# 6. Seebeck Effect

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# Problem

Two long metal strips are bent into the form of an arc and are joined at both ends. One end is then heated. What are the conditions under which a magnetic needle placed between the strips shows maximum deviation?

# Overview

- Seebeck effect
- Theory
- Optimization
- Experiments

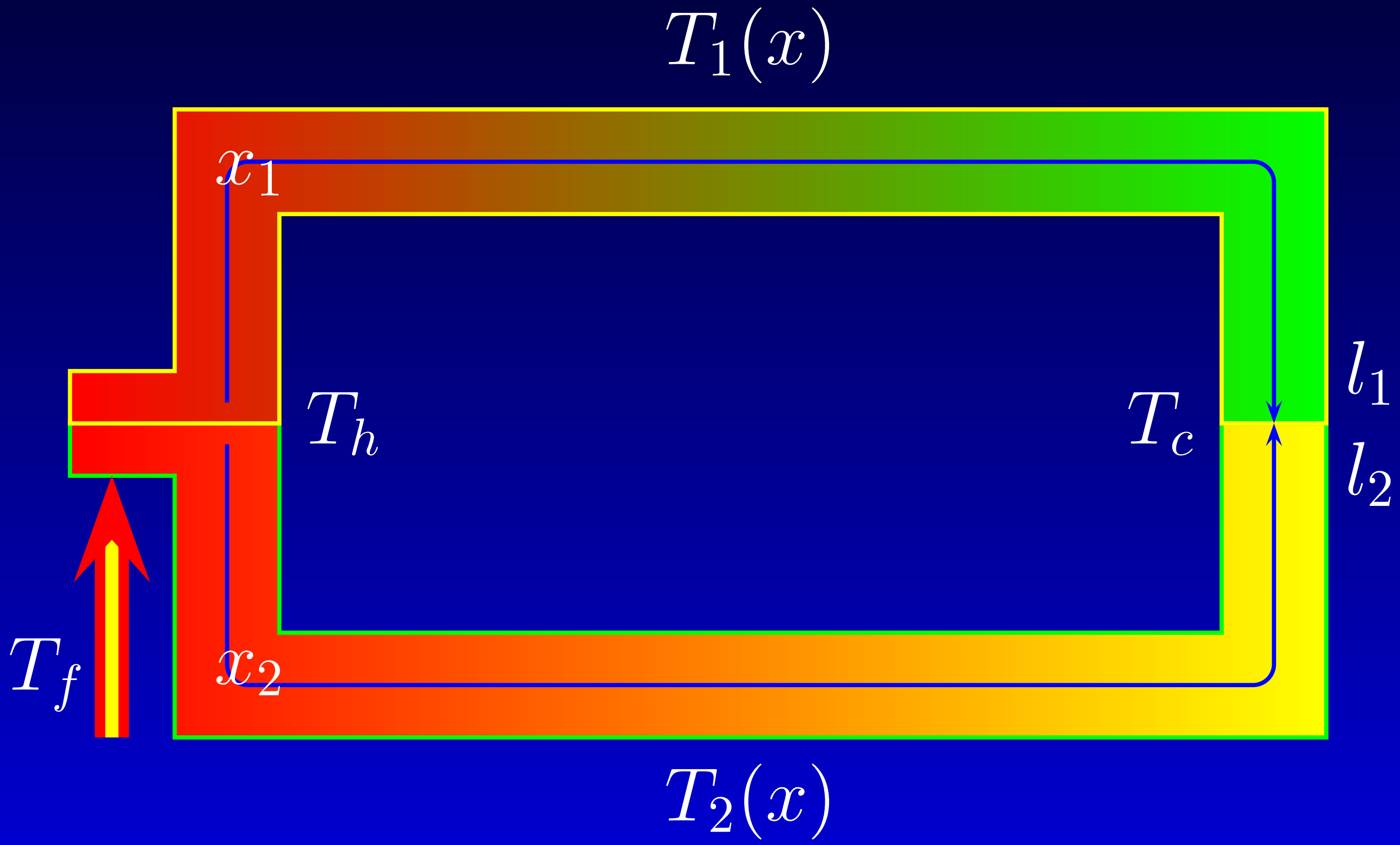
# Seebeck Effect

- Temperature difference $\Delta T$ between contact points due to heating
- No cooling
- Thermoelectric voltage due to electron diffusion
- Voltage $(\Delta\varphi_1 - \Delta\varphi_2) = U \propto \Delta T$
- Current $I$ dependent on voltage $U$, resistance $R$
- Magnetic field $B$ proportional to $I$
- Needle deviation maximal for maximal $B$

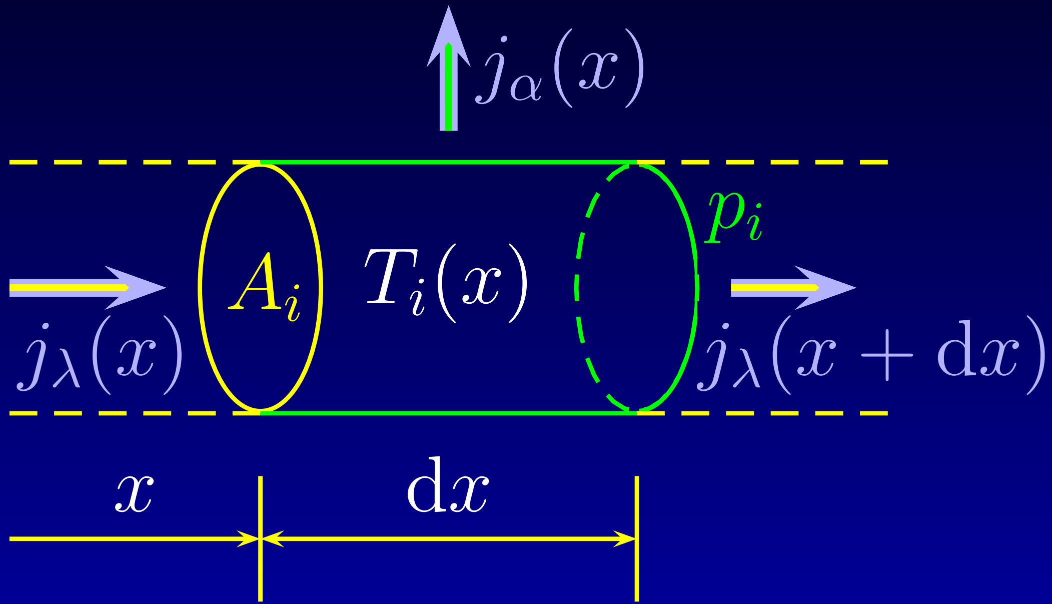
# Theoretical Treatment

- Heat conduction
- Electric current
- Magnetic Field
- Needle Deviation

# Heat Conduction



# Differential Heat Balance




$$
\begin{aligned}
[j] &= \mathrm{W\,m^{-2}} \\
j_\lambda &= \lambda \frac{\partial T}{\partial x} \\
j_\alpha &= \alpha(T - T_0)
\end{aligned}
$$

# Differential Equation

- Stationary state $\Rightarrow T_i(x) = \text{const}$
- $p_i j_\alpha(x)\mathrm{d}x = A_i \left(j_\lambda(x) - j_\lambda(x + \mathrm{d}x)\right)$
- Differential equation:

$$\frac{\partial^2 T_i}{\partial x^2} = \frac{\alpha p_i}{\lambda_i A_i}(T_i - T_0),\ i = 1, 2$$

- Boundary conditions:

$$\begin{aligned} T_1(0) = T_2(0) &= T_f && (1) \\ T_1(l_1) &= T_2(l_2) = T_c && (2) \\ j_1(l_1) + j_2(l_2) &= 0 && (3) \end{aligned}$$

# Solution

$$
\begin{aligned}
T_i(x) - T_0 &= (T_f - T_0)[C_{i,1}\mathrm{e}^{k_i x} + C_{i,2}\mathrm{e}^{-k_i x}] \\
k_i &= \sqrt{\frac{\alpha p_i}{\lambda_i A_i}} \\
C_{i,j} &= f_j(l_i, A_i, p_i, \lambda_i)
\end{aligned}
$$

- Now the difference

$$\Delta T = T_h - T_c$$

can be determined

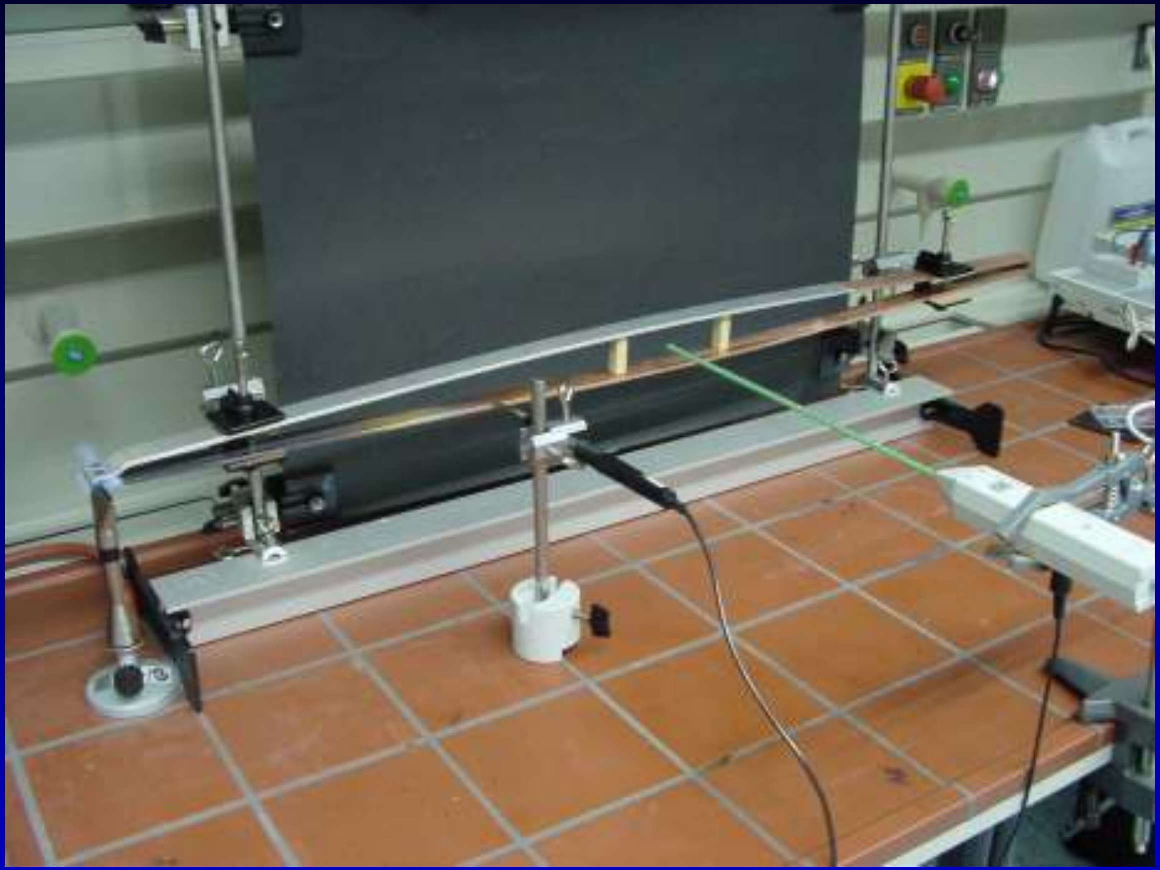
- Heating through electric current can be neglected

# $\alpha$ Measurement

- Linearized cooling law (Newtons law)
- Approx. exponential cooling
- $\alpha$ determined from the experiment

$$\alpha = 9.7\,\mathrm{Wm^{-2}K^{-1}}$$

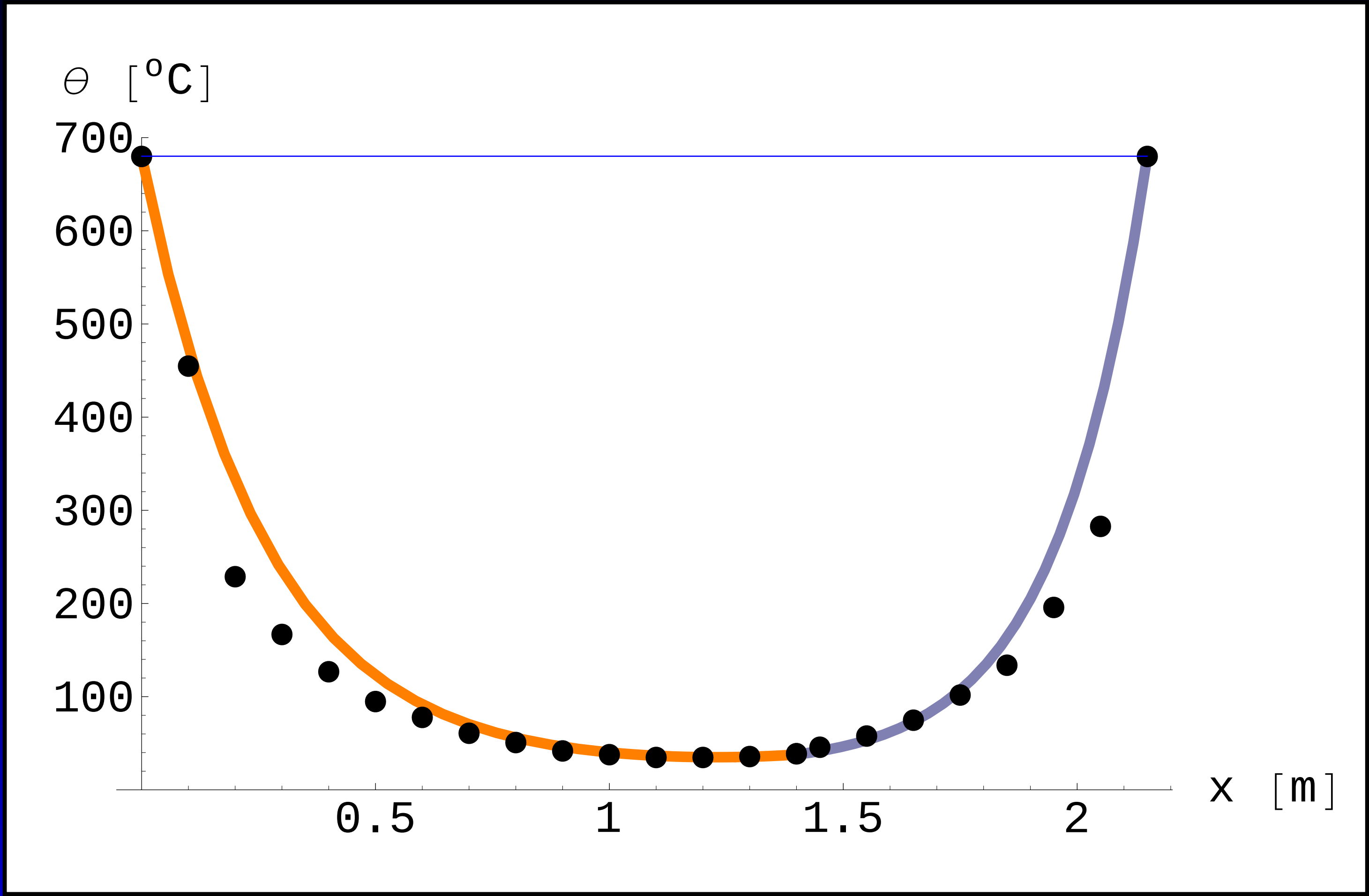
# Experimental Setup



- Device used:

$$l_{\mathsf{Cu}} = 1.4\,\mathrm{m}, \quad l_{\mathsf{Al}} = 0.75\,\mathrm{m}$$

# Comparison - Temperature




- Faster cooling for big $T$
- Good agreement for small $T$

# Electric Current

- Thermoelectric voltage:

$$U = (Q_1 - Q_2)\Delta T$$

- Resistance:

$$R_i = \rho_i \frac{l_i}{A_i} + \rho_i \frac{\beta_i}{A_i} \int_0^{l_i} (T_i(x) - T_0)\mathrm{d}x$$

- Electric current:

$$I = \frac{U}{R_1 + R_2}$$

# Current Optimization

- $T_i(x)$ and $\int T_i(x)\mathrm{d}x$ can be calculated both algebraically and numerically
- Thus current can easily be calculated for different
  - Materials
  - Geometry
- Only the complete model predicts current maximum correctly

# Model Setup

- Materials tested:

| Material | $Q$ | Material | $Q$ |
|---|---|---|---|
| Antimony (Sb) | 43 | Platinum (Pt) | -4.4 |
| Iron (Fe) | 15 | Nickel (Ni) | -20.8 |
| Copper (Cu) | 6.4 | Constantan | -38 |
| Aluminium (Al) | -0.4 | Bismuth (Bi) | -68 |

$[Q] = \mu\mathrm{V/K}$ (compared to Pb)

- Metal stripes: thickness $d_i$, width $b_i$
  - $p_i = 2(d_i + b_i)$
  - $A_i = d_i b_i$

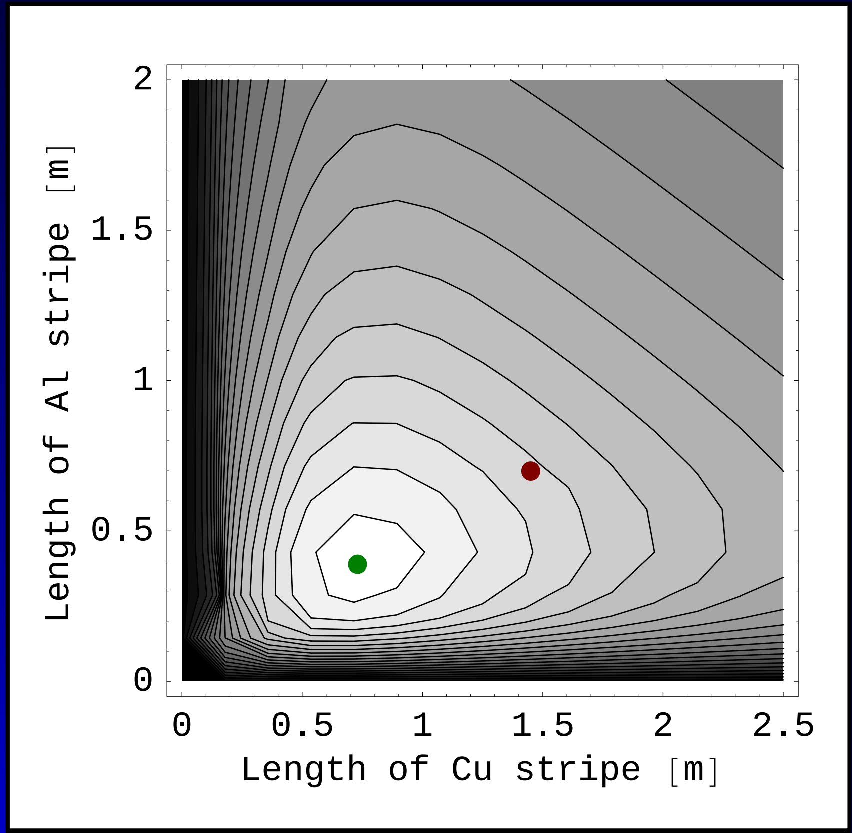
# Optimal Material

- $T_f - T_0 = 660\,\mathrm{K}$
- $d_1 = d_2 = 4\,\mathrm{mm}$, $b_1 = b_2 = 3\,\mathrm{cm}$
- $l_1 = l_2 = 1\,\mathrm{m}$
- Best pairs:

| Pair | Cu-Ni | Al-Ni | Cu-Al | Sb-Ni | Sb-Const |
|---|---|---|---|---|---|
| $I$ [A] | 9.1 | 6.0 | 4.9 | 3.9 | 3.0 |

# Optimal Length

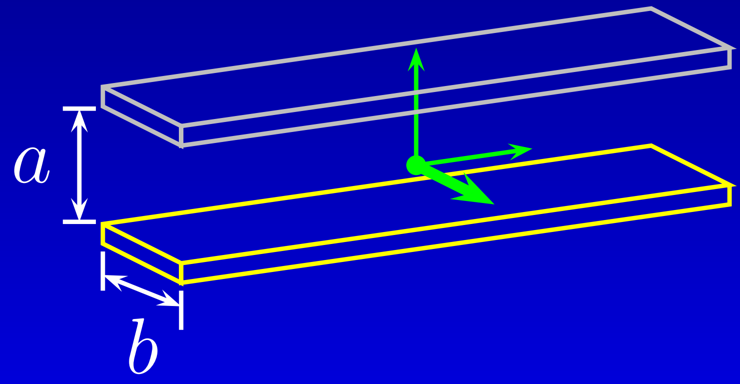
- Cu-Al-Pair:




- $I_{\text{max}} = 8.66\,\text{A}$ at
  $l_{\text{Cu}} = 0.73\,\text{m}$, $l_{\text{Al}} = 0.39\,\text{m}$

# Magnetic Field

- $\vec{B} = \vec{B}(x, y, z)$
- $B \propto I$ (for given geometry)
- $\vec{B}$ depends on geometry
- Long (straight) stripes give maximal $B$ if placed closely:



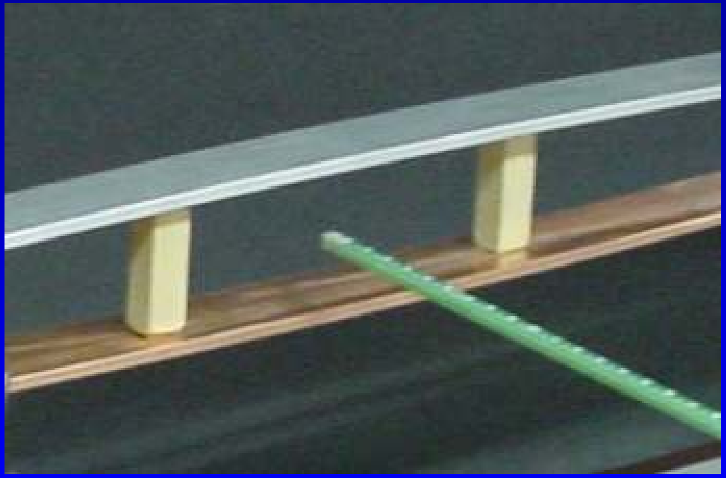

$$B = \frac{2}{\pi} \frac{\mu_0 I}{b} \arctan \frac{b}{a}$$

# Magnetic Field Measurement

- Hall probe placed in field direction
- Results:

| $a$ | Measured value | Calculated value |
|---|---|---|
| $1\,\mathrm{cm}$ | $B = 0.18 \pm 0.02\,\mathrm{mT}$ | $B = 0.18\,\mathrm{mT}$ |
| $4\,\mathrm{cm}$ | $B = 0.08 \pm 0.01\,\mathrm{mT}$ | $B = 0.09\,\mathrm{mT}$ |

- Hall probe $\longrightarrow$ compass needle



# Needle Deviation

- Small needle ($l_n \ll L_i, 2d$)
- $\vec{B} \approx \mathrm{const}$ along the needle
- $I = I_{max}$, $d = 1\,\mathrm{cm}$ yields

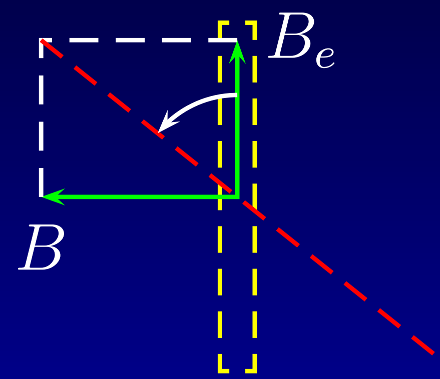
$$B = 180\,\mu\mathrm{T} \gg B_e = 20\,\mu\mathrm{T}$$

- Any deviation $\varphi \in [-180°, 180°]$ possible

# Needle Deviation

- Special case: $B \perp B_e$

$$\begin{aligned} \tan\varphi &= B/B_e \\ \tan\left(\frac{\pi}{2}-\varphi\right) &= B_e/B \\ \varphi &\approx \frac{\pi}{2} - B_e/B \end{aligned}$$




- Under same conditions we get: $\varphi = 83.7^\circ$
- Measured: $\varphi = 80 - 85^\circ$

# Summary

- High precision current calculation
  - Optimal conditions
- Simplified field description
  - Reasonable assumptions
  - Exact treatment possible
- $90^\circ - \varphi$ small

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