1. Misty

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Problem

Invent and construct a device that would allow the size of a droplet of a mist to be determined using a sound generator.
Overview

- Device
- General characteristics
- Absorption - theoretical treatment
- Experimental results
Device
General Characteristics

- Fog in a sound field as *aerosol in oscillating fluid*
- Droplet size $2 - 200 \, \mu m$
- Water concentration below $0.2\% \ (1.5 \, g \, m^{-3})$
- Wave length $1.7 \, cm - 17 \, m$
Droplet Motion

- Flow around velocity

\[ v_{\text{rel}}(t) = v_{\text{wave}}(t) - v_{\text{drop}}(t) \]

- Frictional force:

\[ F(t) = 6\pi \eta rv_{\text{rel}}(t) \]

- Forced oscillation
Droplet Motion

- **Sound wave:** $\nu_{\text{wave}} = \hat{v}_{\text{wave}} e^{i\omega t}$
- **Acceleration:**
  \[
a(t) = i\omega \nu_{\text{drop}} = \frac{F}{m} \propto \nu_{\text{rel}}
  \]
- **Phase shift:**
  \[
  \tan \varphi = \frac{|\nu_{\text{rel}}|}{|\nu_{\text{drop}}|} = \frac{2\rho_{\text{drop}}}{9\eta} r^2 \omega
  \]
  \[
  \hat{\nu}_{\text{rel}} = \hat{\nu}_{\text{wave}} \sin \varphi = \hat{\nu}_{\text{wave}} \left(1 + \tan^{-2} \varphi \right)^{-\frac{1}{2}}
  \]
Absorption

- Energy absorbed by one droplet:

\[ \frac{dE_0}{dt} = -Fv_{rel} = -6\pi \eta rv_{rel}^2 \quad \langle v_{rel}^2 \rangle = \frac{1}{2} \hat{v}_{rel}^2 \]

- Intensity loss:

\[ \frac{dI}{dx} = \frac{dE_0}{dt} n \quad I = \frac{1}{2} \rho_{air} c \hat{v}_{wave}^2 \]

\[ \frac{dI}{dx} \propto \hat{v}_{rel}^2 \propto \hat{v}_{wave}^2 \propto I \]
Intensity Loss

\[ \frac{dI}{dx} = -6 \frac{\pi \eta}{\rho_{\text{wave}} c} \frac{r}{(1 + \tan^{-2} \varphi)} nI = -kI \]

transmitted intensity after distance \( x \):

\[ \frac{I}{I_0} = e^{-kx} = \gamma_{\text{fog}} \]

\[ k = 6 \frac{\pi \eta}{\rho_{\text{wave}} c} \left( r^4 + \left( \frac{9 \eta}{4 \pi \rho_{\text{drop}}} \right)^2 \frac{1}{f^2} \right) n \]
Transmission Dependent on $r$, $f$

- Good measurability for small $r$
- Best range for measurement: $3 \text{ kHz} < f < 9 \text{ kHz}$
Other Absorption Mechanisms

- Intensity loss due to
  - Damping in air
  - Geometrical reasons (e.g. spherical wave)
- Correct formula:

\[
\frac{I}{I_0} = \gamma_{\text{fog}} \gamma_0
\]

- \(\gamma_0\) not dependent on fog parameters
Main Idea

• Eliminating $\gamma_0$ by measuring intensity loss with/without fog:

\[
\frac{I_{\text{fog}}}{I_{\text{air}}} = \gamma_{\text{fog}}; \quad -\ln \frac{I_{\text{fog}}}{I_{\text{air}}} = kx
\]

• Eliminating $n, x, c$ by measuring at two different frequencies

• End formula for $r$
Radius of Droplet

\[ r = \frac{3}{2} \left( \frac{\eta}{\pi \rho_{\text{drop}}} \right)^{\frac{1}{2}} \left( \frac{f_2^{-2} \ln(I_{\text{fog2}}/I_{\text{air2}}) - f_1^{-2} \ln(I_{\text{fog1}}/I_{\text{air1}})}{\ln(I_{\text{fog1}}/I_{\text{air1}}) - \ln(I_{\text{fog2}}/I_{\text{air2}})} \right)^{\frac{1}{4}} \]
Device for Indoor Measurement

- Geometrical characteristics captured by $\gamma_0$
- Ultrasonic fog generator (“artificial” mist)
- Measurement with a fog “flow”
- Warming up the tube to prevent “condensation”
Receiver
Intensity Measurement

![Graph showing microphone output over time](image)

- **Microphone output [V]**
  - 4.2
  - 4.1
  - 4.0
  - 3.9
  - 3.8
  - 3.7
  - 3.6

- **Time [s]**
  - 10
  - 15
  - 20
  - 25
  - 30

Note: The graph illustrates the change in microphone output over time, showing a drop and then a rise.
Results - Linear Regression

\[ u := \ln \frac{U_{\text{fog}}}{U_{\text{air}}} \]

\[ w := \frac{1}{f^2} \ln \frac{U_{\text{fog}}}{U_{\text{air}}} \]

\[ w \cdot 10^9 \text{[s}^{-2}\text{]} \]
**Result - Droplet Size**

- The method of least squares yields:

<table>
<thead>
<tr>
<th>Measured value</th>
<th>Literature value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = (2.3 \pm 0.2) \mu m$</td>
<td>$r = 2 - 4 \mu m$</td>
</tr>
</tbody>
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Summary

• Assumptions:
  • No dispersion
  • Droplets of constant size
• Higher precision can be achieved:
  • More powerful fog generator
• Outdoor measurement hardly feasible
• Test series verifies theory