3. Electric Pendulum

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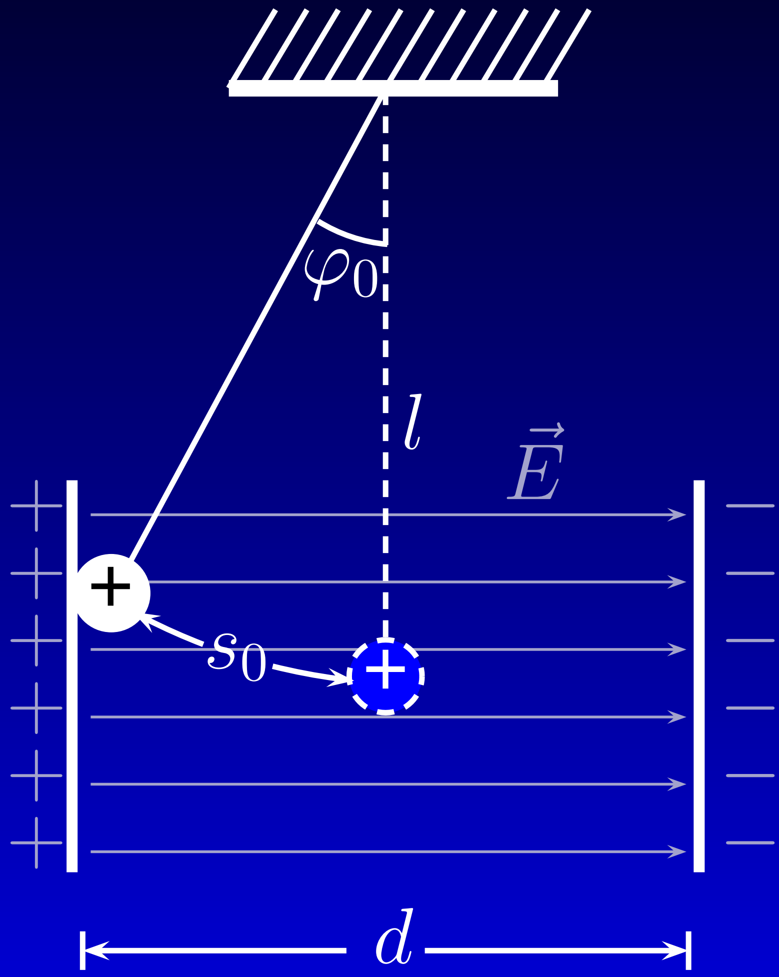
Task

Use a thread to suspend a ball between the plates of a capacitor. When the plates are charged the ball will start to oscillate. What does the period of the oscillations depend on?

Overview

- Experiments
  - Influences on Charge and Frequency
  - Measurements of Dependencies
- Theory
  - Description of Oscillation
  - Frequency
  - Charge

Experimental set-up



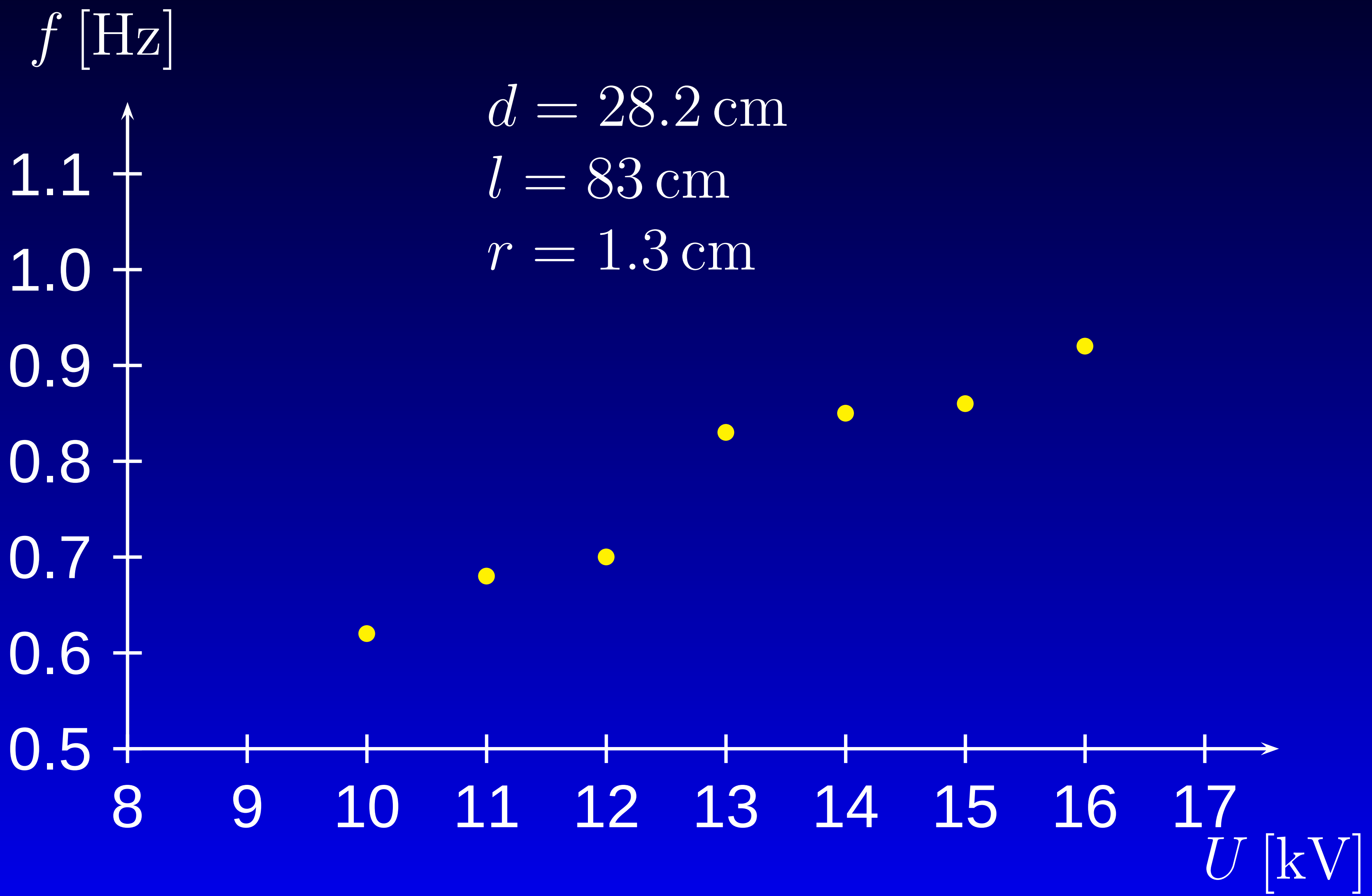
Influence on frequency

- Voltage U between plates
- Distance d of plates
- length l of thread

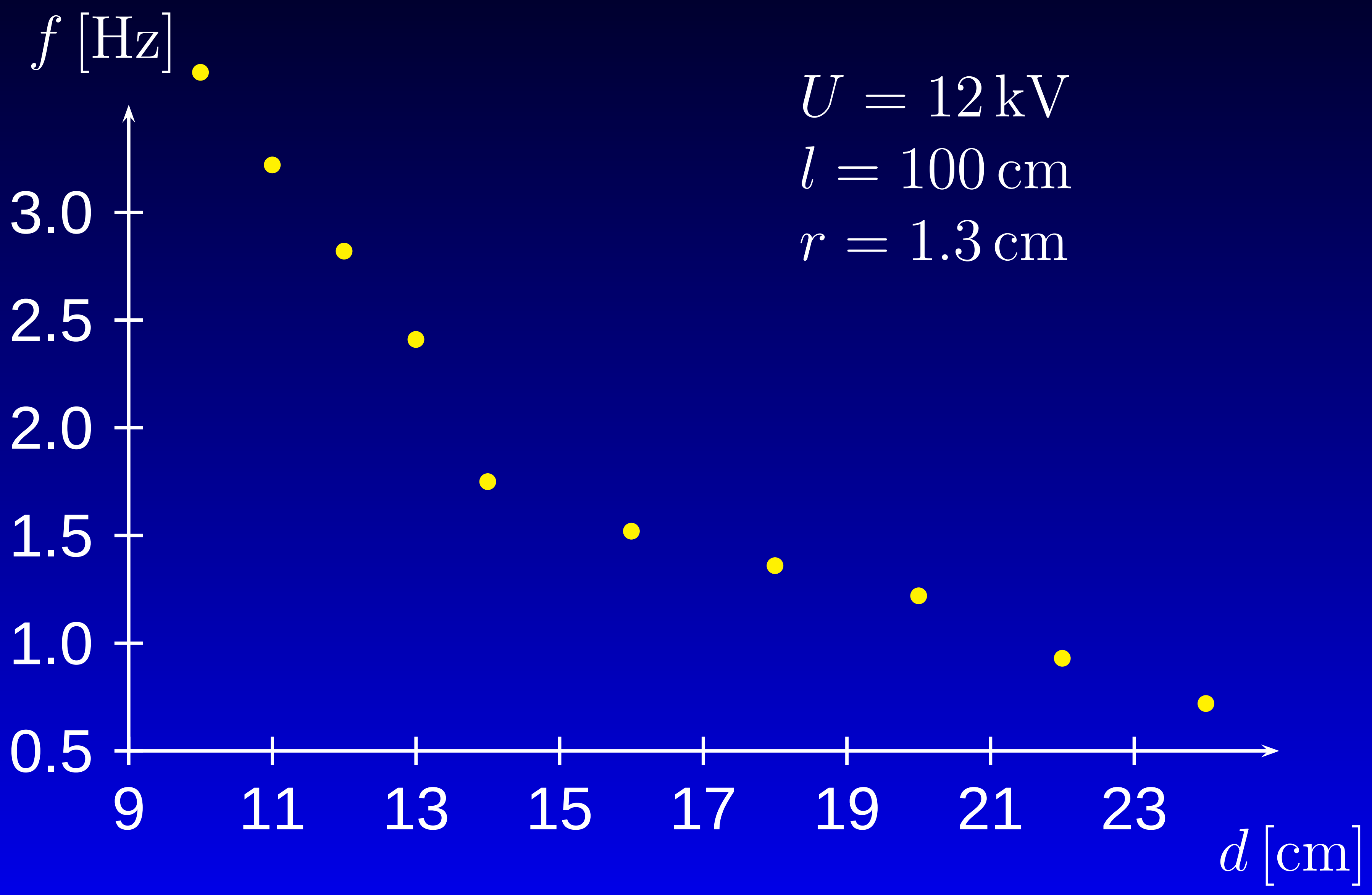
Influence on frequency

- Ball (radius r, mass m, material)
- Coefficient of restitution e
- Charge of ball
  - d, U, r

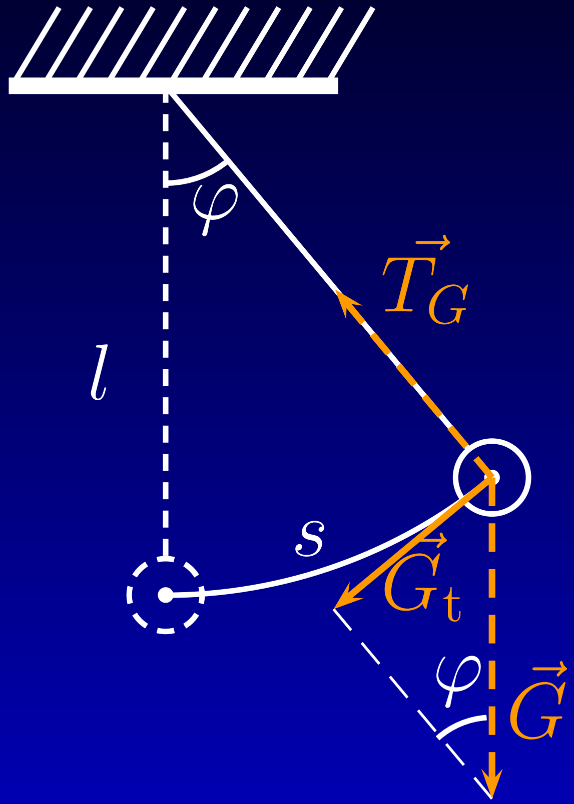
Dependence on Voltage



Dependence on Plate Distance

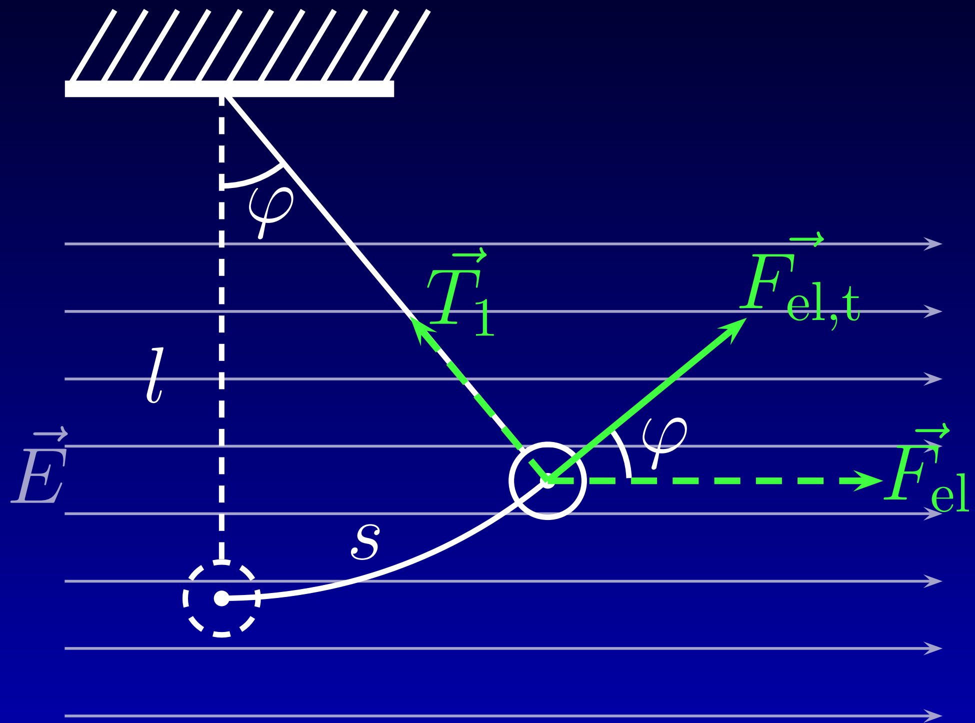


Gravitational Force on Ball



Tangential Component: $G_{\mathrm{t}} = mg \sin\varphi \approx mg\frac{s}{l}$

Electrical Force on Ball



Tangential Component: $F_{\mathrm{el,t}} = \frac{Uq}{d} \cos\varphi \approx \frac{Uq}{d}$

Equation of Motion

Tangential components of Forces:

- Gravitational Force $G_{\mathrm{t}} \approx mg\frac{s}{l}$
- Electrical Force $F_{\mathrm{el,t}} \approx \frac{Uq}{d}$

- Differential equation of motion

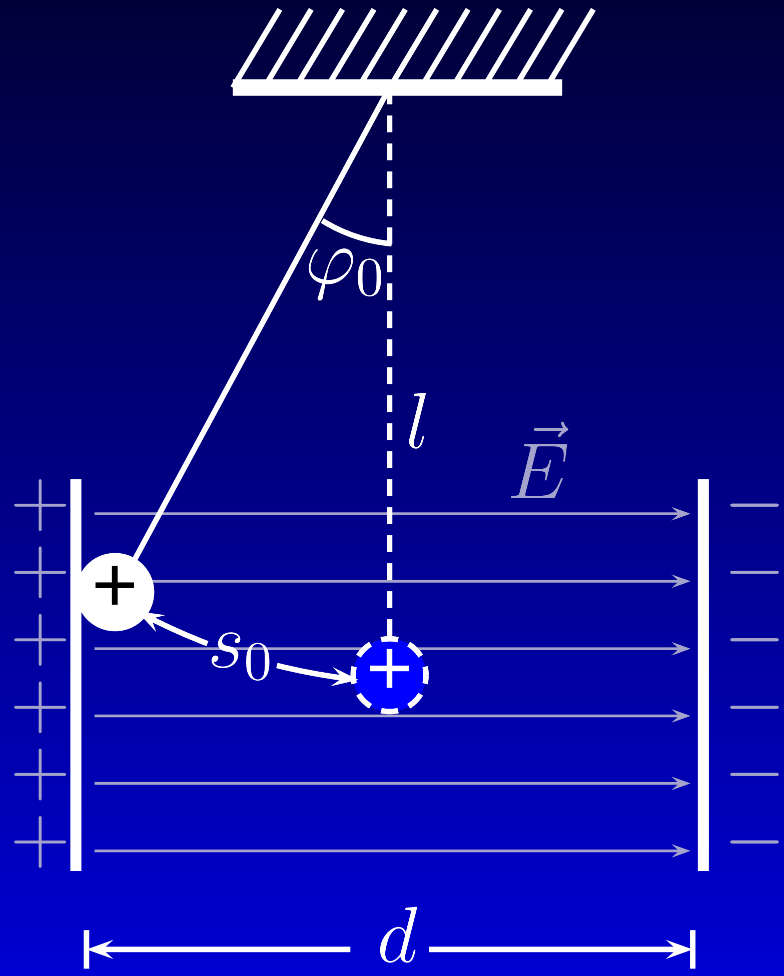
$$m\ddot{s} = -mg\frac{s}{l} + \frac{Uq}{d}$$

Boundary Conditions

- Period only from $-s_0$ to s_0
- Reflection at plates

$$v_{\text{refl}} = e \cdot v_{\text{inc}}$$

- electrical force change sign at collision with plate



Coefficient of restitution

Collision with non-charged plate

$$\frac{mg(1-\cos\varphi)}{mg(1-\cos\varphi')} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}m(ev)^2}$$

$$e = \sqrt{\frac{1-\cos\varphi'}{1-\cos\varphi}}$$

$\Rightarrow e \approx 0.86$

Velocity when Leaving Plate

v_0: velocity when leafing plate
v_1: velocity when arriving at other plate
After some collisions the energy loss equals energy gain of ball ($v_0 = ev_1$):

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_0^2 + q_n E(d-2r)$$

$$\Rightarrow v_1 = \sqrt{v_0^2 + 2qE\frac{d-2r}{m}}$$

$$\Rightarrow v_0 = e\sqrt{\frac{2qE(d-2r)}{(1-e^2)m}}$$

Frequency of Oscillation

Solving for frequency delivers:

$$f = \frac{\omega_0}{2} \frac{1}{\arctan \frac{2s_0{\omega_0}^2 - \chi}{2\omega_0\sqrt{2s_0\chi + {v_0}^2}} + \arctan \frac{\chi + 2s_0{\omega_0}^2}{2\omega_0 v_0}}$$

$\omega_0^2 = \frac{g}{l}$

$\chi = 2\frac{Uq}{md}$

$s_0 \approx (d - 2r)/2$ maximum elongation

$v_0 = e\sqrt{\frac{2qE(d-2r)}{(1-e^2)m}}$ velocity when leaving plate

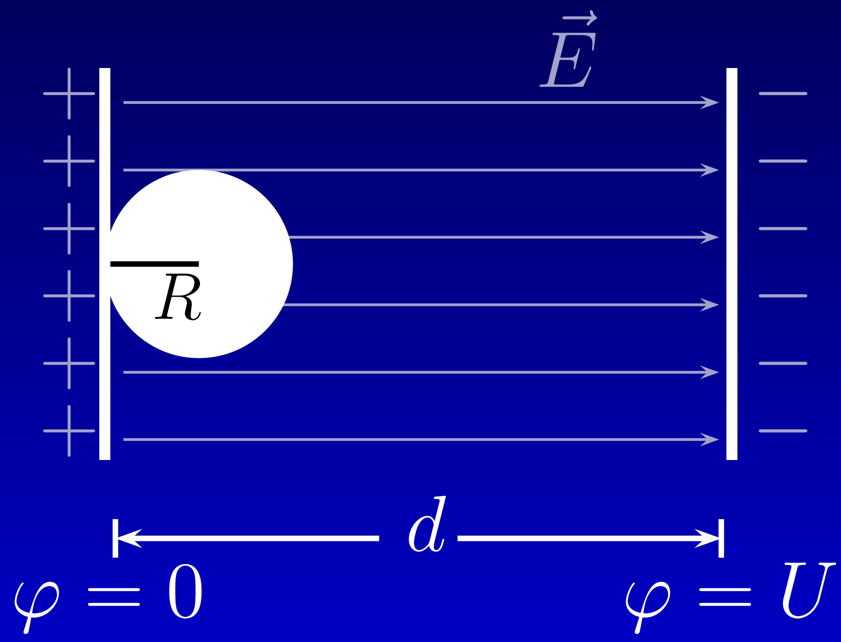
Maximum Charge of Ball

Charge in homogeneous field $\frac{U}{d}$

center of ball:

$$\varphi = \Delta\varphi_{\text{Plate}} + \Delta\varphi_{\text{Ball}} \ = 0$$

$$\Rightarrow \boxed{q_{\text{Ball}} = 4\pi\varepsilon_0 R^2 \frac{U}{d}}$$



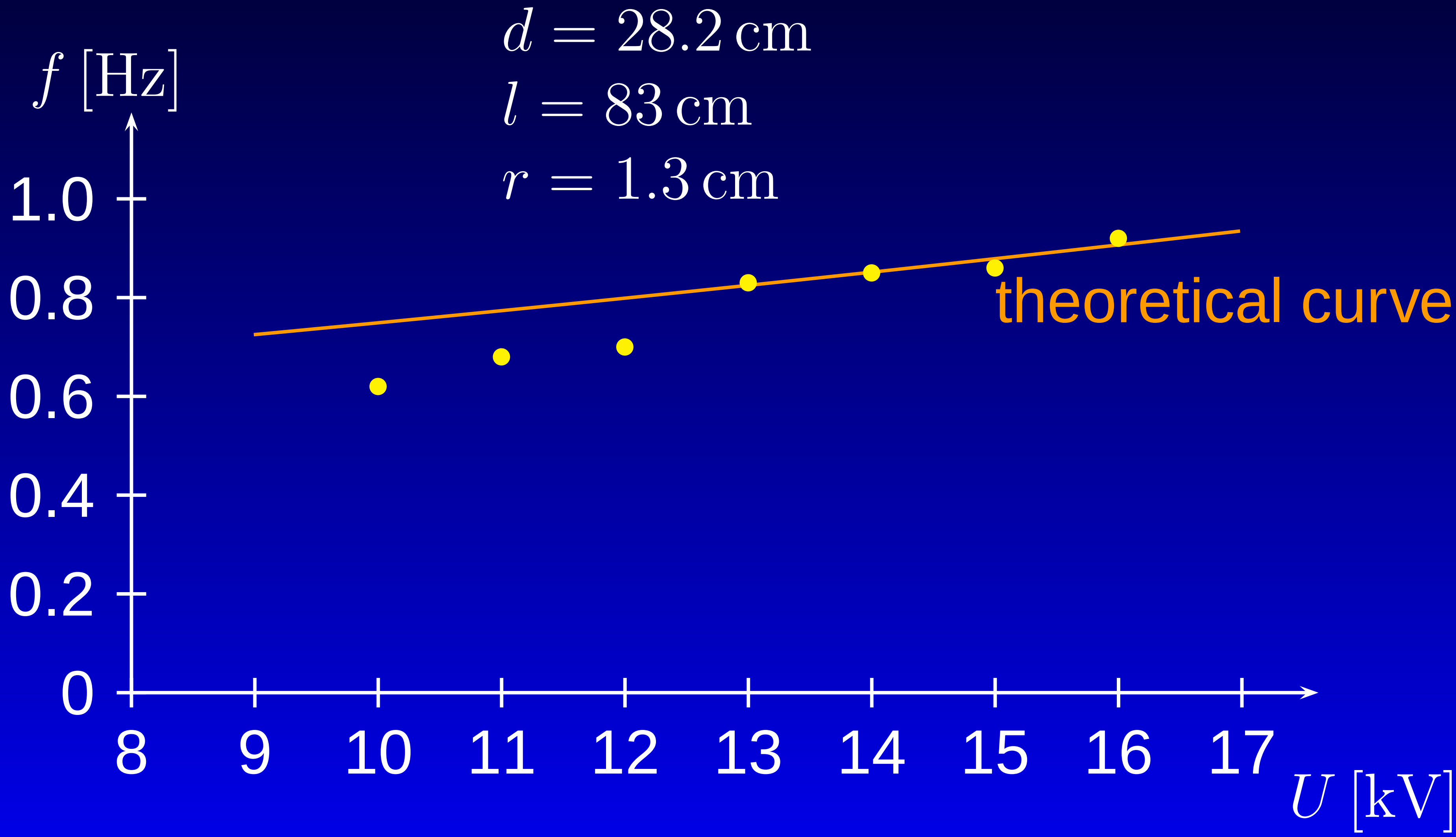
Ball Completely Charged

Theory

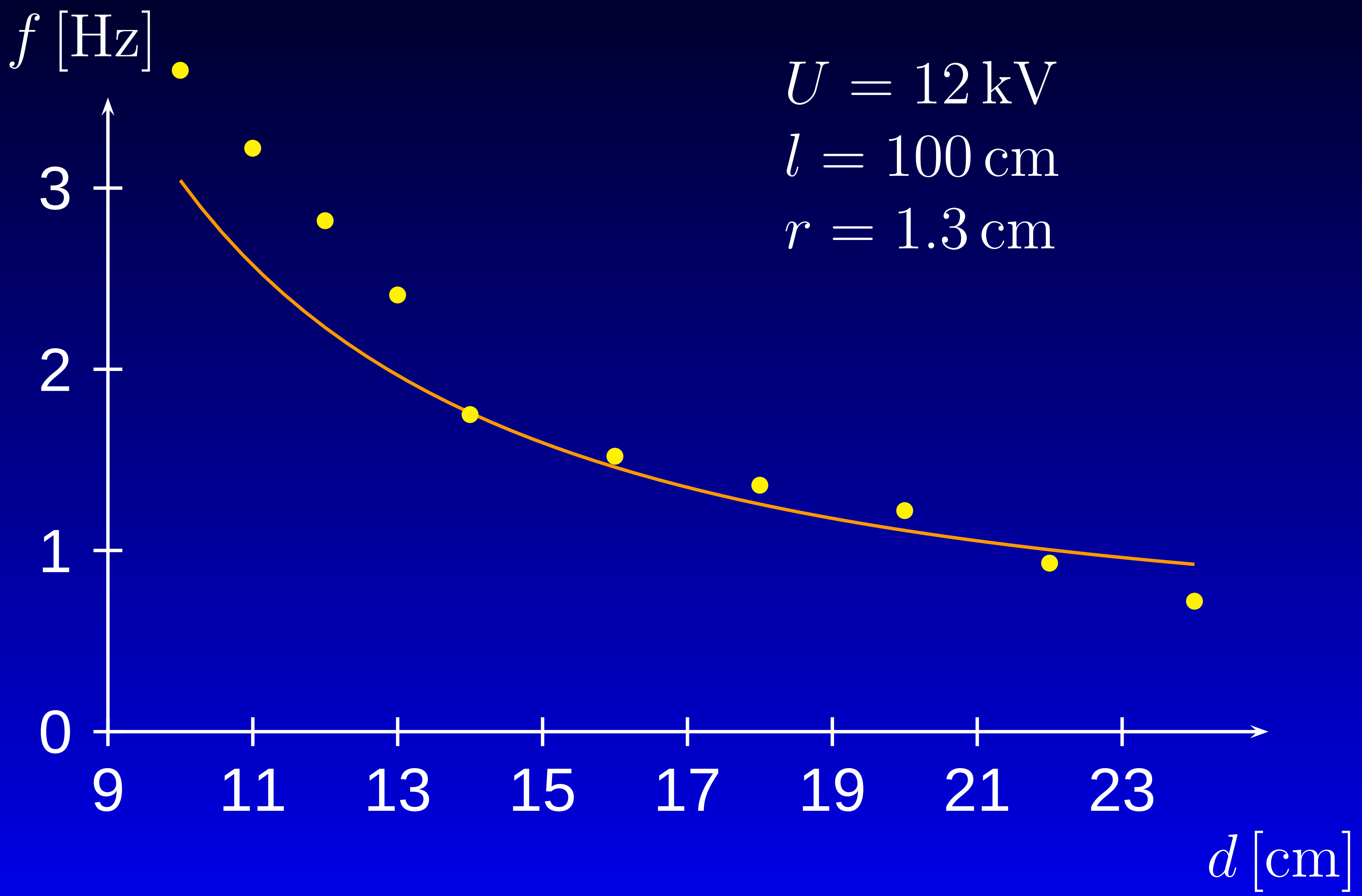
compared to

Experiments

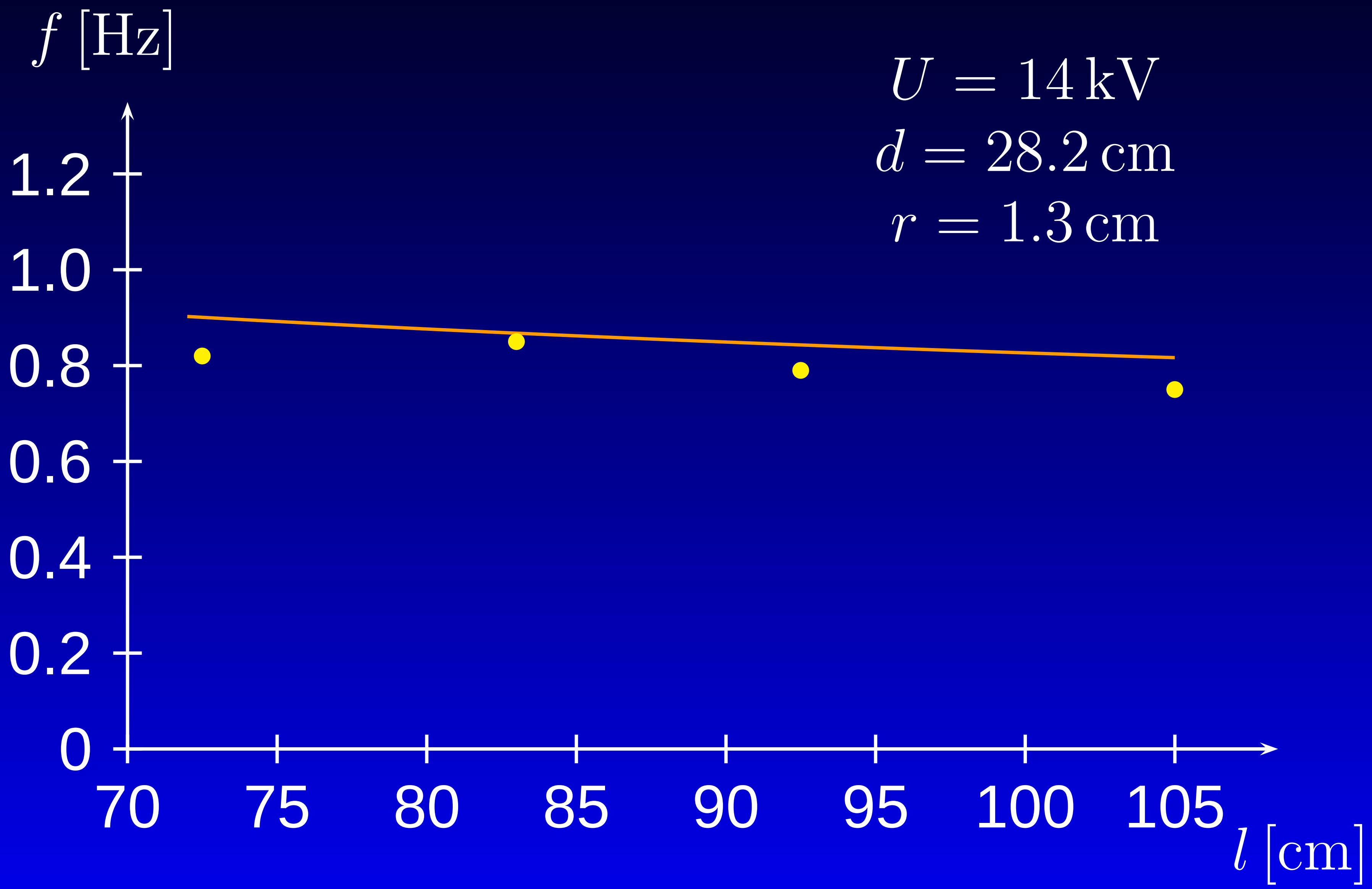
Dependence on Voltage



Dependence on Plate Distance



Dependence on Length of String



Conclusions

- Complete formula for frequency for long threads
- Theoretical approximation of charge
- Measurement confirming theory

Appendix

Coefficient of restitution

Collision with non-charged plate

$$\frac{mg(1-\cos\varphi)}{mg(1-\cos\varphi')} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}m(ev)^2}$$

$$e = \sqrt{\frac{1-\cos\varphi'}{1-\cos\varphi}}$$

Charge of the Ball

Charge in homogeneous field $\frac{U}{d}$
Middle of the ball:

$$\varphi = \Delta\varphi_{\text{Plate}} + \Delta\varphi_{\text{Ball}} = 0$$

$$-\frac{U}{d}R + \frac{1}{4\pi\varepsilon_0}\oint\frac{\mathrm{d}q}{R} = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{Ball}}}{R} = \frac{U}{d}R$$

$$\Rightarrow \boxed{q_{\text{Ball}} = 4\pi\varepsilon_0 R^2\frac{U}{d}}$$