7. Coin

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Problem

Stand a coin on its edge upon a horizontal surface. Gently spin the coin and investigate the resulting motion as it settles.

Overview

- Video
- Experiments
- Theory
- Simulation
- Experiment vs. Theory

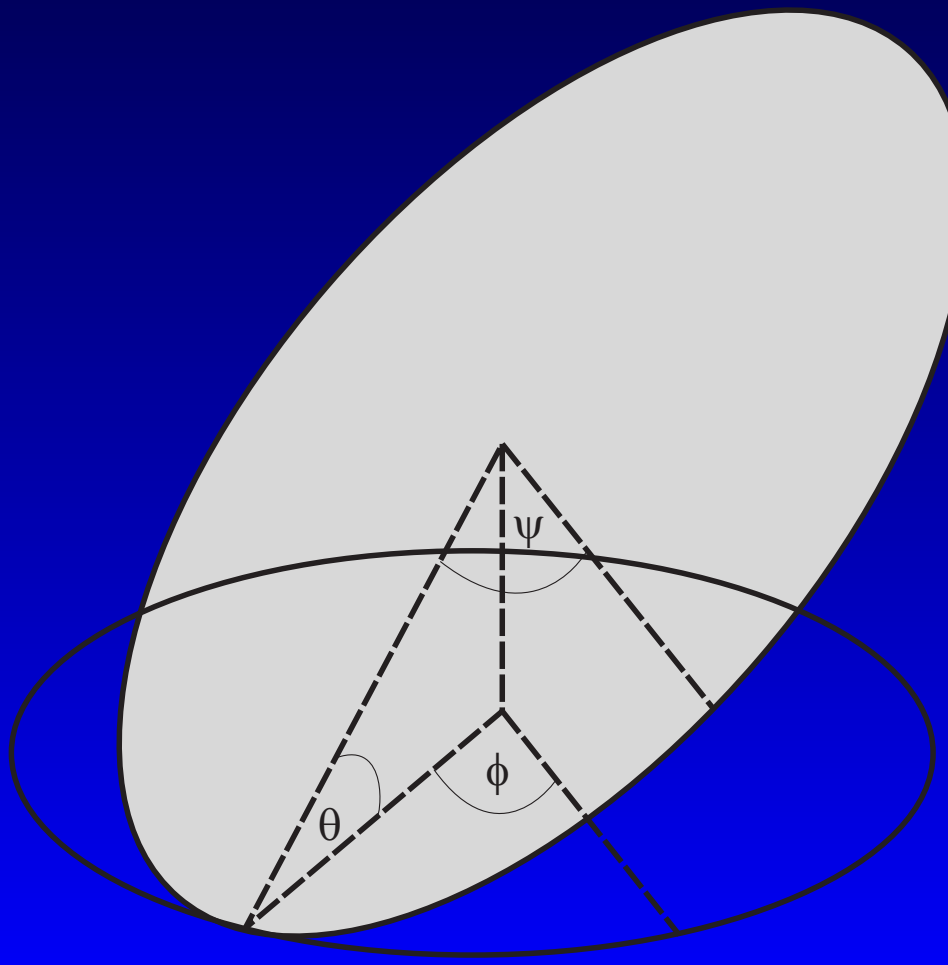
Video

Experiments

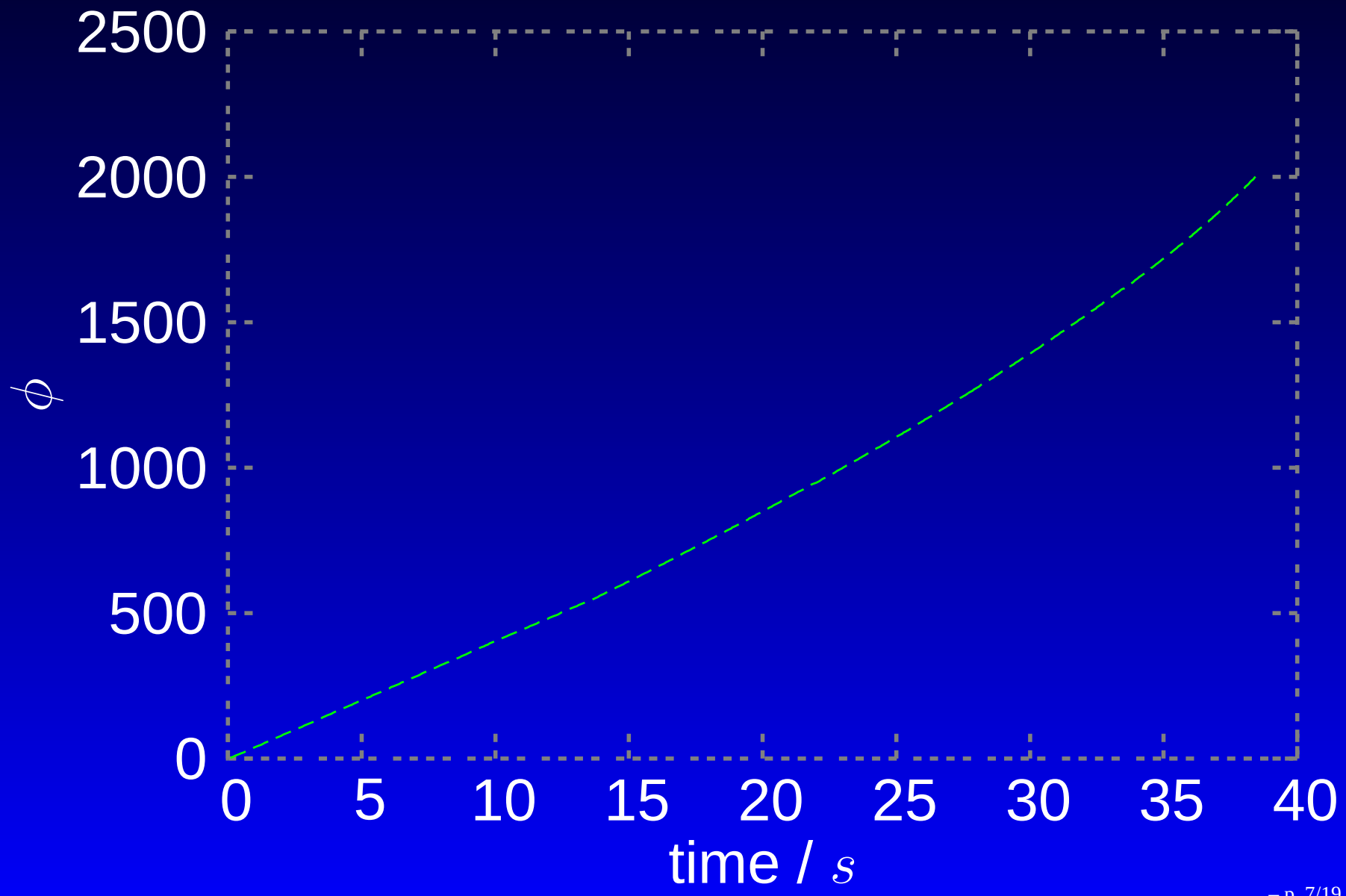
- Several disks, variation in diameter, thickness, mass, material
- Spun on table, granite
- Captured with high-speed video (100fps)

The Coin's Motion

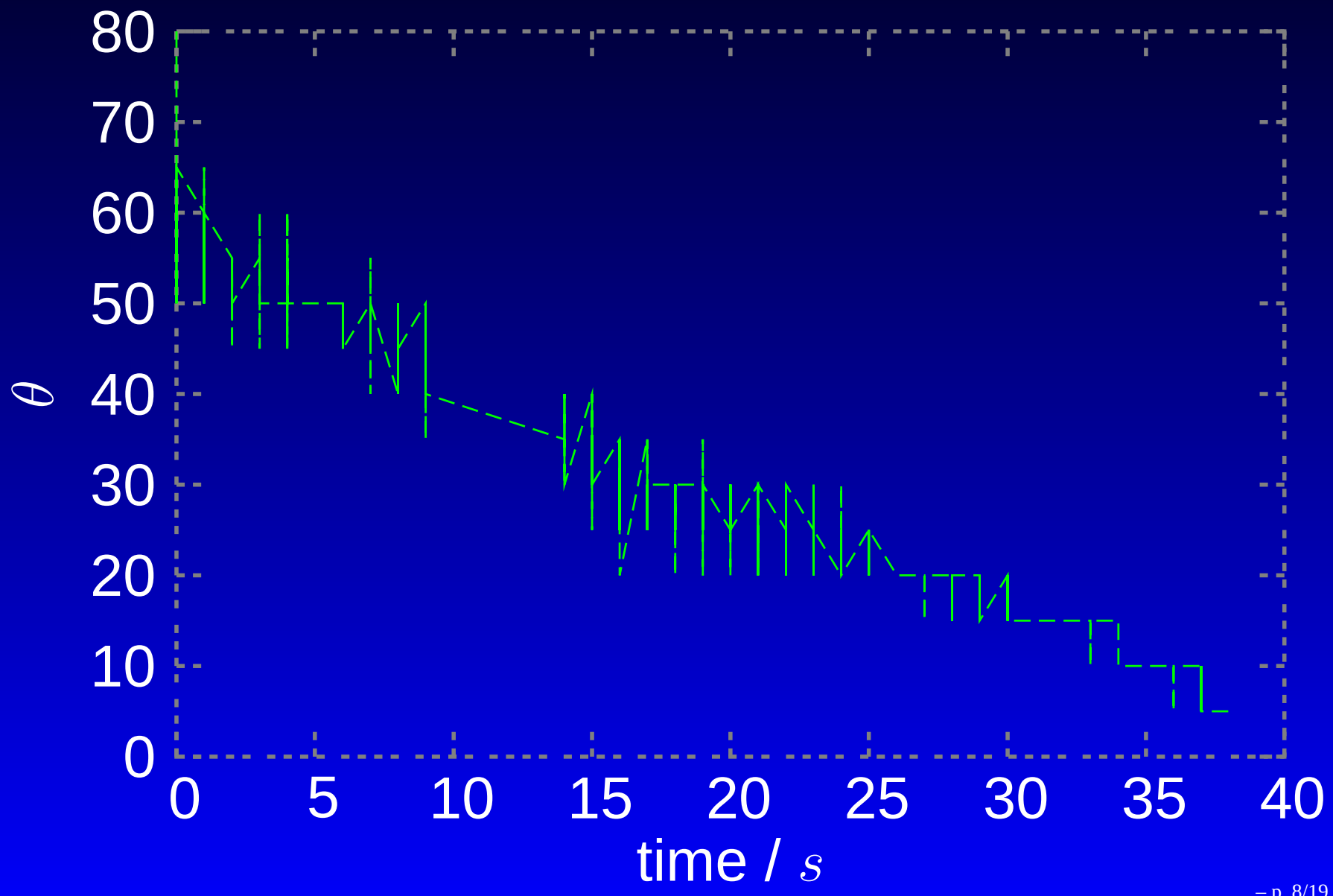
- Motion due to initial momentum
- Rotation



ϕ-t (experiment)



θ-t (experiment)



Equations of Motion

- Lagrange formalism
- Kinetic energy of the system:

$$T = \frac{\Theta_1}{2}\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right) + \frac{\Theta_3}{2}\left(\dot{\psi} + \dot{\phi}\cos\theta\right)^2$$

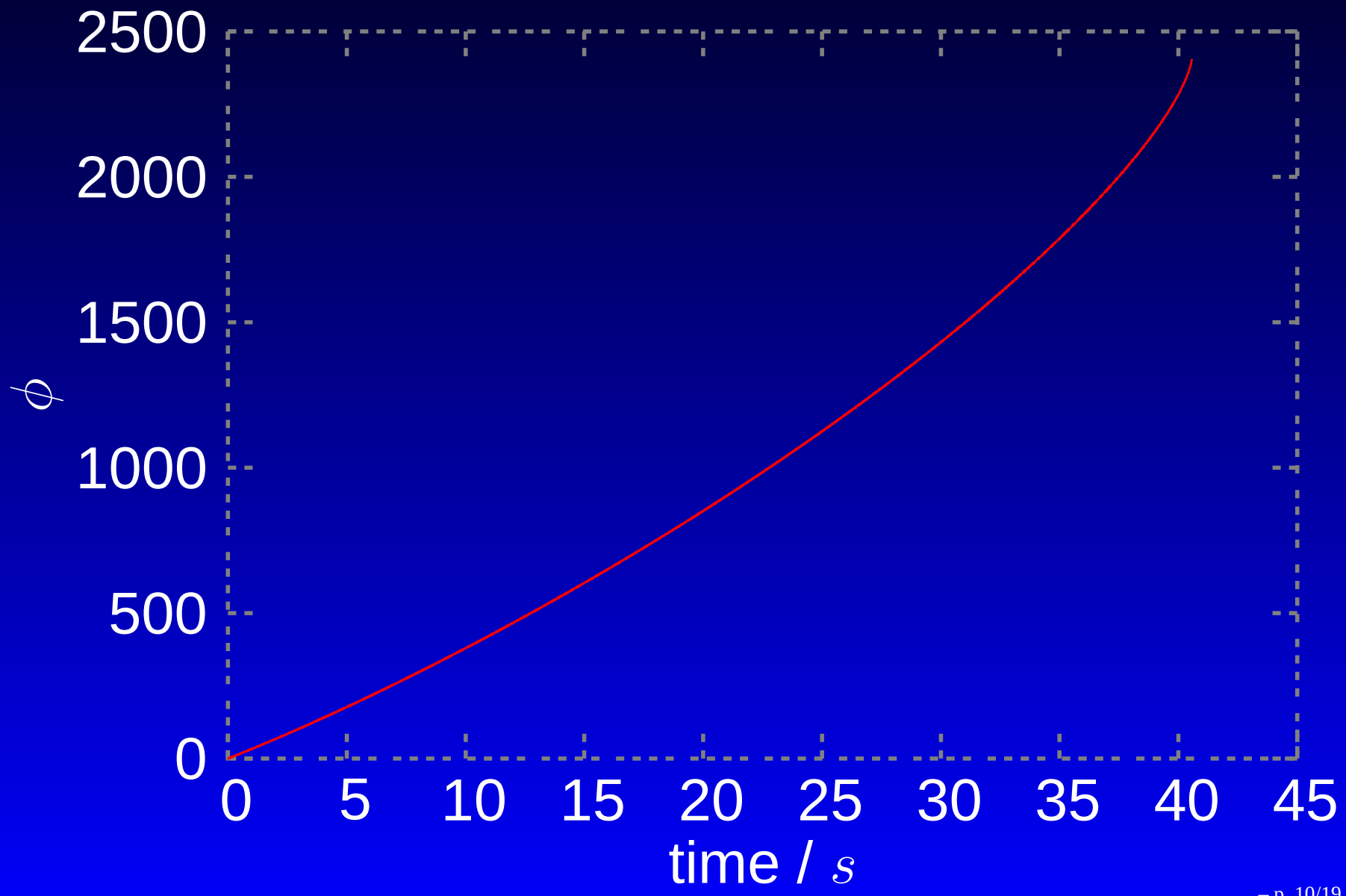
- Potential energy of the system:

$$U = mgr\sin\theta$$

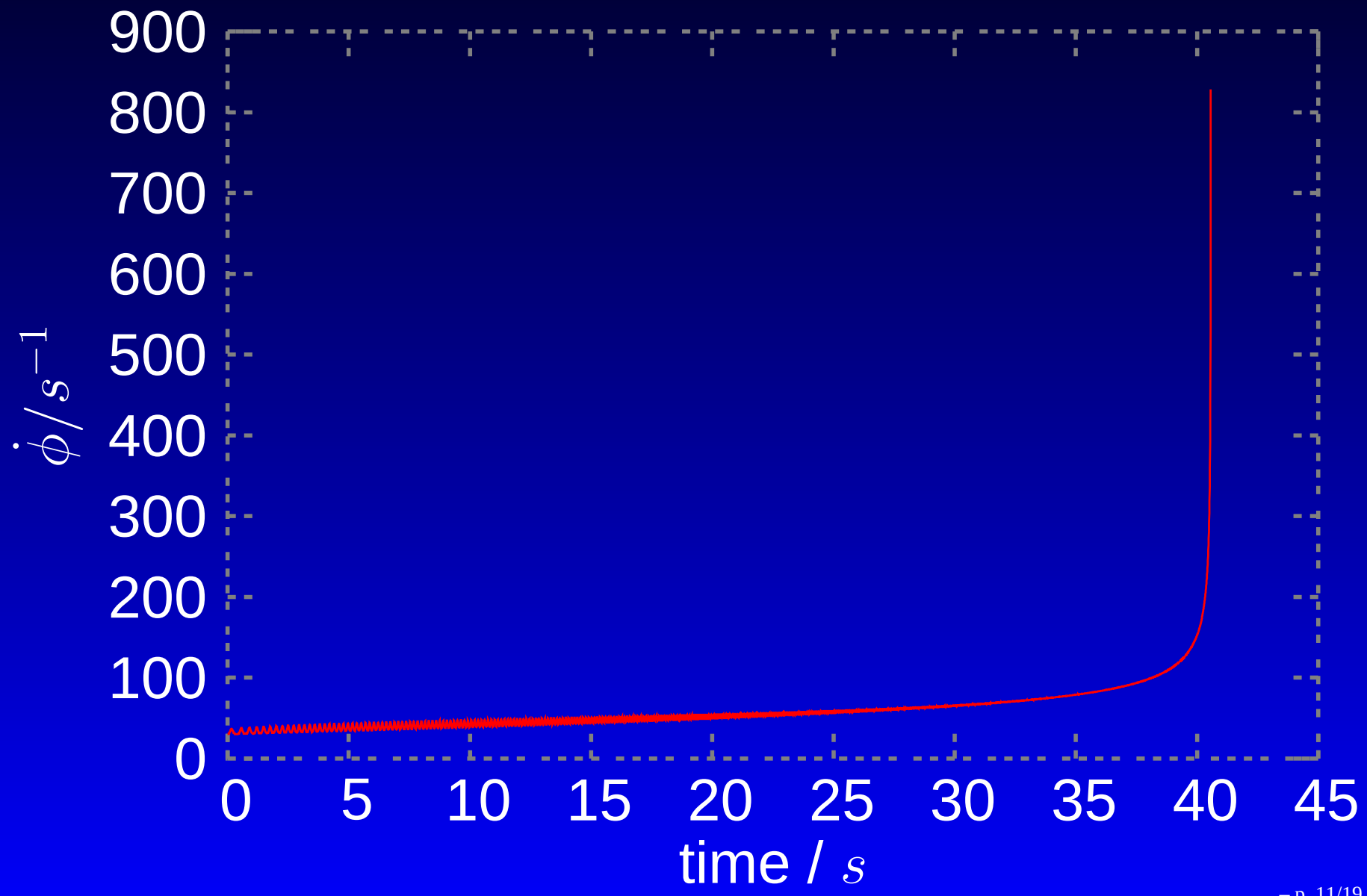
- Rolling friction

$$F_{\mathrm{R}} = \mu r m\left(g + r\frac{\partial^2 \sin\theta(t)}{\partial t^2}\right)$$

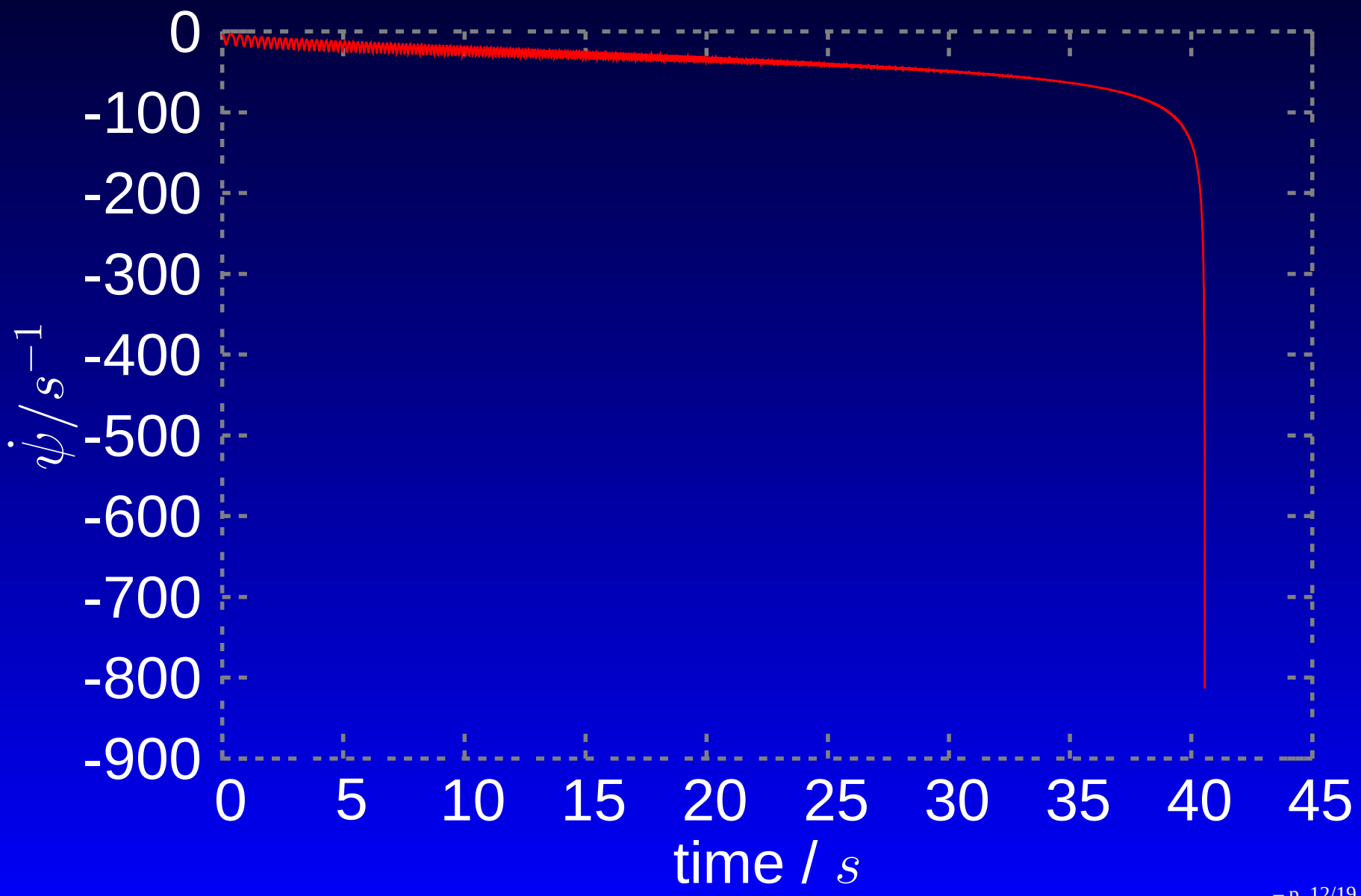
ϕ-t (theory)



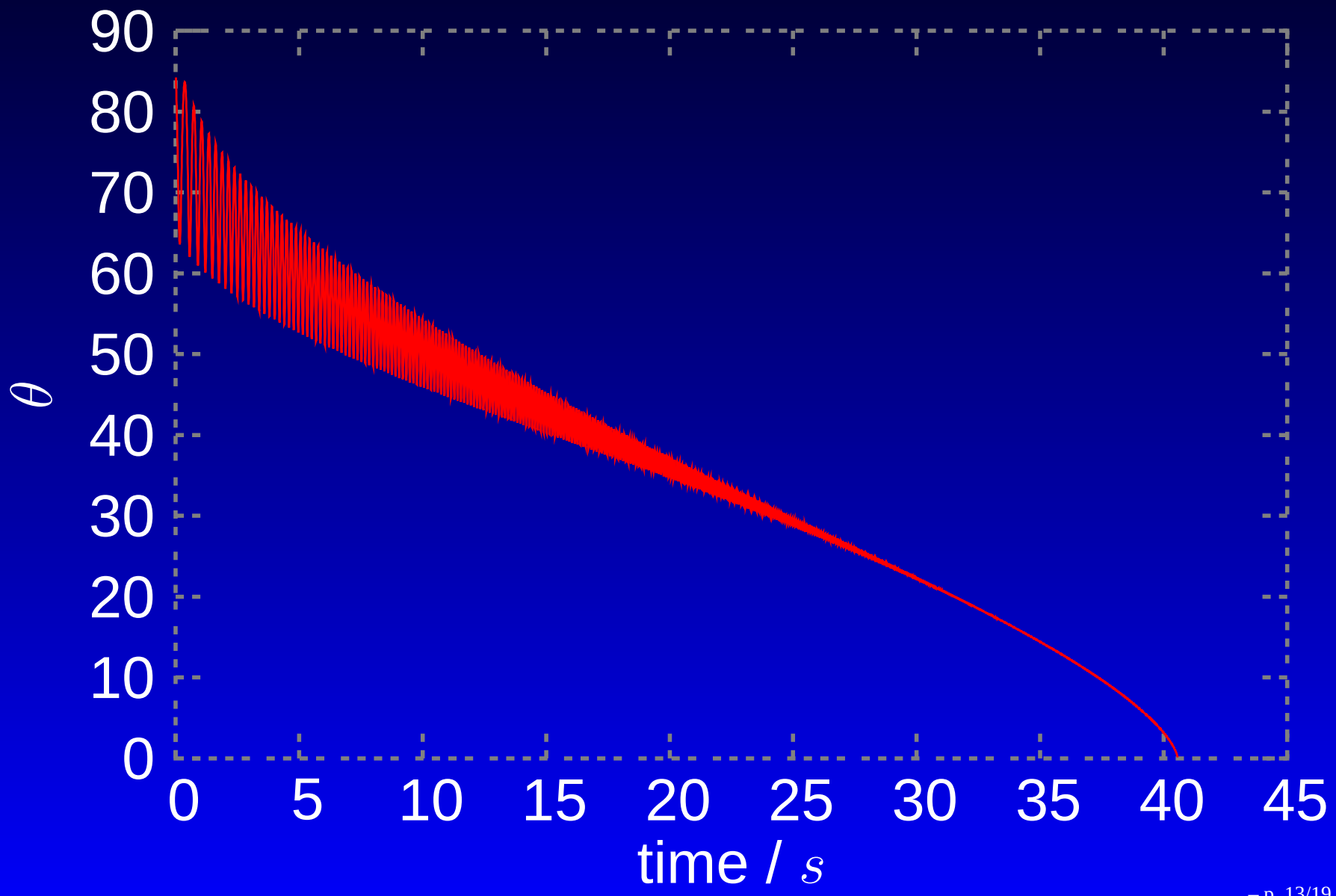
$\dot{\phi}$-t (theory)



$\dot{\psi}$-t (theory)



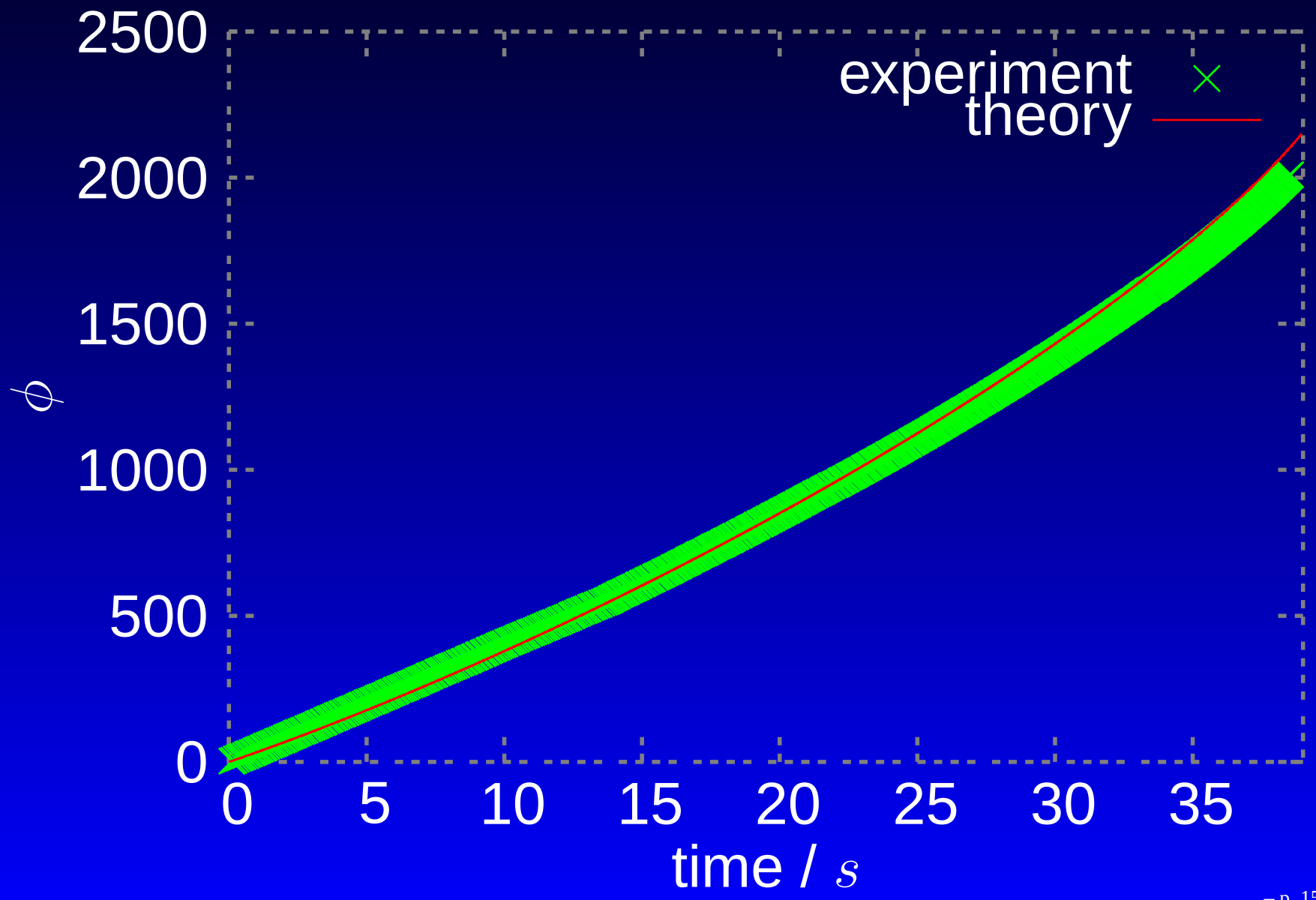
θ-t (theory)



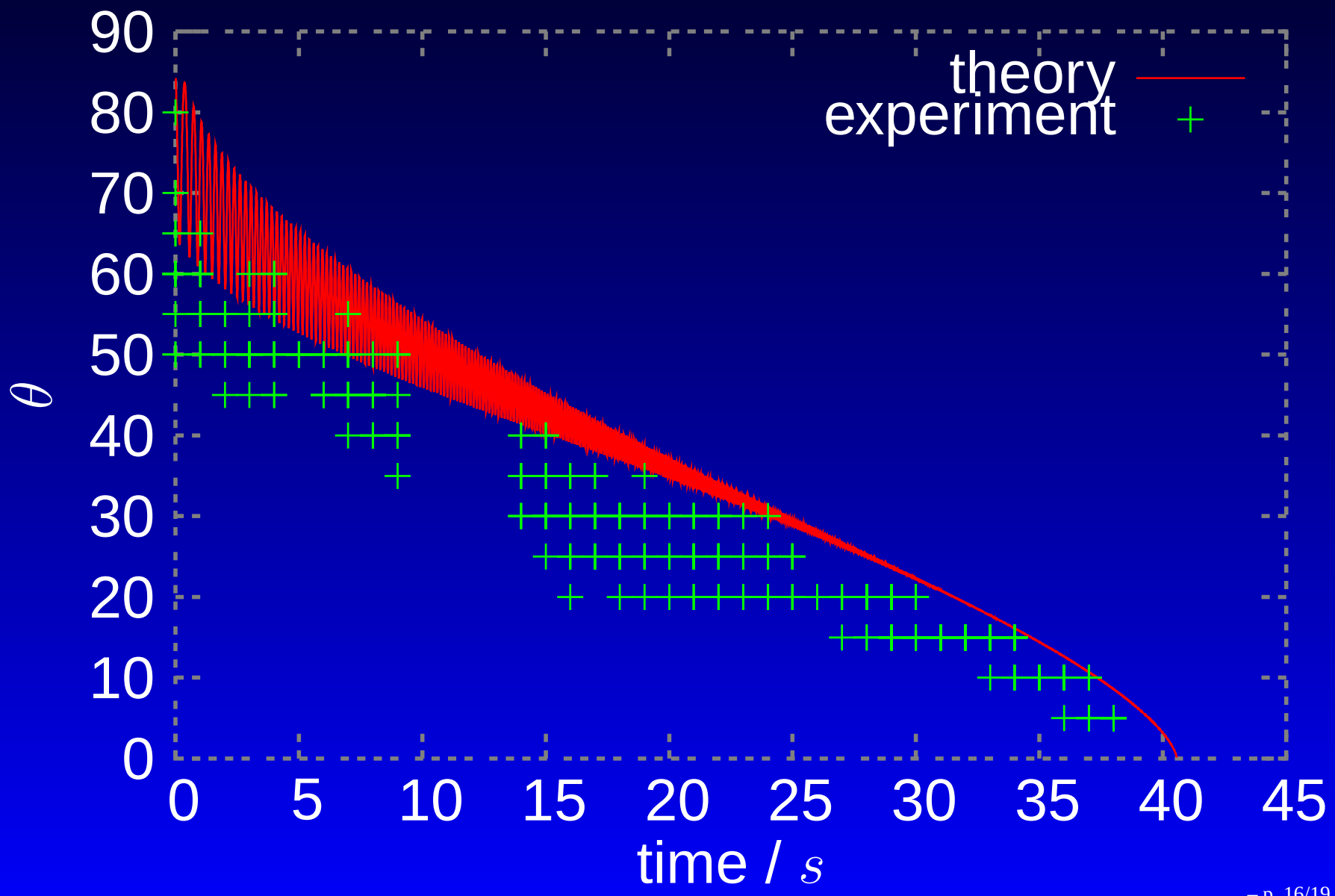
Simulation

- Lagrange formalism: System of coupled differential equations
- Assumption for simulation:
  - quasi-statical approach
  - no oscillations of θ
  - projection of center of mass of the disk constant

Experiment vs. Theory: ϕ



Experiment vs. Theory: θ



Visibility of Image

- Image not visible at beginning
- Image visible in the end
  $\longrightarrow$ although velocity increases

Summary

- Nutation
- Angular velocity of precession goes towards infinity
- Image on coin is visible in the end although $\lim_{t \to t_{\text{still}}} \dot{\psi} = \infty$

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