15. Invent Yourself: Fractals

REPORT

Greece - Anatolia High School

Hellenic Physical Society

I.Y.N.T. - Minsk 2019

Problem to be investigated

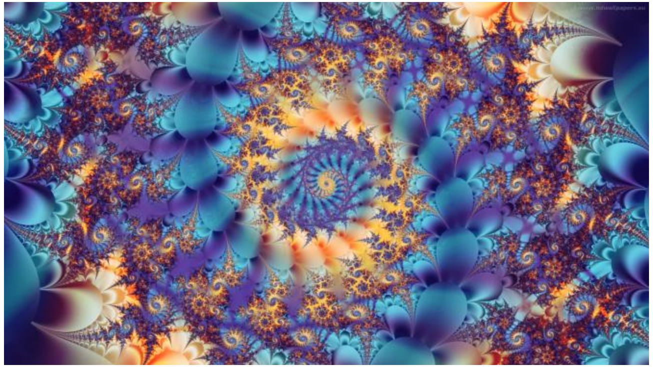
Propose an interesting experimental and theoretical investigation involving fractal geometry.

Investigate the structure and production of fractal patterns in plants.

Contents

Introduction

- Fractals are a relatively new and promising field in modern geometry.
- Although difficult to represent and elusive in their properties, fractals can lead to a better understanding of the world we live.



Fractal Geometry

- **Fractals** (although they may look like it) are not shapes.
- They are defined to be a subset of Euclidean geometry space, with the exception of their dimension being an irrational number.

Fractal properties

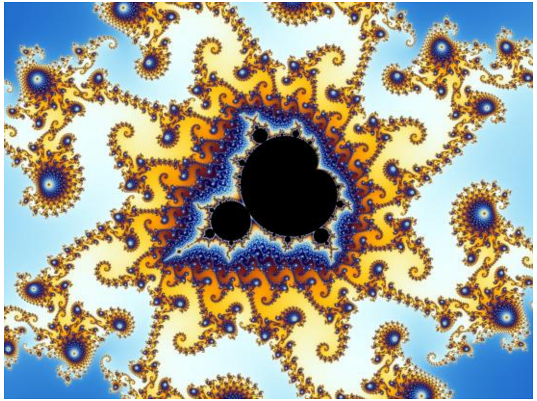
- **Self-similarity in any of its forms is one of the most recognizable attribute of fractals.**
- **Relatively simple and recursive definition**

Fractal Generation Techniques

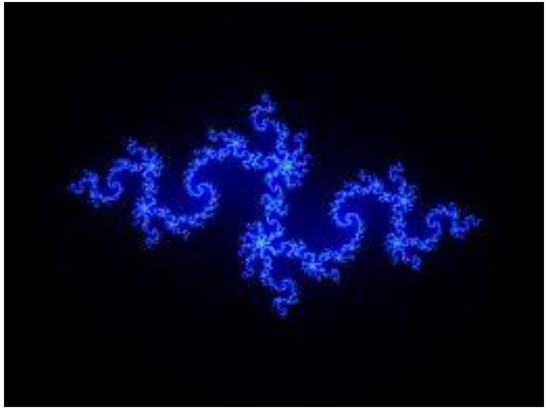
- Escape Time
- Iterated Function Systems (IFS)
- L - systems

Escape - Time

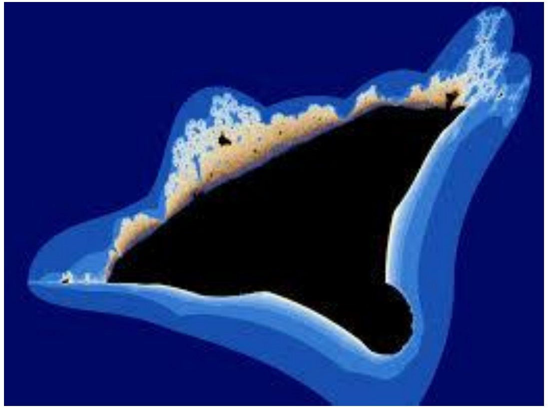
- A formula is applied to every point in a space.
- They tend to be quasi-self-similar: in different scales the pattern does not appear identical.



Mandelbrot Set



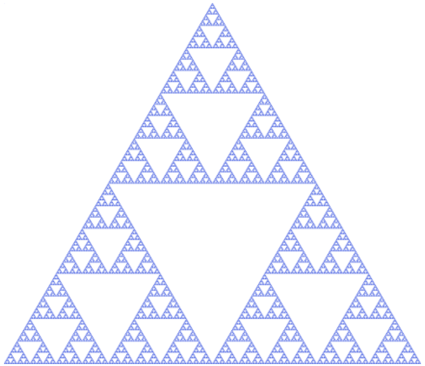
Julia Sets



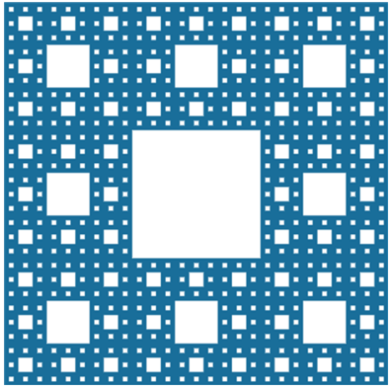
Burning Ship

Iterated Function Systems - IFS

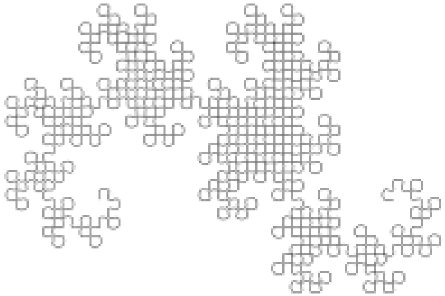
- They are generated by multiple iterations of a function.
- They are made of copies of themselves.



Sierpinski Gasket



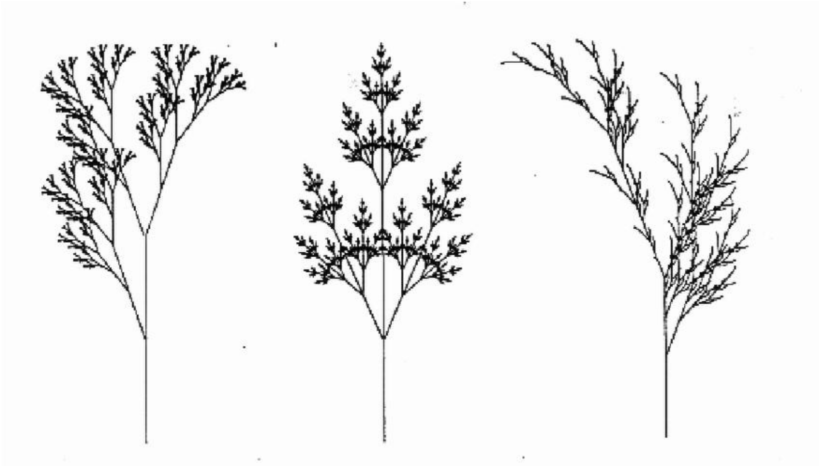
Sierpinski Carpet



Dragon Curve

L-systems

- A type of rewriting system and a formal grammar.
- Its components are
  - a set of symbols,
  - a starting word, and
  - a set of production (rewriting) rules for new words to be generated.
- Fractals can be drawn using specific words and rules, because the rewriting can lead to iteration. *The symbols are then translated into drawing commands.*



Fractal patterns in nature

- Fractal-like patterns can be spotted in nature.
- Many natural constructions resemble or can be modeled using fractals:
  - branching patterns,
  - vascular networks,
  - leaves,
  - mountain landscapes
  - clouds
  - lightning bolts
  - Inflorescence patterns

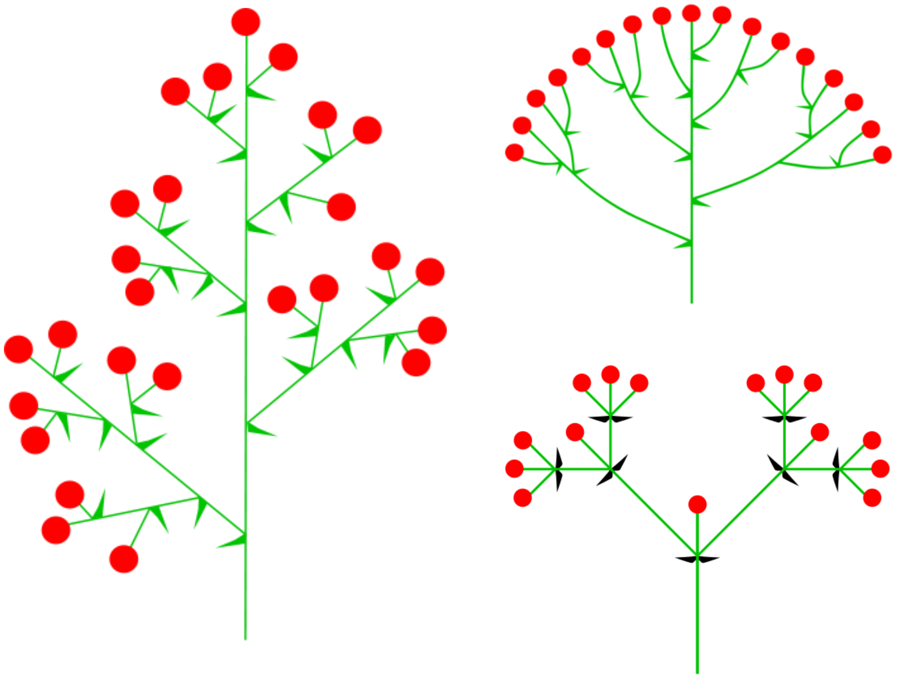
Fractals in Plants

There are various plants, trees and herbs that sport fractal-like patterns.

However, those patterns can not be named “fractals” because they do not continue to be self similar in every scale.

Inflorescence and fractals

- An inflorescence is a group of flowers arranged on a stem comprised of a central branch or an arrangement of branches.
- Many inflorescence patterns resemble fractals.



Experiment

Hypothesis

Inflorescence of

- Bougainvillea and
- Tomato

follow fractal-like patterns.

Experimental procedure

- We observed a **Bougainvillea** glabra plant and a **Solanum lycopersicum (tomato)** plant.
- We noted their inflorescences and made schematic representations of them.
- Finally, we developed L-system code to represent the main inflorescence patterns of the plants.

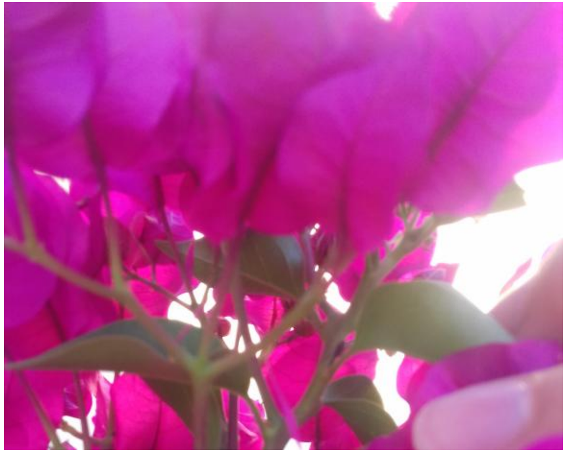
Bougainvillea

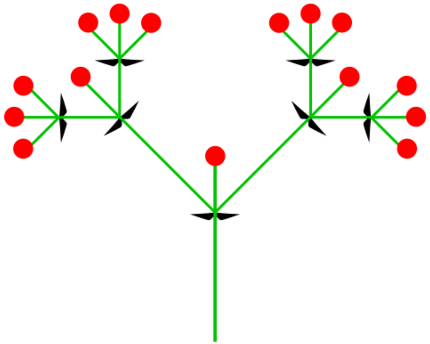
B. glabra (paper flower) has a dichasium cymose inflorescence.

The bracts surrounding the flower will be considered a part of the flower.

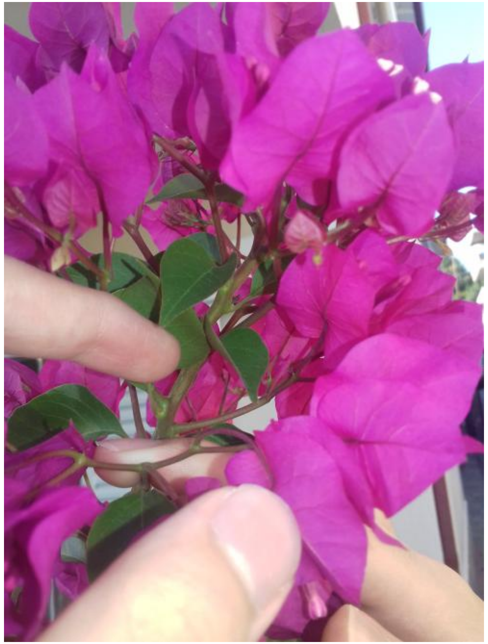
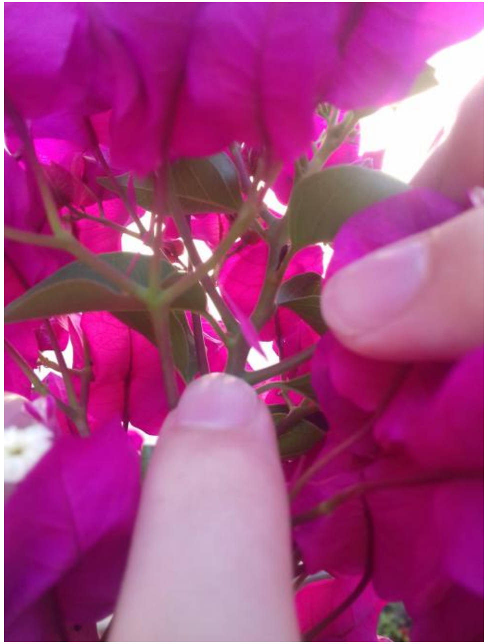
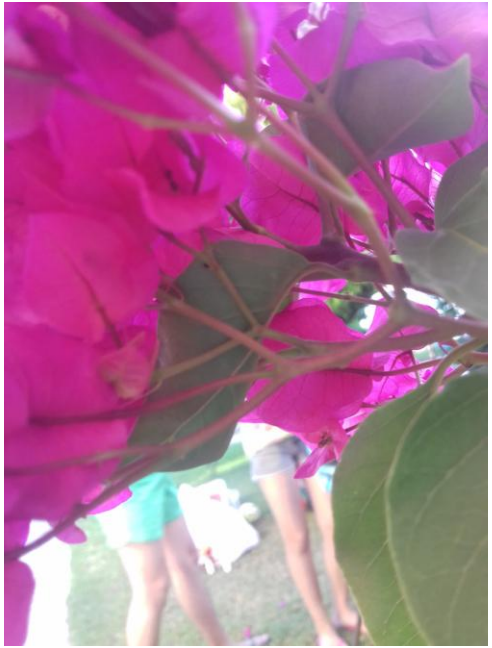
Analysis of the Bougainvillea inflorescence

- Dichasium inflorescence can be deemed as fractal-like, because it can be described by the L-system technique in a simple manner.
- The pattern of the main axis terminating in a flower and the two lateral ones continuing is repeated ⇒ quasi-self-similar structure.

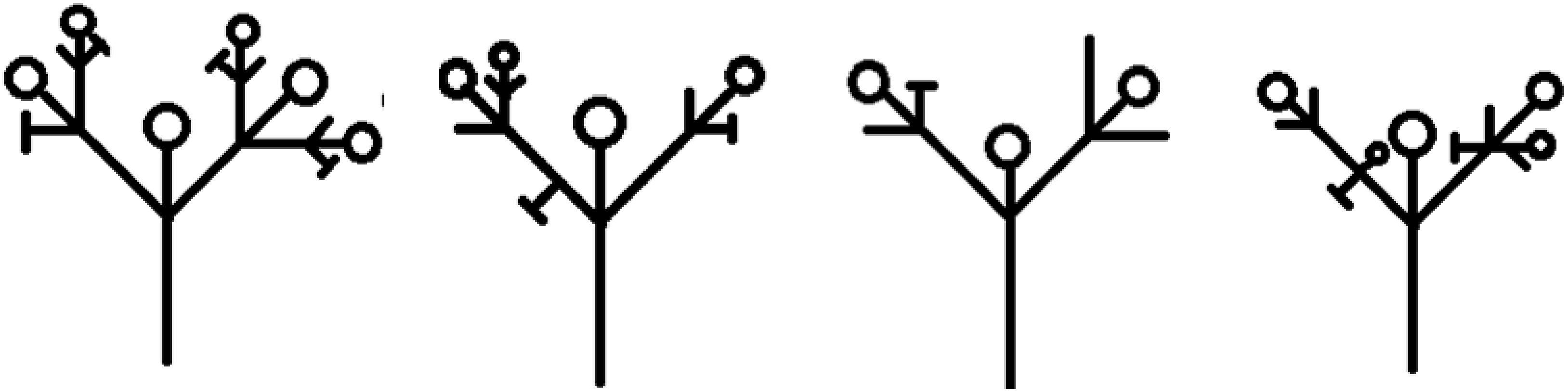




Highlights of the experiment (1)

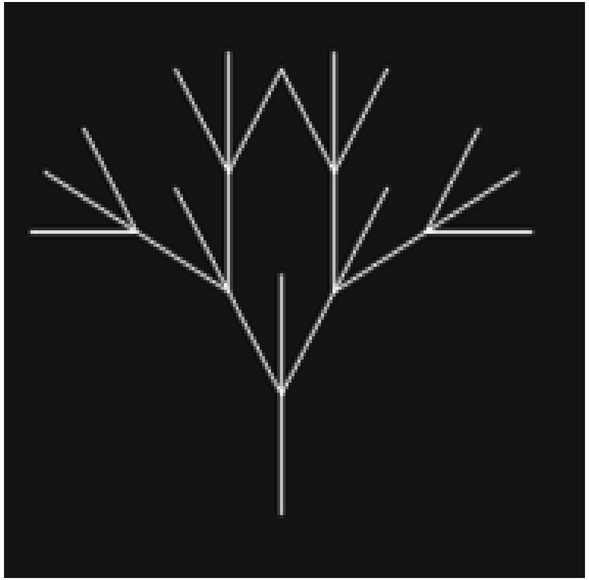
Schematics of some Bougainvillea inflorescences



The inflorescences mainly follow the dichasium pattern, but there are deviations (4 branchings, flowers where there should be none).

Code for modelling the inflorescence types

Angle	30
Starting Word	F
Production Rule	F=X[-F+][+F-]X



Bougainvillea

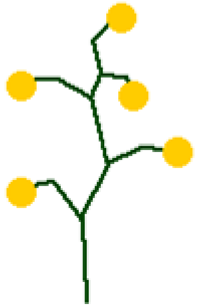
Tomato

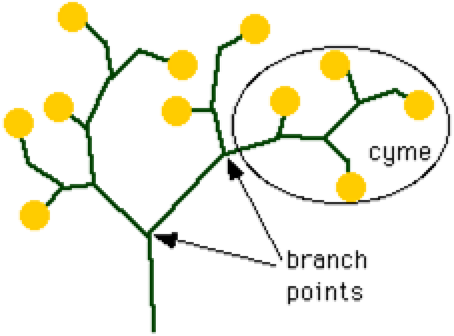
S. Lycopersicum (domesticated tomato) has a compound dichasium inflorescence, solitary inflorescence or scorpioid cyme inflorescence.

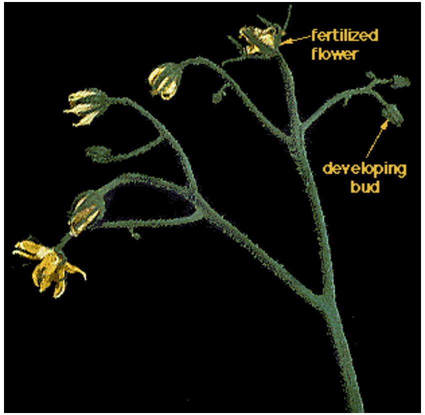
Analysis of the tomato inflorescences

The scorpioid cyme and compound dichasium could be considered fractal-like, in that

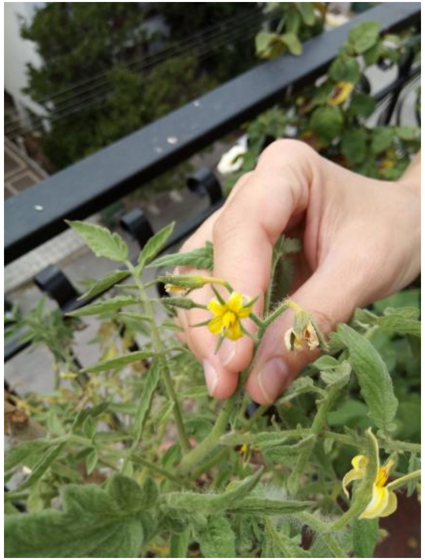
- they are quasi-self-similar and
- have simple L-system definitions.



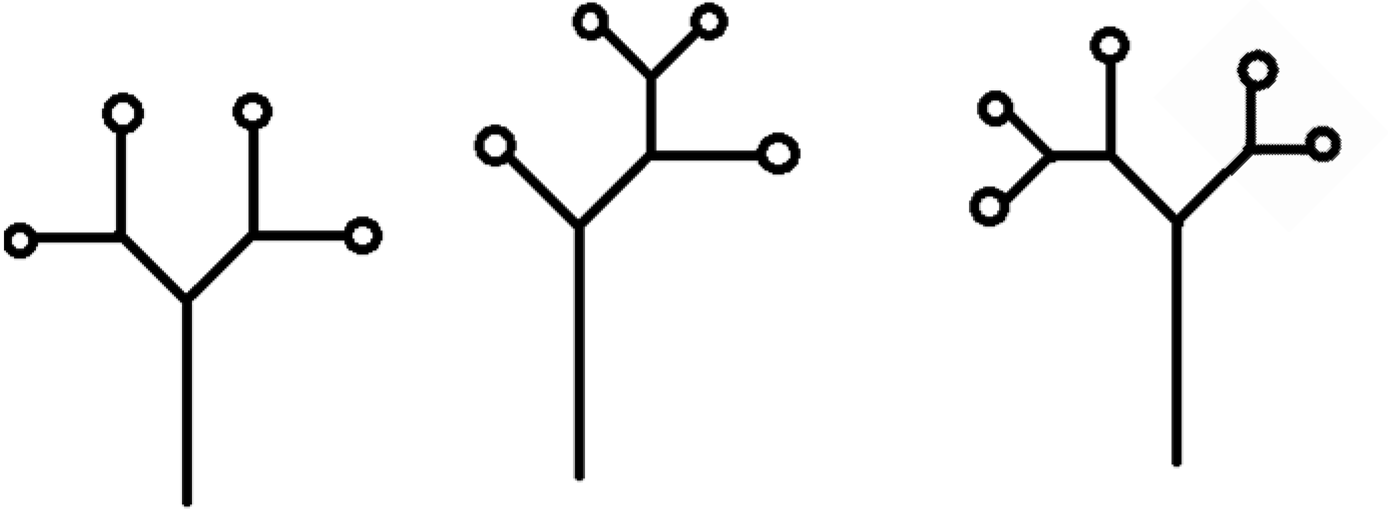




Highlights of the experiment (2)



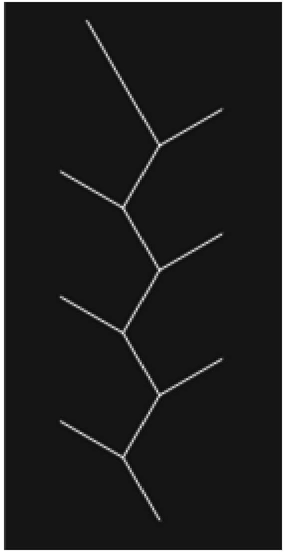
Schematics of tomato inflorescences



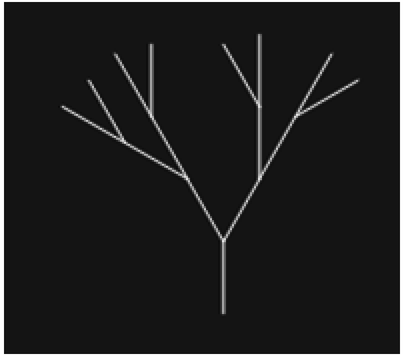
The tomato inflorescences studied all followed the component dichasium pattern.

Code for modelling the inflorescence types

Angle	30
Starting Word	-FA
Production Rules	A=[-F]++F[B] B=[+F]--F[A]



Angle	30
Starting Word	F[-A][+A]
Production Rules	A=F[-A]A, F[+A]A



Tomato

Results summary

- The Bougainvillea inflorescences followed the main pattern without any element missing (rather many extra elements were found).
- The tomato inflorescences studied followed the general pattern(s) exactly.
- In both cases, the L-systems could represent the patterns accurately.

Conclusions

- The tomato and bougainvillea plants’ inflorescences can be described by L-Systems and have fractal qualities ⇒ they are fractal-like.
- However, real-life plants do not strictly follow the expected inflorescence patterns.

References

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- Lindermayer, Arstid and Prusinkiewicz, Przemyslaw. *The Algorithmic Beauty of plants,* Springer-Verlag, New York 1990.
- https://www.researchgate.net/publication/228484251_Inflorescence_development_in_two_tomato_species
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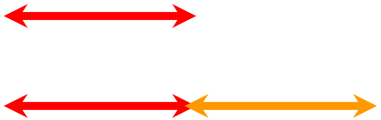
Thank you for your attention!

Appendix

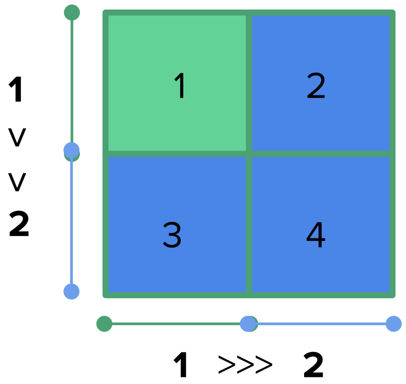
Dimension formula

- When a shape of a **dimension *d*** is being scaled (i.e. lengthened in all directions possible) by a factor of *x*, the resulting size *s* can be represented by the formula: $s = x^d$

Dimension formula

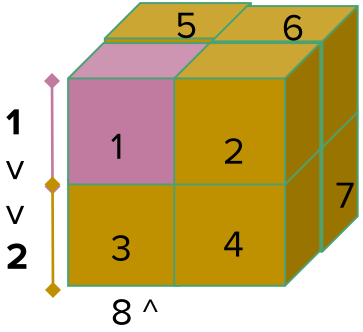


- If the formula is tested for exemplar shapes, then the topological dimension they belong to can be calculated:
- Line: $log_2 2 = d.$, $2^1 = 2$, $d = 1$
- Square: $log_2 4 = d.$, $2^2 = 4$, $d = 2$
- Cube: $log_2 8 = d.$, $2^3 = 8$, $d = 3$



(x=2, here meaning *doubling* ***every*** *line*)

- It is proven thus that a line is one-dimensional, a square is 2-dimensional and a cube is 3-dimensional.



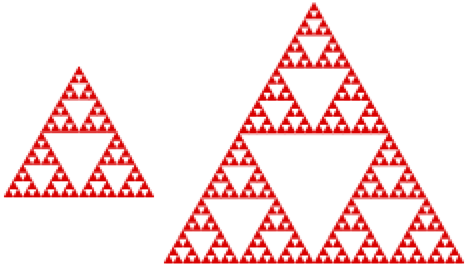
(x=2, again doubling every line)

Fractal Geometry (2)

- If we apply this method to a fractal called the Sierpinski Gasket, we can see that when it is scaled by a factor of 2, its content changes by 3. Following the formula, we can see that:

  $log_2 3 = d.$, so $2^d = 3$

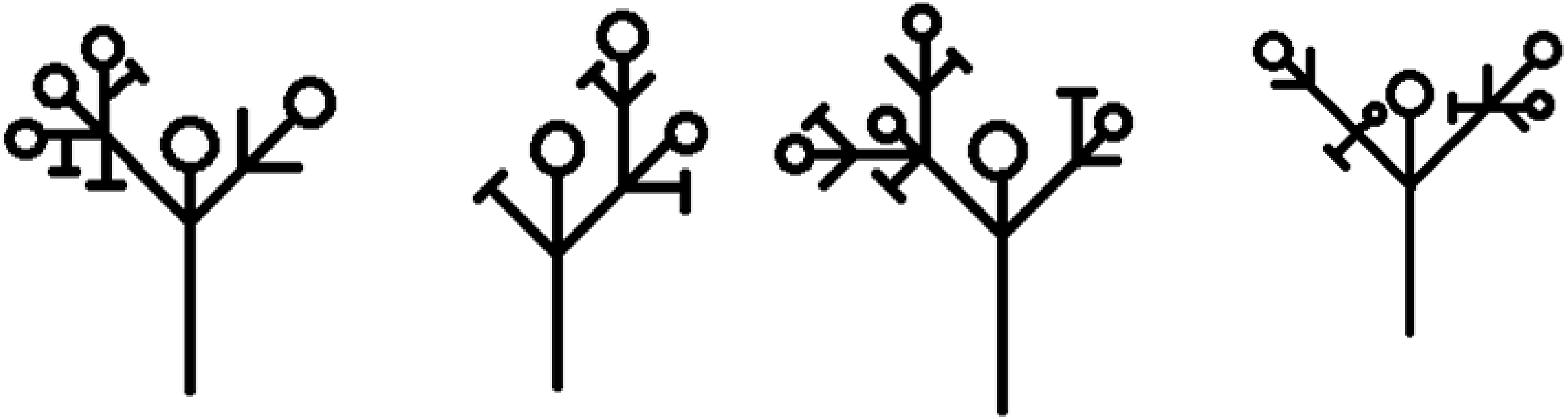
- Since $2^1 = 2$ and $2^2 = 4$, ***d*** must be a number between 1 and 2.
- The dimension of this fractal is an irrational number: ***d***=1.585
- This quality applies to most of the fractals.



Efficiency of fractal inflorescence in plants

- Fractal inflorescence increases the amount of flowers that can be in one spot in a plant, thus increasing the probabilities of pollination.
- Plants are also exposed to different types of loads. Fractal inflorescence branching helps them maintain a shape by distributing the loads and stresses evenly.

More Bougainvillea Schematics



More Bougainvillea photographs