1. Misty

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Problem

Invent and construct a device that would allow the size of a droplet of a mist to be determined using a sound generator.

Overview

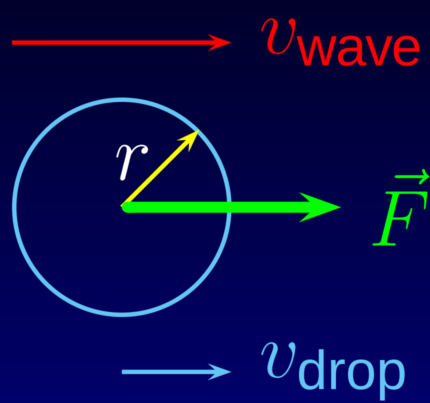
- Device
- General characteristics
- Absorption - theoretical treatment
- Experimental results

Device

General Characteristics

- Fog in a sound field as *aerosol in oscillating fluid*
- Droplet size $2 - 200\,\mu\mathrm{m}$
- Water concentration below 0.2% ($1.5\,\mathrm{g}\,\mathrm{m}^{-3}$)
- Wave length $1.7\,\mathrm{cm} - 17\,\mathrm{m}$

Droplet Motion



- Flow around velocity

$$v_{\text{rel}}(t) = v_{\text{wave}}(t) - v_{\text{drop}}(t)$$

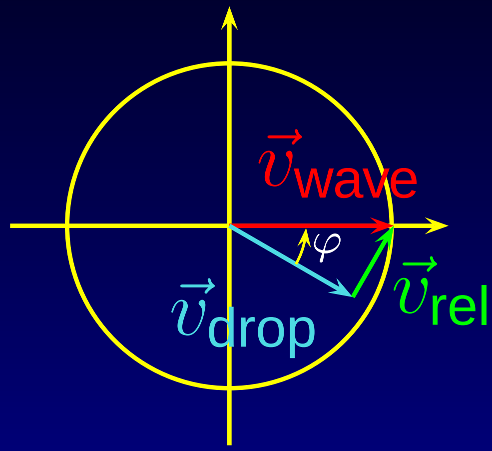
- Frictional force:

$$F(t) = 6\pi\eta r v_{\text{rel}}(t)$$

- Forced oscillation

Droplet Motion

- Sound wave: $v_{\text{wave}} = \hat{v}_{\text{wave}} e^{i\omega t}$
- Acceleration:



$$a(t) = i\omega v_{\text{drop}} = \frac{F}{m} \propto v_{\text{rel}}$$

- Phase shift:

$$\tan\varphi = \frac{|v_{\text{rel}}|}{|v_{\text{drop}}|} = \frac{2\rho_{\text{drop}}}{9\eta} r^2 \omega$$

$$\hat{v}_{\text{rel}} = \hat{v}_{\text{wave}} \sin\varphi = \hat{v}_{\text{wave}} (1 + \tan^{-2}\varphi)^{-\frac{1}{2}}$$

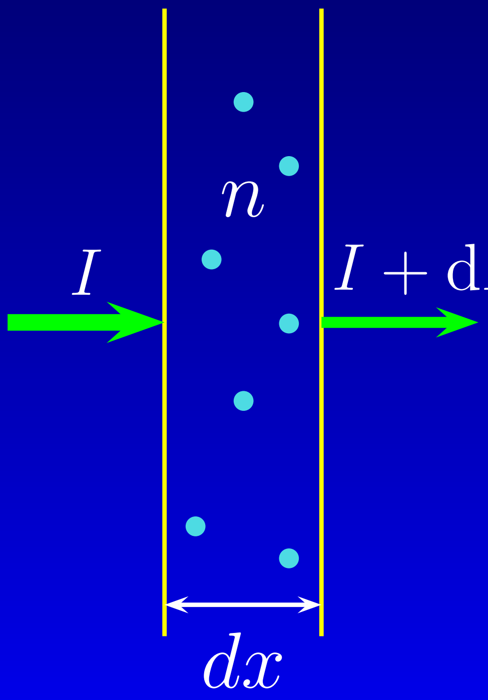
Absorption

- Energy absorbed by one droplet:

$$\frac{\mathrm{d}E_0}{\mathrm{d}t} = -Fv_{\text{rel}} = -6\pi\eta r v_{\text{rel}}^2 \qquad \langle v_{\text{rel}}^2 \rangle = \frac{1}{2}\hat{v}_{\text{rel}}^2$$

- Intensity loss:

$$\frac{\mathrm{d}I}{\mathrm{d}x} = \frac{\mathrm{d}E_0}{\mathrm{d}t} n \qquad I = \frac{1}{2}\rho_{\text{air}} c \hat{v}_{\text{wave}}^2$$



$$\frac{\mathrm{d}I}{\mathrm{d}x} \propto \hat{v}_{\text{rel}}^2 \propto \hat{v}_{\text{wave}}^2 \propto I$$

Intensity Loss

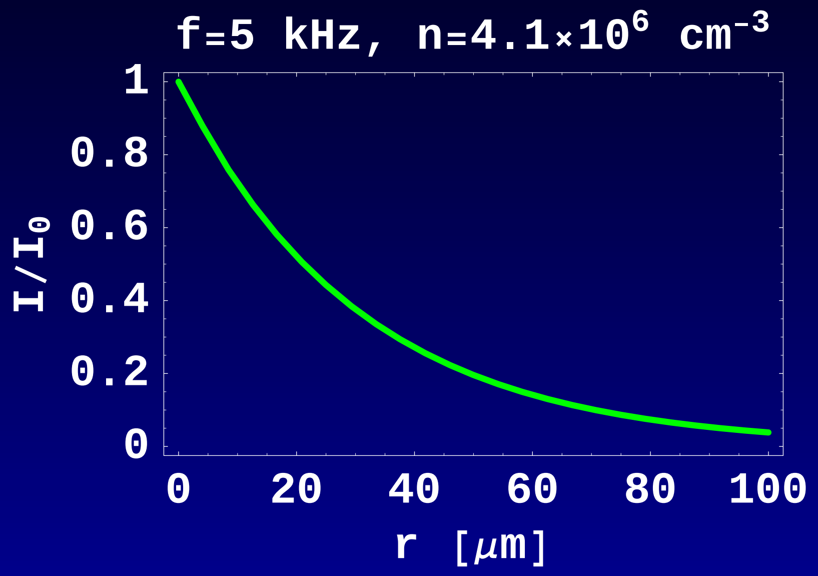
$$\frac{\mathrm{d}I}{\mathrm{d}x} = -6\frac{\pi\eta}{\rho_{\text{wave}}c}\frac{r}{(1+\tan^{-2}\varphi)}nI = -kI$$

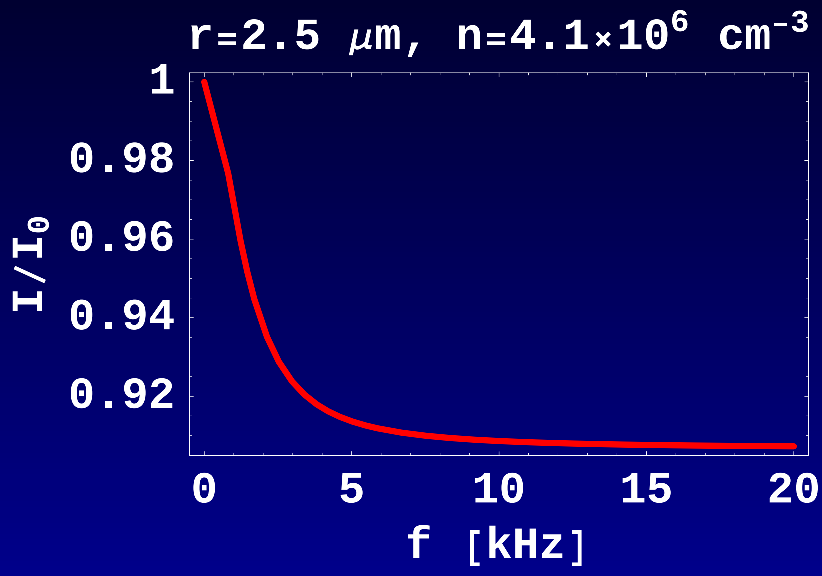
transmitted intensity after distance x:

$$\frac{I}{I_0} = e^{-kx} = \gamma_{\text{fog}}$$

$$k = 6\frac{\pi\eta}{\rho_{\text{wave}}c}\frac{r^5}{\left(r^4 + \left(\frac{9\eta}{4\pi\rho_{\text{drop}}}\right)^2\frac{1}{f^2}\right)}n$$

Transmission Dependent on r, f





- Good measurability for small r
- Best range for measurement: $3\,\mathrm{kHz} < f < 9\,\mathrm{kHz}$

Other Absorption Mechanisms

- Intensity loss due to
  - Damping in air
  - Geometrical reasons (e.g. spherical wave)
- Correct formula:

$$\frac{I}{I_0} = \gamma_{\text{fog}}\gamma_0$$

- γ_0 not dependent on fog parameters

Main Idea

- Eliminating γ_0 by measuring intensity loss with/without fog:

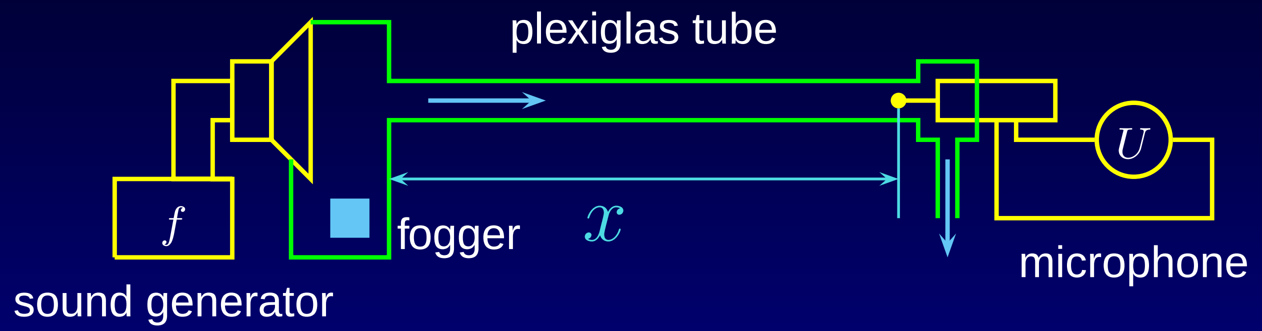
$$\frac{I_{\text{fog}}}{I_{\text{air}}} = \gamma_{\text{fog}}; \qquad -\ln \frac{I_{\text{fog}}}{I_{\text{air}}} = kx$$

- Eliminating n, x, c by measuring at two different frequencies
- End formula for r

Radius of Droplet

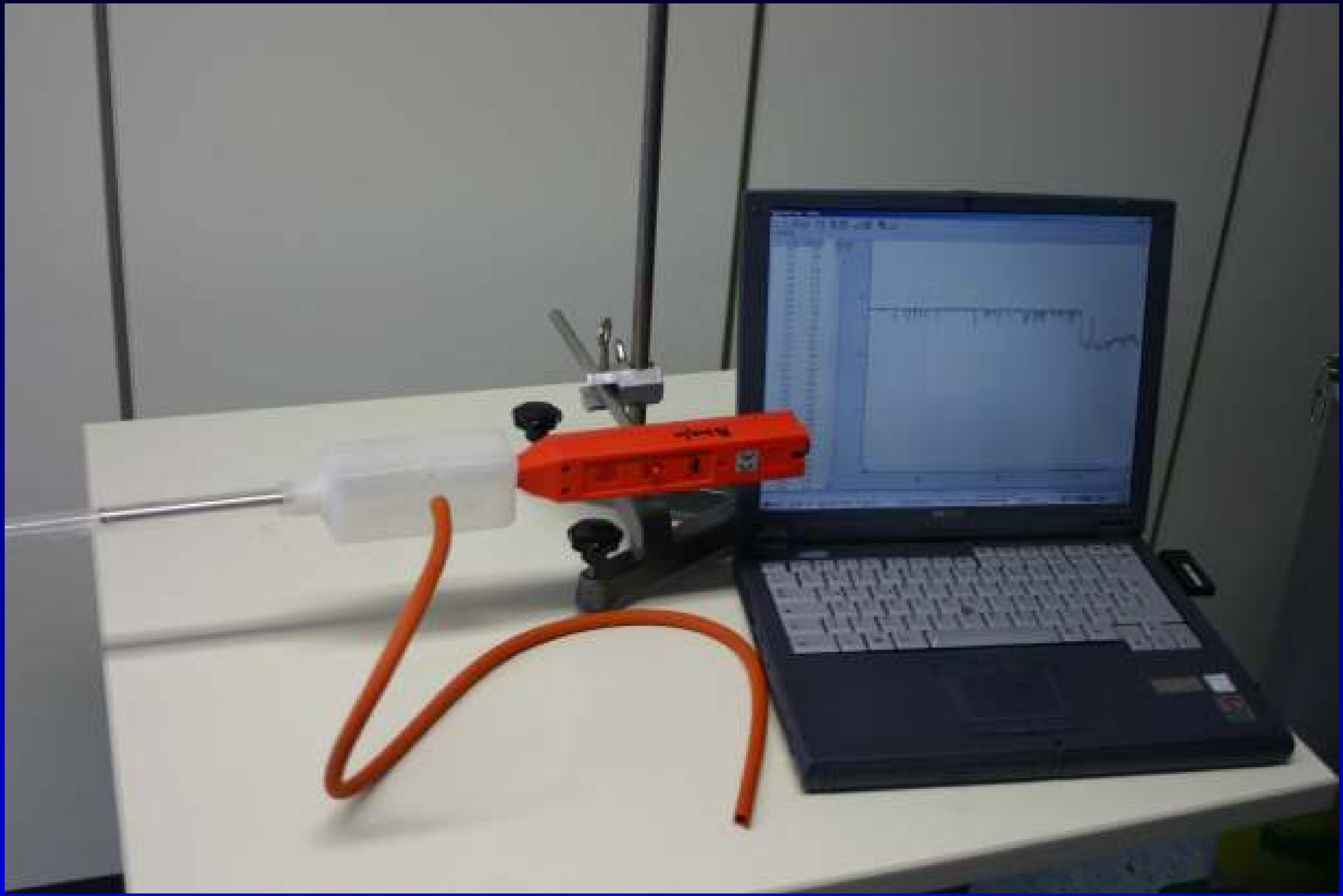
$$r = \frac{3}{2}\left(\frac{\eta}{\pi\rho_{\mathsf{drop}}}\right)^{\frac{1}{2}}\left(\frac{f_2^{-2}\ln(I_{\mathsf{fog}2}/I_{\mathsf{air}2}) - f_1^{-2}\ln(I_{\mathsf{fog}1}/I_{\mathsf{air}1})}{\ln(I_{\mathsf{fog}1}/I_{\mathsf{air}1}) - \ln(I_{\mathsf{fog}2}/I_{\mathsf{air}2})}\right)^{\frac{1}{4}}$$

Device for Indoor Measurement

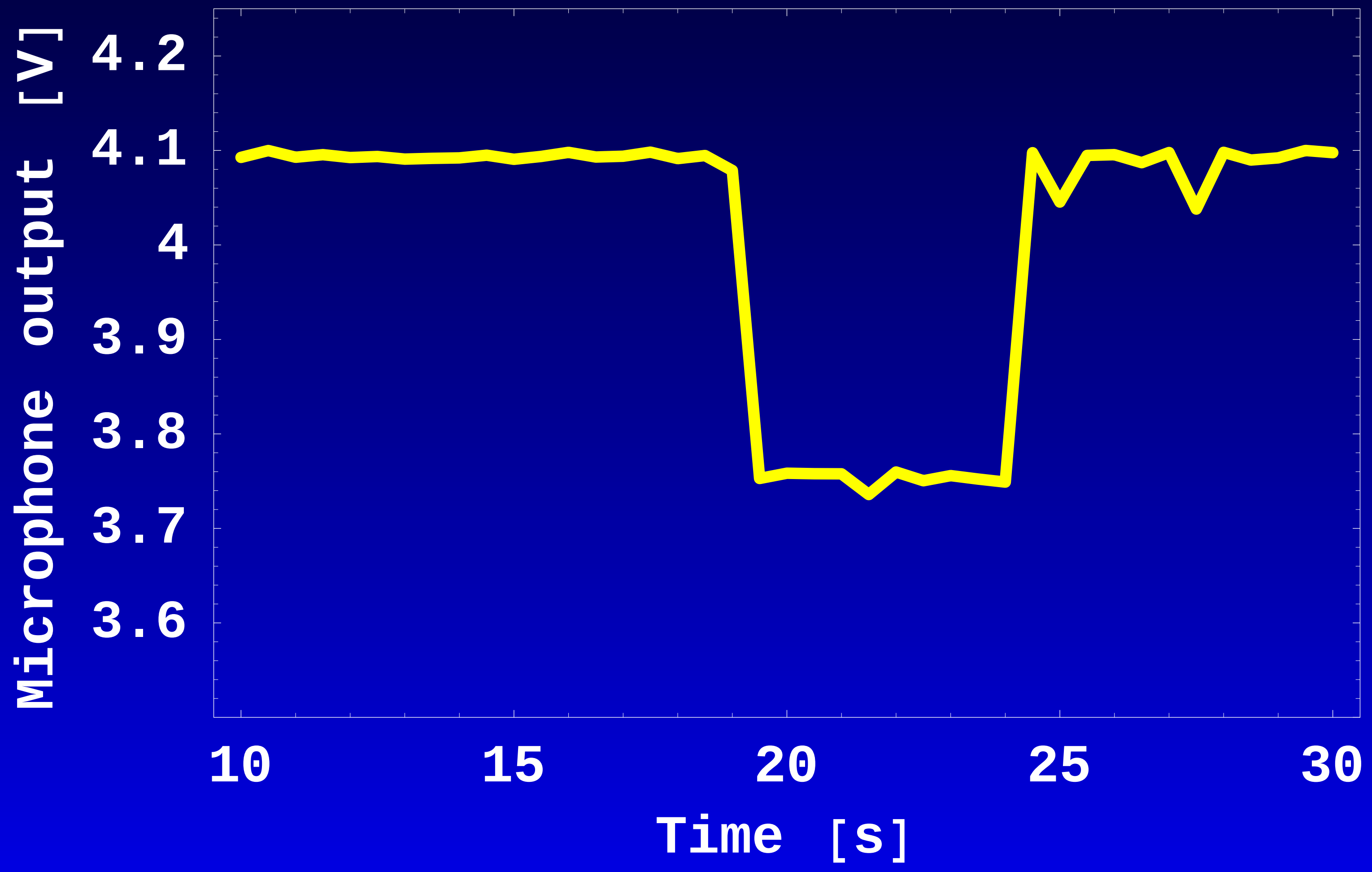


- Geometrical characteristics captured by γ_0
- Ultrasonic fog generator (“artificial” mist)
- Measurement with a fog “flow”
- Warming up the tube to prevent “condensation”

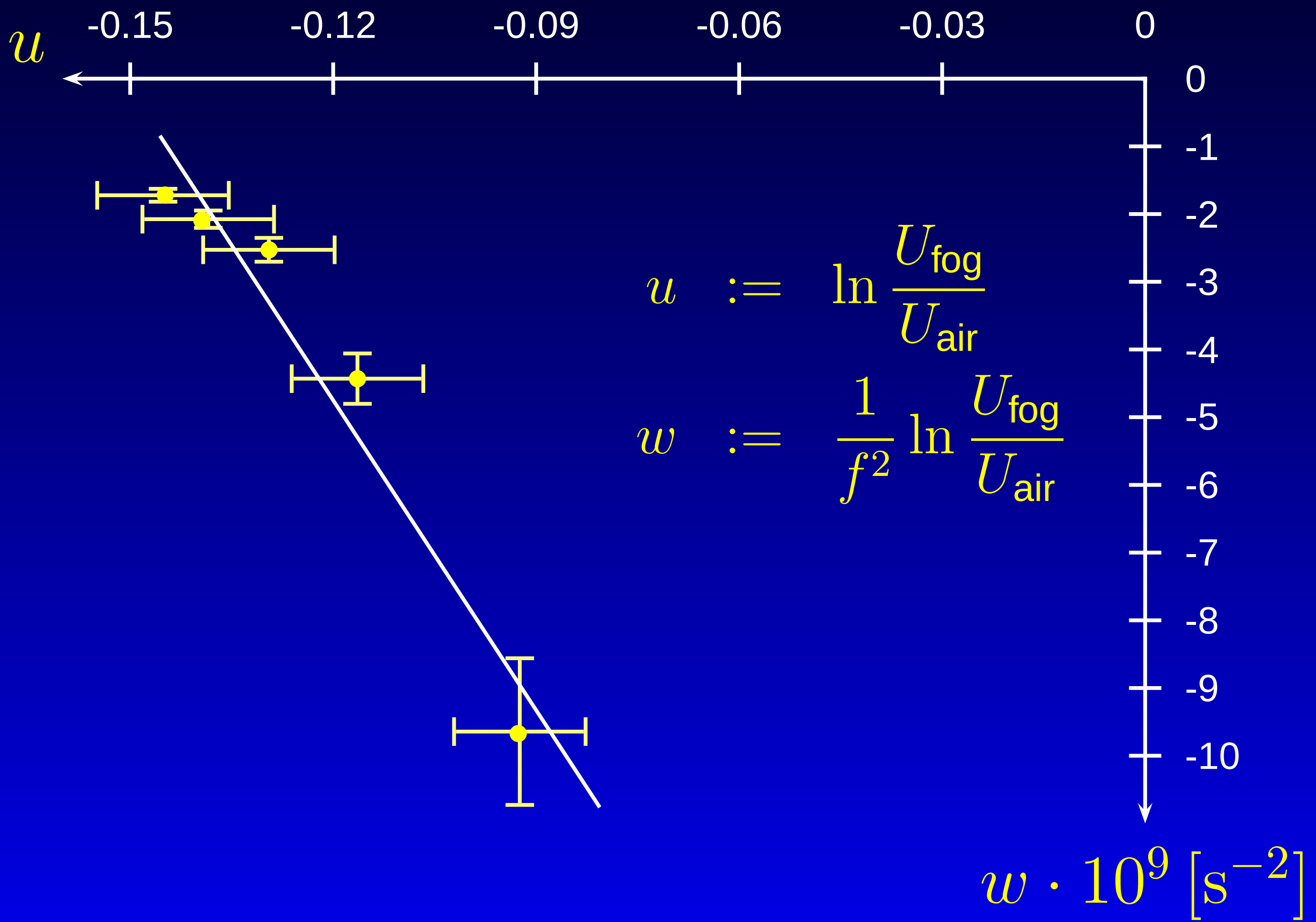
Receiver



Intensity Measurement



Results - Linear Regression



$$u := \ln \frac{U_{\text{fog}}}{U_{\text{air}}}$$

$$w := \frac{1}{f^2} \ln \frac{U_{\text{fog}}}{U_{\text{air}}}$$

Result - Droplet Size

- The method of least squares yields:

Measured value	Literature value
$r = (2.3 \pm 0.2)\,\mu\mathrm{m}$	$r = 2 - 4\,\mu\mathrm{m}$

Summary

- Assumptions:
  - No dispersion
  - Droplets of constant size
- Higher precision can be achieved:
  - More powerful fog generator
- Outdoor measurement hardly feasible
- Test series verifies theory