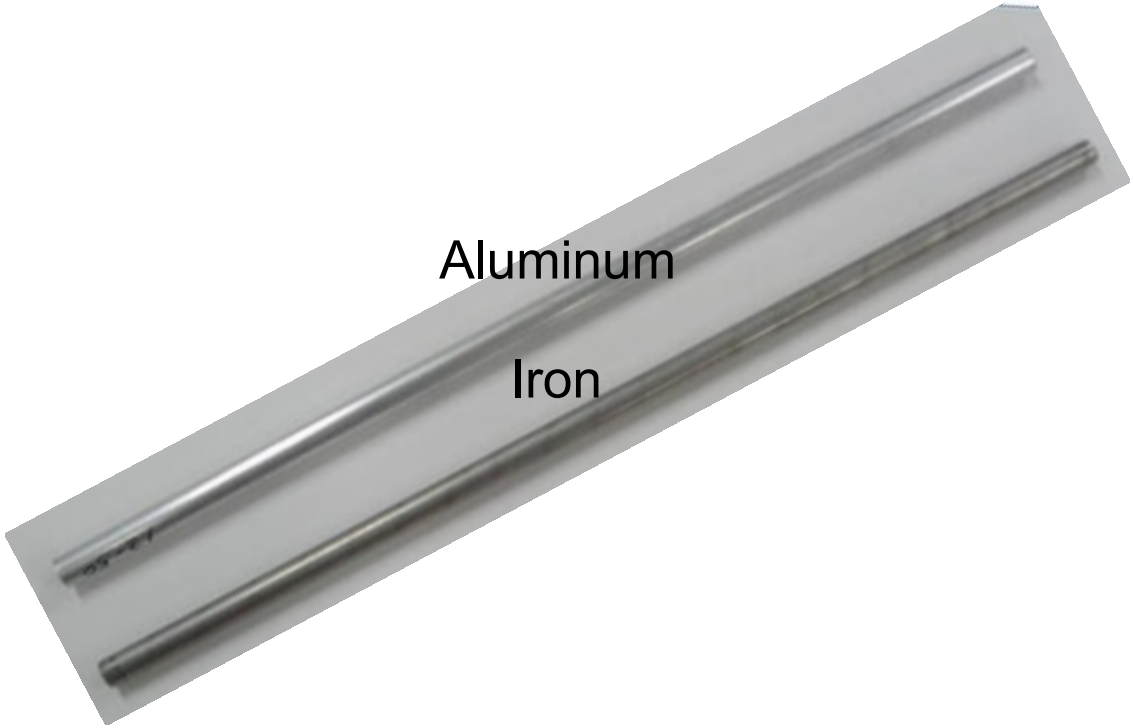
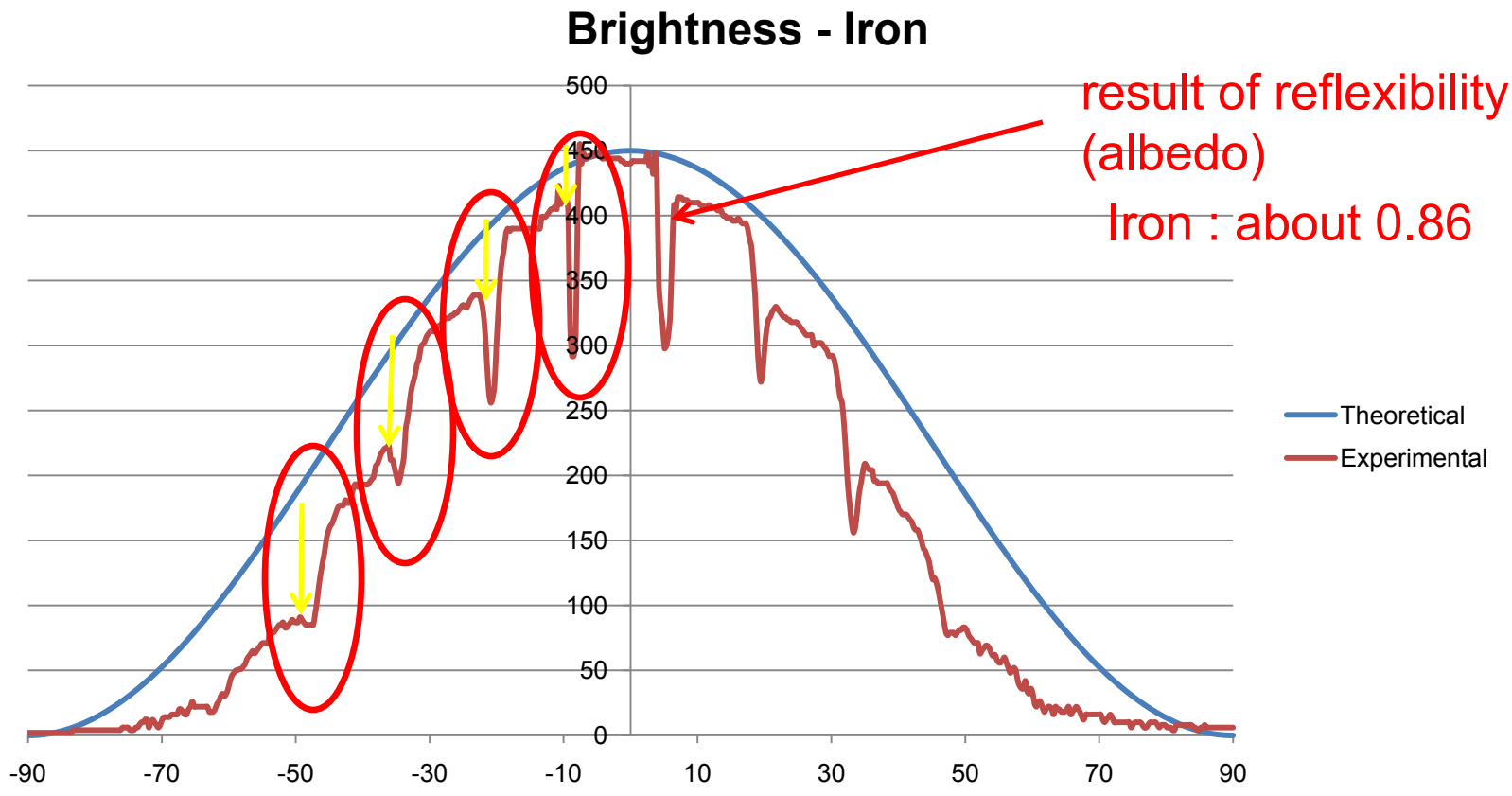
- **Tubes used**
  - Variable : Types of metal



DIFFERENT TUBES

Republic of Korea

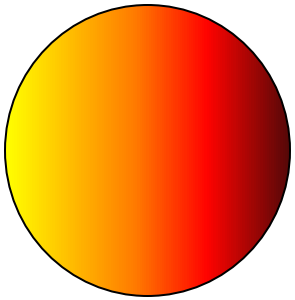
- **Iron tube**

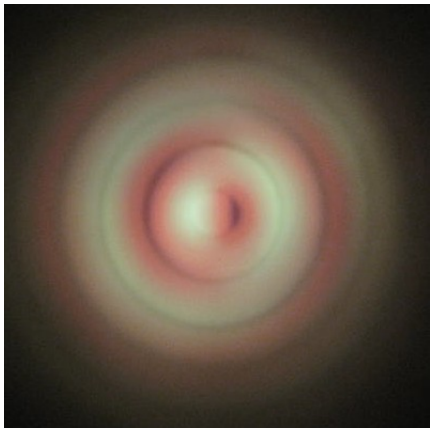


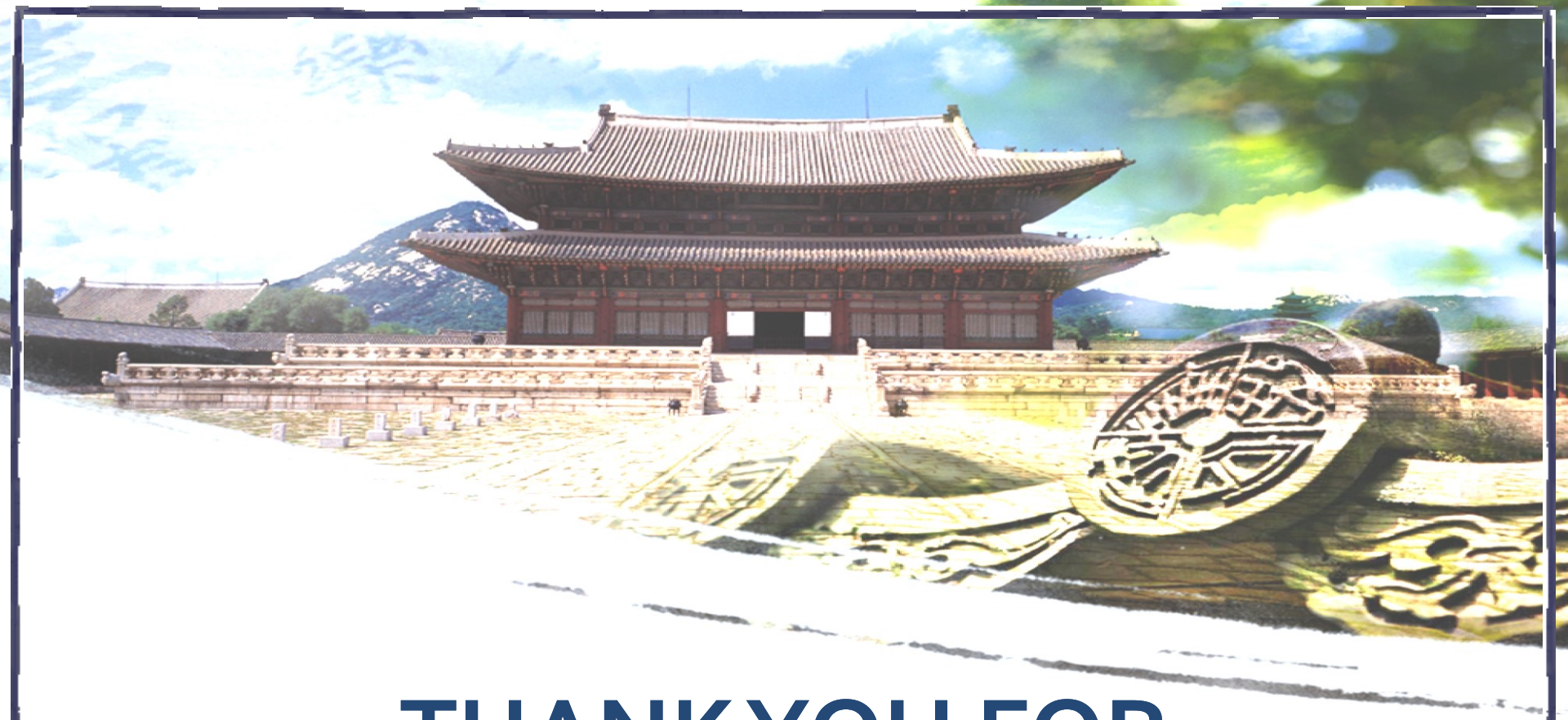
CONCLUSION

Republic of Korea

- ***1. Phenomenon related to reflection inside the metal tube.***
- ***2. Edge of the metal tube causes the dark bands to be formed.***
- ***3. Light bands have similar thickness near the center, and get darker as it gets farther from the center, because the length of path of light increases.***







THANK YOU FOR YOUR ATTENTION!

EQUATIONS

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when $L >> d$, which means $\frac{L}{d} >> 1$ and $\frac{d}{L} << 1$, then

$$\theta_n = \arctan\left(\frac{1}{\frac{L}{d} + \frac{d}{L}\frac{(2n-1)(2n+1)}{4}}\right) \approx \frac{1}{\frac{L}{d} + \frac{d}{L}\frac{(2n-1)(2n+1)}{4}}$$

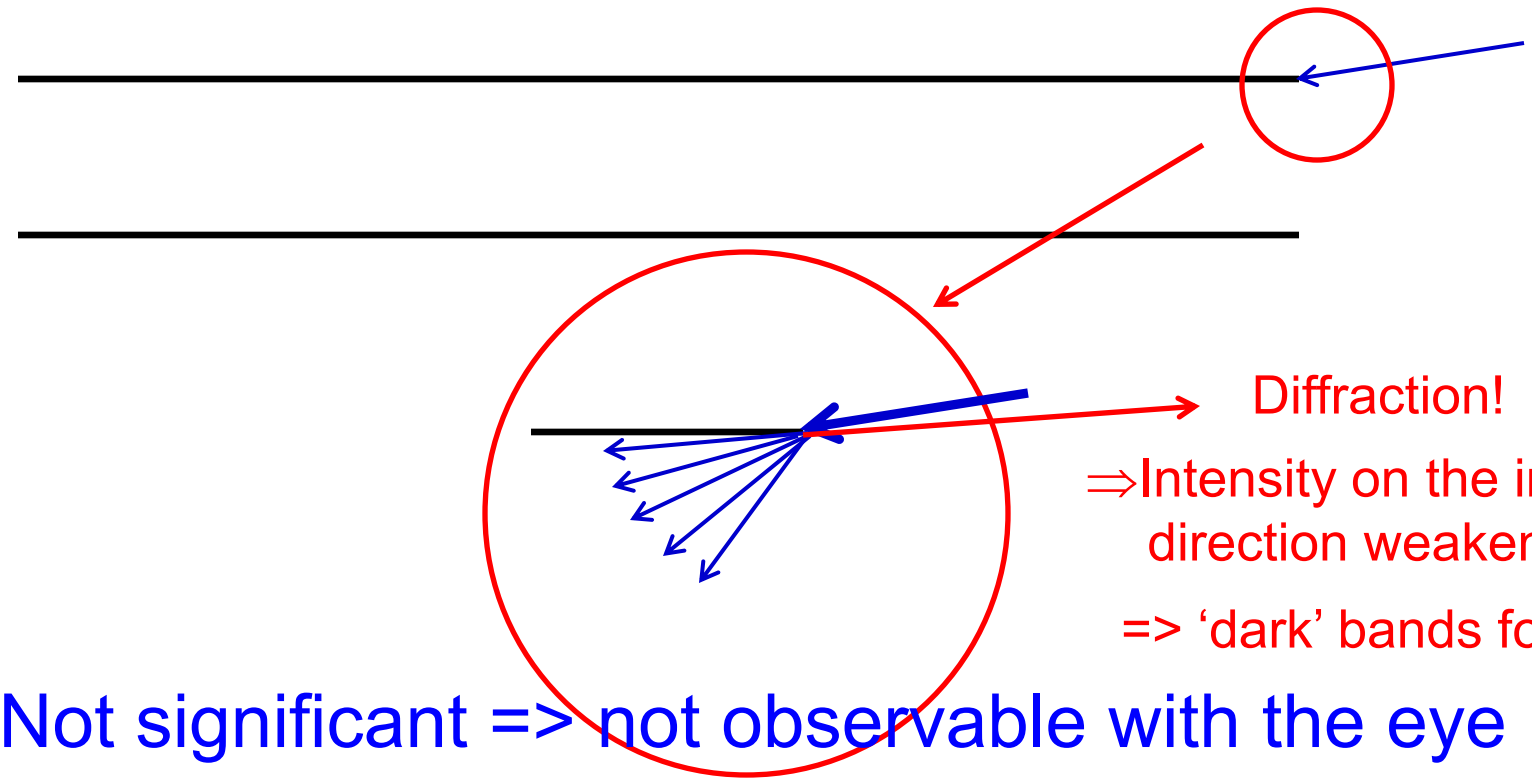
$$= \frac{d}{L}\left(\frac{1}{1 + \frac{d^2}{L^2}\frac{(2n-1)(2n+1)}{4}}\right) \approx \frac{d}{L}\left(1 - \frac{d^2}{L^2}\frac{(2n-1)(2n+1)}{4}\right)$$

$$= \frac{d}{L} - \frac{d^3}{L^3}\frac{(2n-1)(2n+1)}{4}$$

EDGE

Republic of Korea

- **ANOTHER REASON**
  - **Diffraction at the edge**
    - **Dark bands should be made in principle**
    - **Not observable with eye**

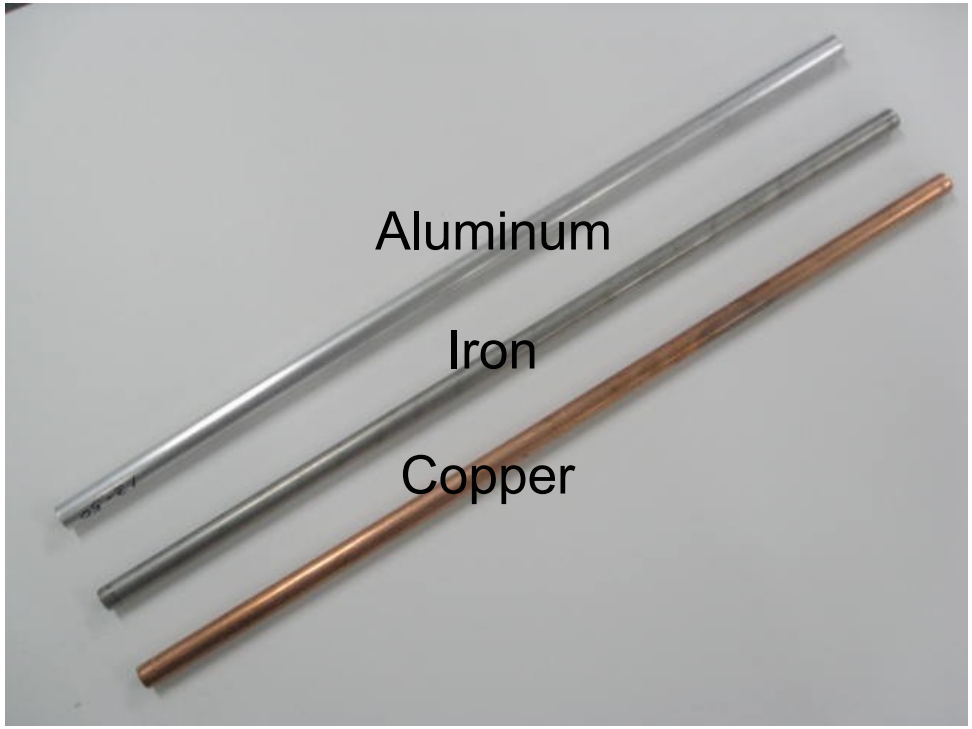


Not significant => not observable with the eye

DIFFERENT TUBES

Republic of Korea

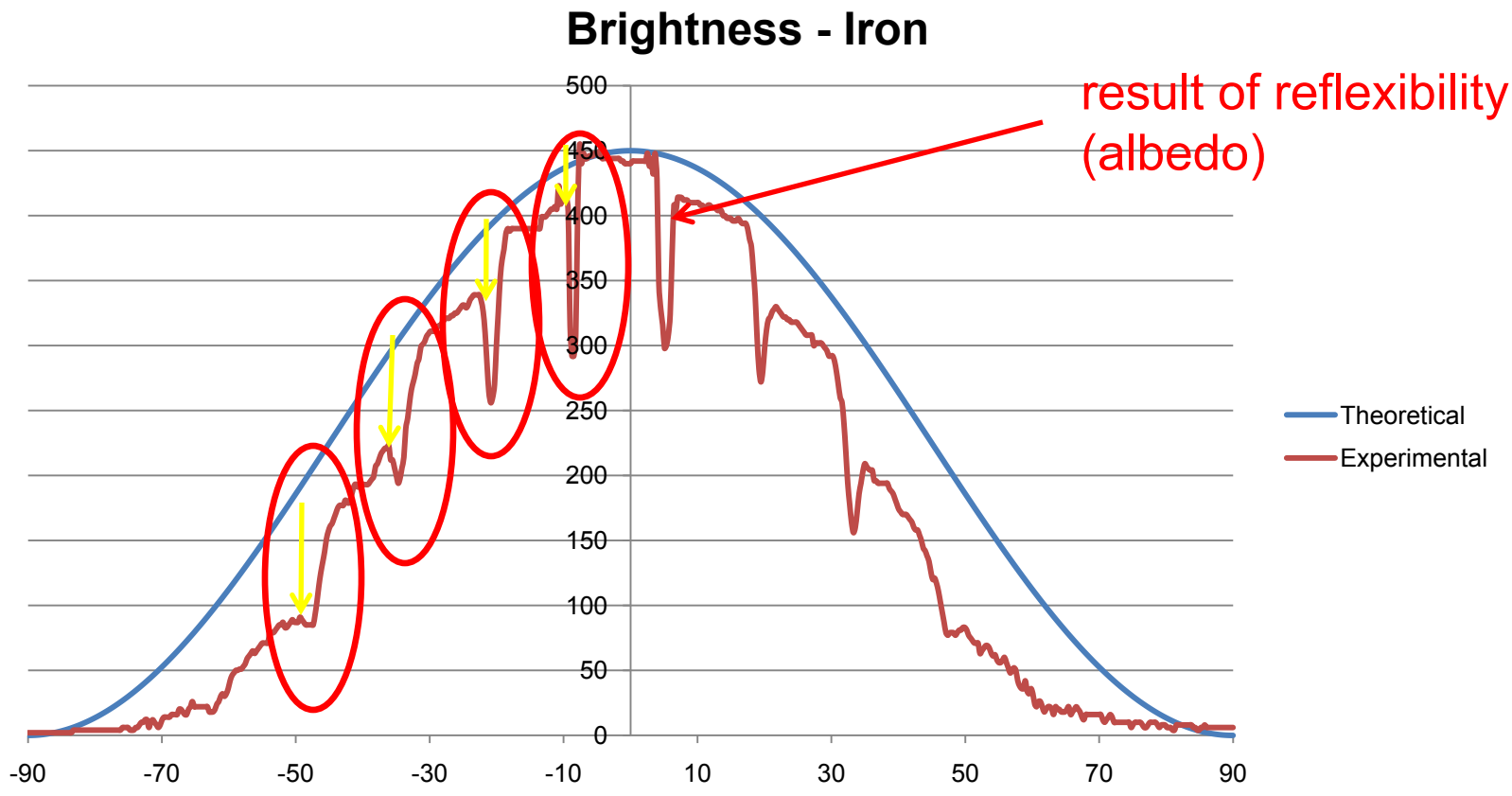
- **Tubes used**
  - Variable : Types of metal



DIFFERENT TUBES

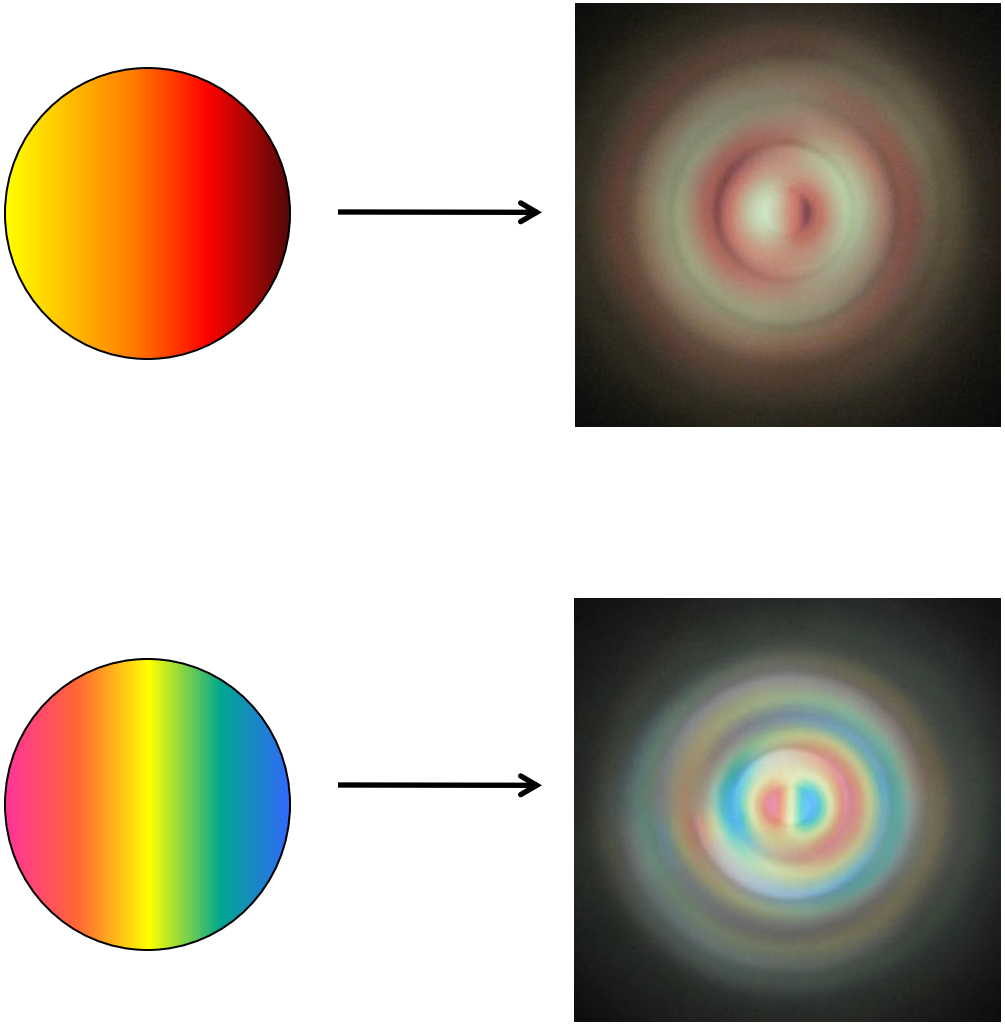
Republic of Korea

- **Iron tube**



DIFFERENT PATTERNS

Republic of Korea



EYE'S POSITION

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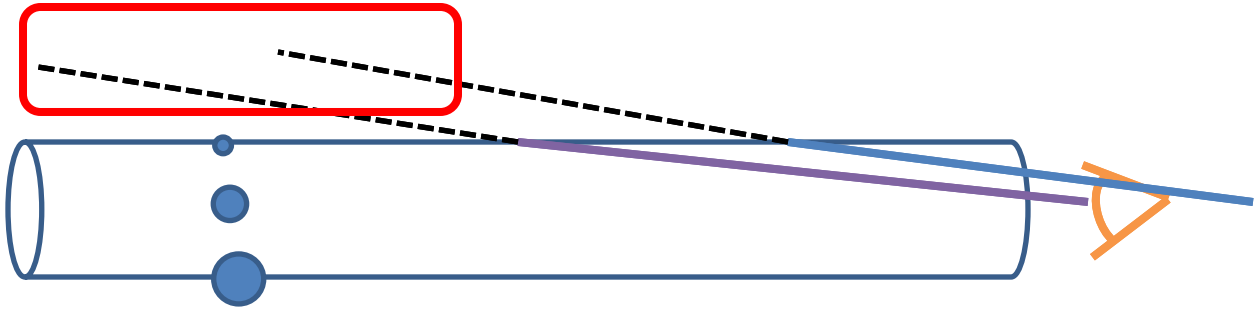
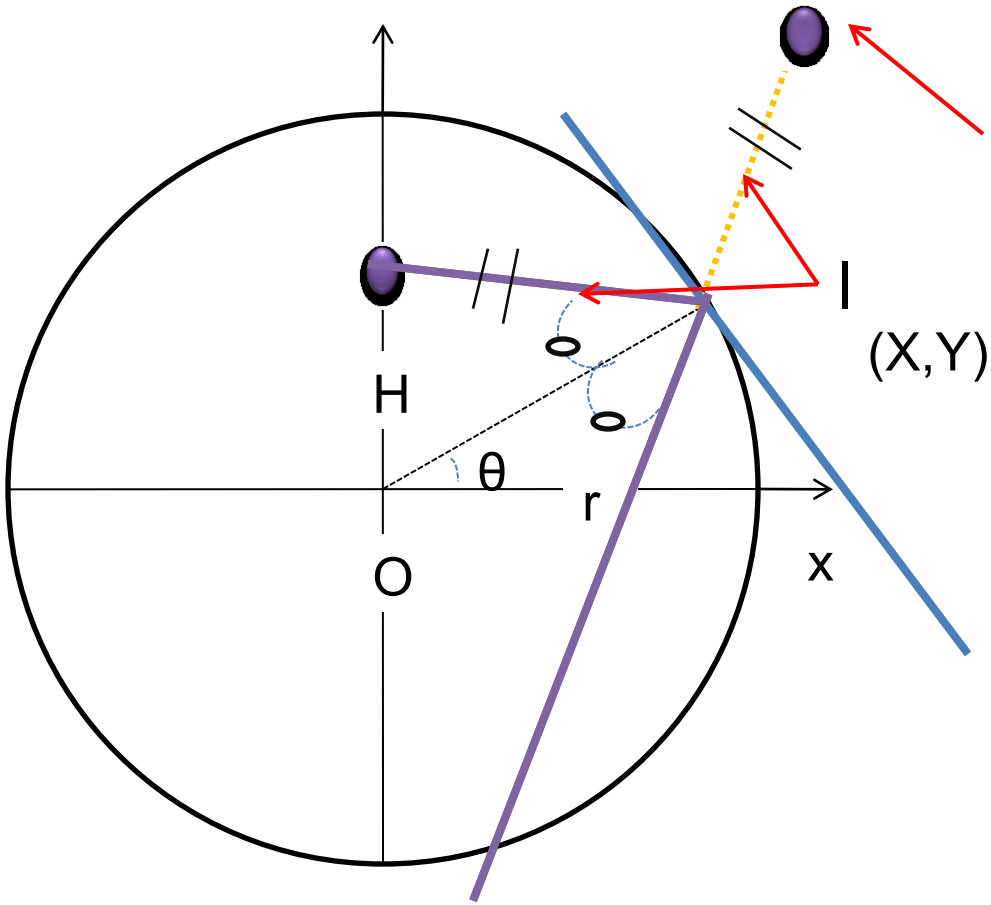


Ring Shifts

HEART SHAPE



(X,Y) = (rcosθ, rsinθ)

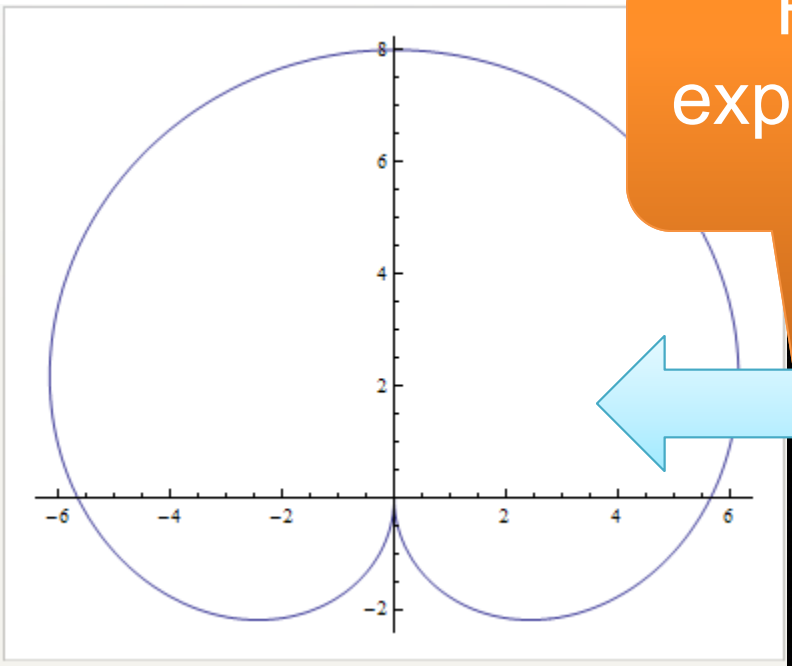
$$0 \le \theta \le 2\pi$$

$$P(x,y) = (X(\theta), Y(\theta))$$

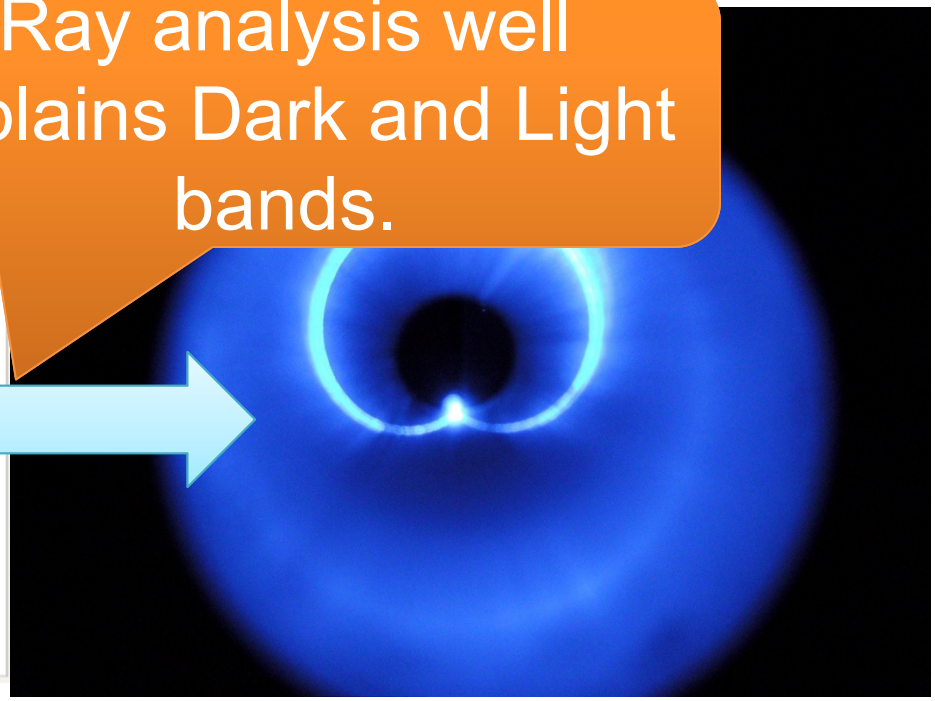
HEART SHAPE

Republic of Korea

Theoretical Graph vs. Photo

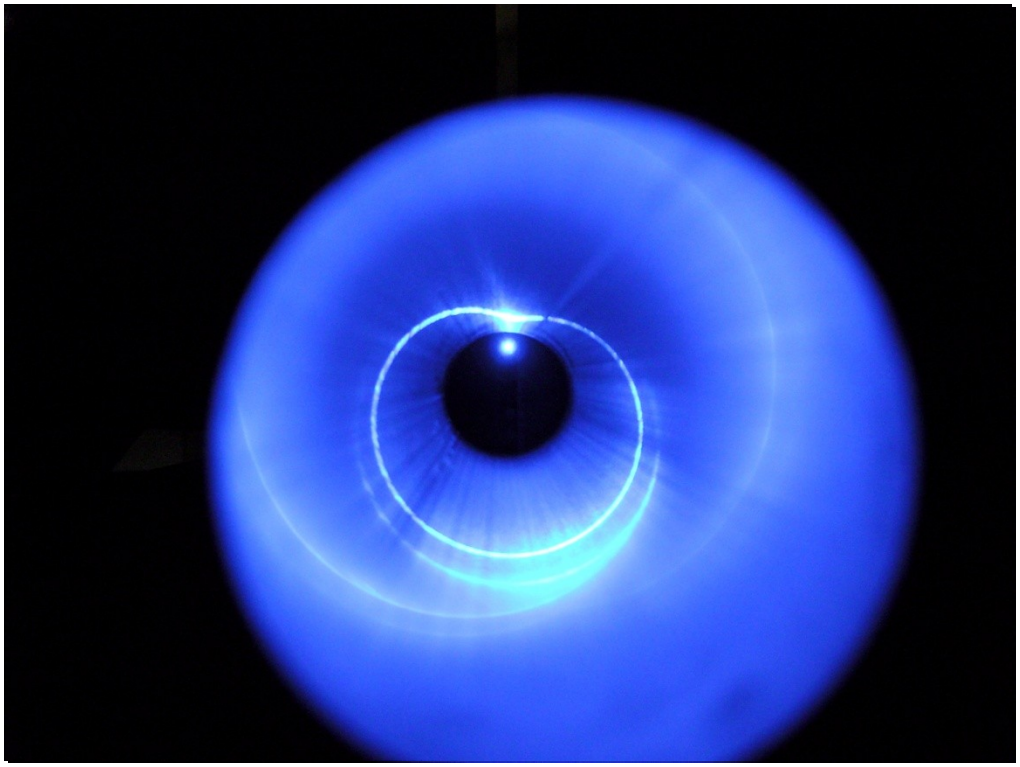


Ray analysis well explains Dark and Light bands.



Radius = 2cm,
H = -2cm

Radius = 2cm,
H = -2cm

HEART SHAPE

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