10. Two Chimneys

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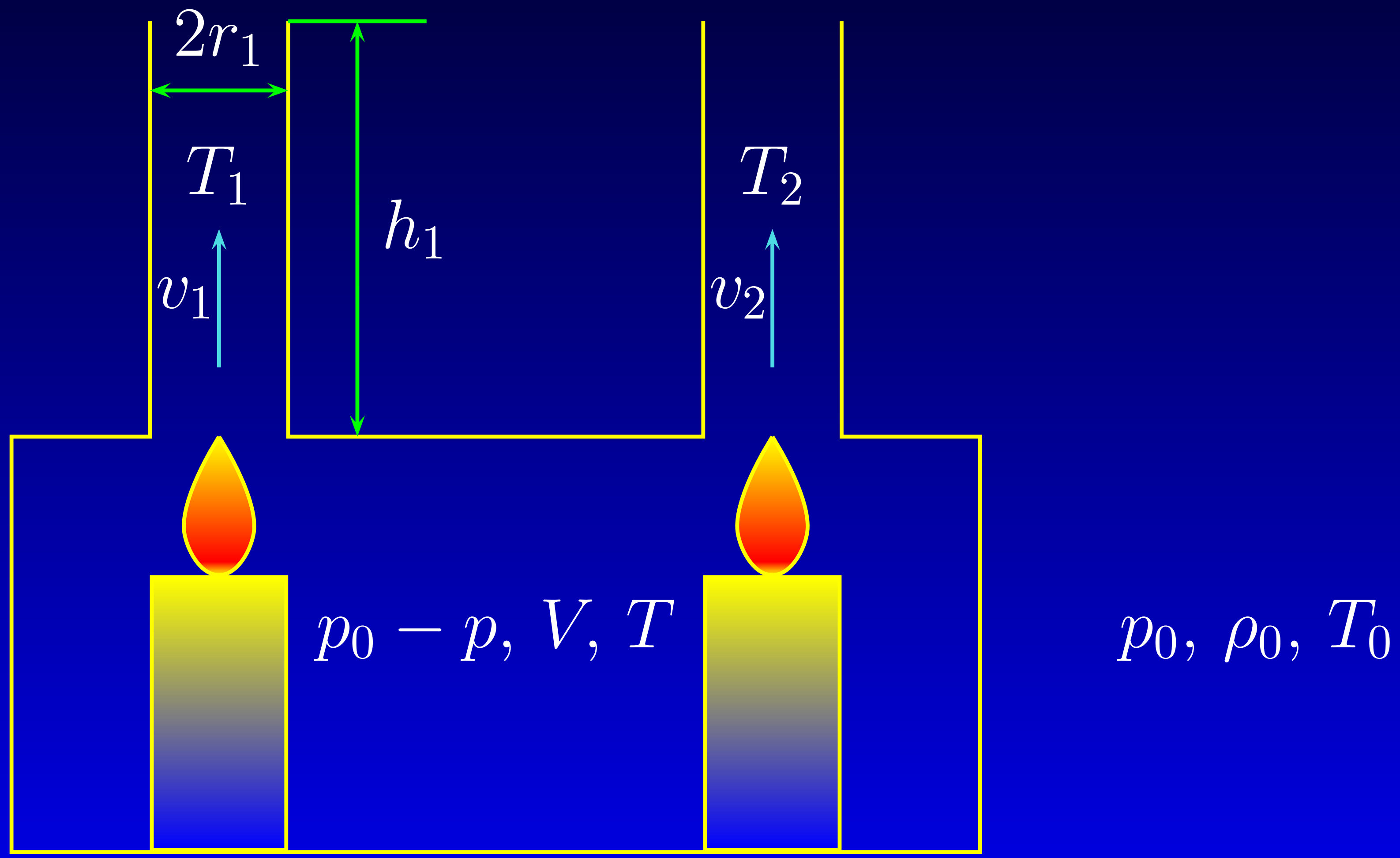
Problem

Two chimneys stand on a box with one transparent side. Under each chimney there is a candle. A short period after the candles are lit one flame becomes unstable. Examine the case and present your own theory of what is happening.

Overview

- Experiments
- Flame behavior
- Theoretical explanation
- Numerical estimates

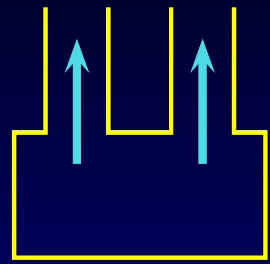
Model



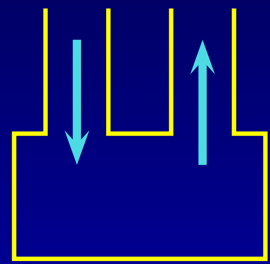
Experiments

- Varying chimneys
  - radius
  - length
- Temperature distribution measurement
- Visualizing air flow with smoke

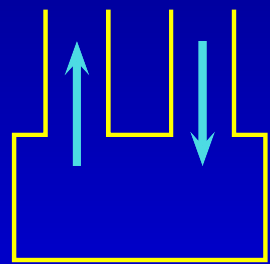
Flame Development



1. Both flames stable



2. One flame becomes unstable



3. Alternating jitter

Air Flow Inside a Chimney

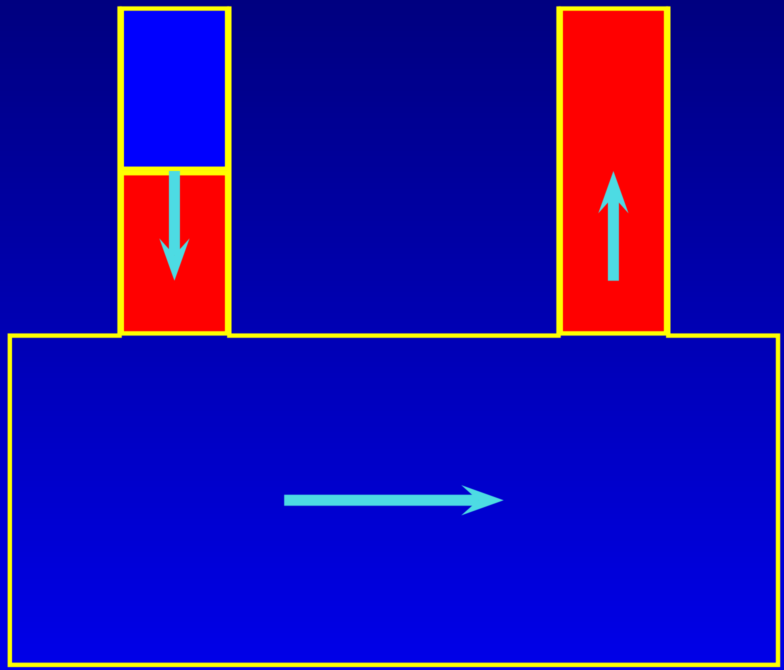
- Air density: $\rho_c = \frac{\rho_0}{x}$, $\quad x = \frac{T_i}{T_0}$
- Pressure: $\Delta p_c = (\rho_0 - \rho_c) g h_c - p$
- Friction can be neglected
- Torricellian formula: $v_c = \pm\sqrt{\frac{2\Delta p_c}{\rho_c}}$

$$v_c = \sqrt{2 g h_c \left(x - 1\right) - 2 \frac{p}{\rho_0} x}$$

- $p = 0 \Rightarrow v_c > 0$
- $p = p_{\text{cr}} = \rho_0 g h_c \left(1 - \frac{1}{x}\right) \Rightarrow v = 0$

Instability

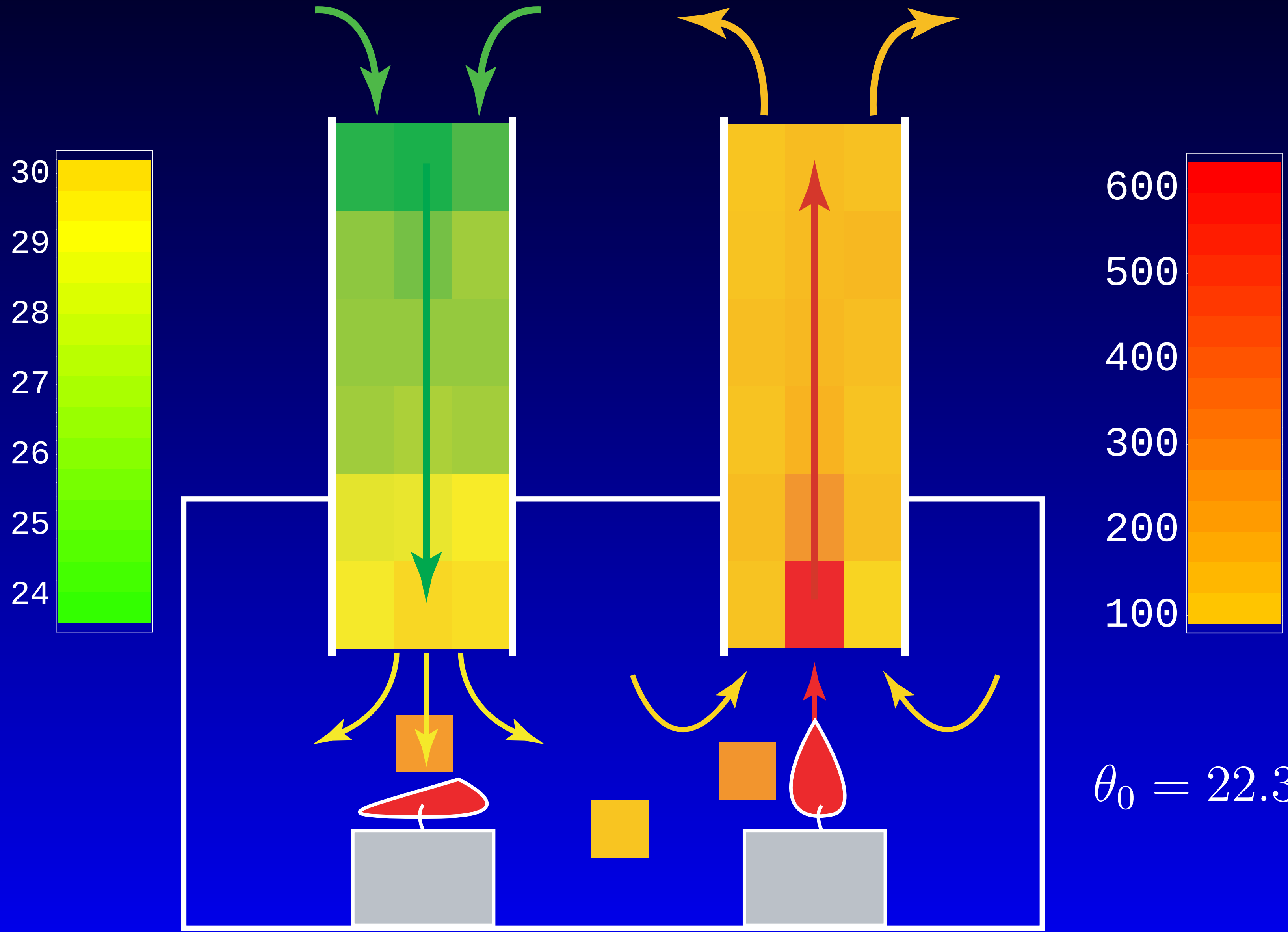
- h_1 decreases because of fluctuations
- $\Rightarrow \quad v_1 < 0$
- $\Rightarrow \quad p$ decreases
- $\Rightarrow \quad v_2 > 0$



Stable Flow

- $r_1^2 v_1(p) + r_2^2 v_2(p) = 0$
- $p_{\text{St}} = \rho_0 g h \frac{x-1}{x+1} \left(\frac{r_{\text{out}}}{r_{\text{in}}} \right)^4$
- $p_{\text{St}} < p_{\text{Cr}}$ for $r_{\text{out}} < r_{\text{in}} \sqrt[4]{1 + \frac{1}{x}}$
- Otherwise only stable flow in reverse direction possible
- Experimental confirmation: flame becomes unstable under the thicker chimney

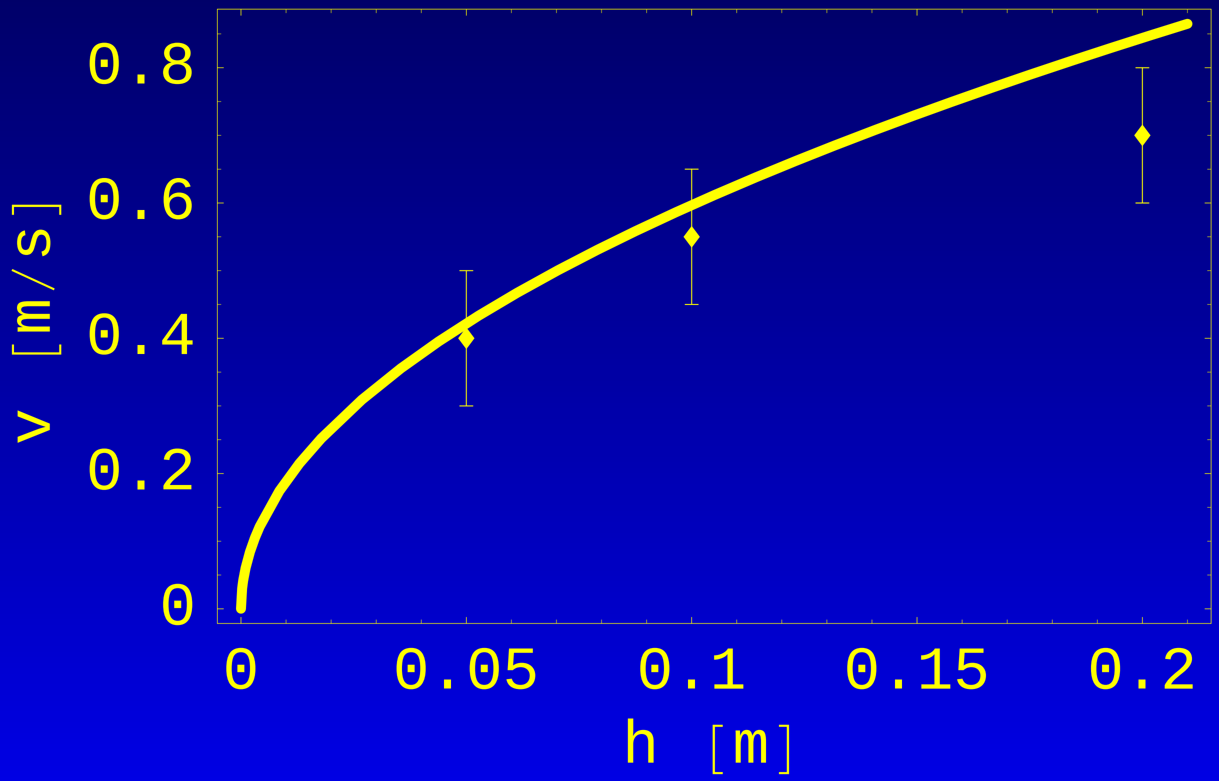
Measurement Results



$\theta_0 = 22.3\,°\mathrm{C}$

Experimental Confirmation

- Flow velocity measured with hand held anemometer
- Comparison with $v(p_{\mathrm{St}})$



Pressure Decrease

- Gas equation:

$$p_i V_i = N_i R T_i$$

- Pressure change:

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \dot{p} = \frac{p_0 T_{\mathsf{box}}}{V_{\mathsf{box}}} \sum_{i=1,2} \frac{A_i v_i}{T_i}$$

Time Constants

- Time delay till flame instability

Pressure equalization

$$\tau_p \approx \frac{2p_{\mathrm{cr}}}{\dot{p}}$$

$$\tau_p \sim 10^{-8}\,\mathrm{s}$$

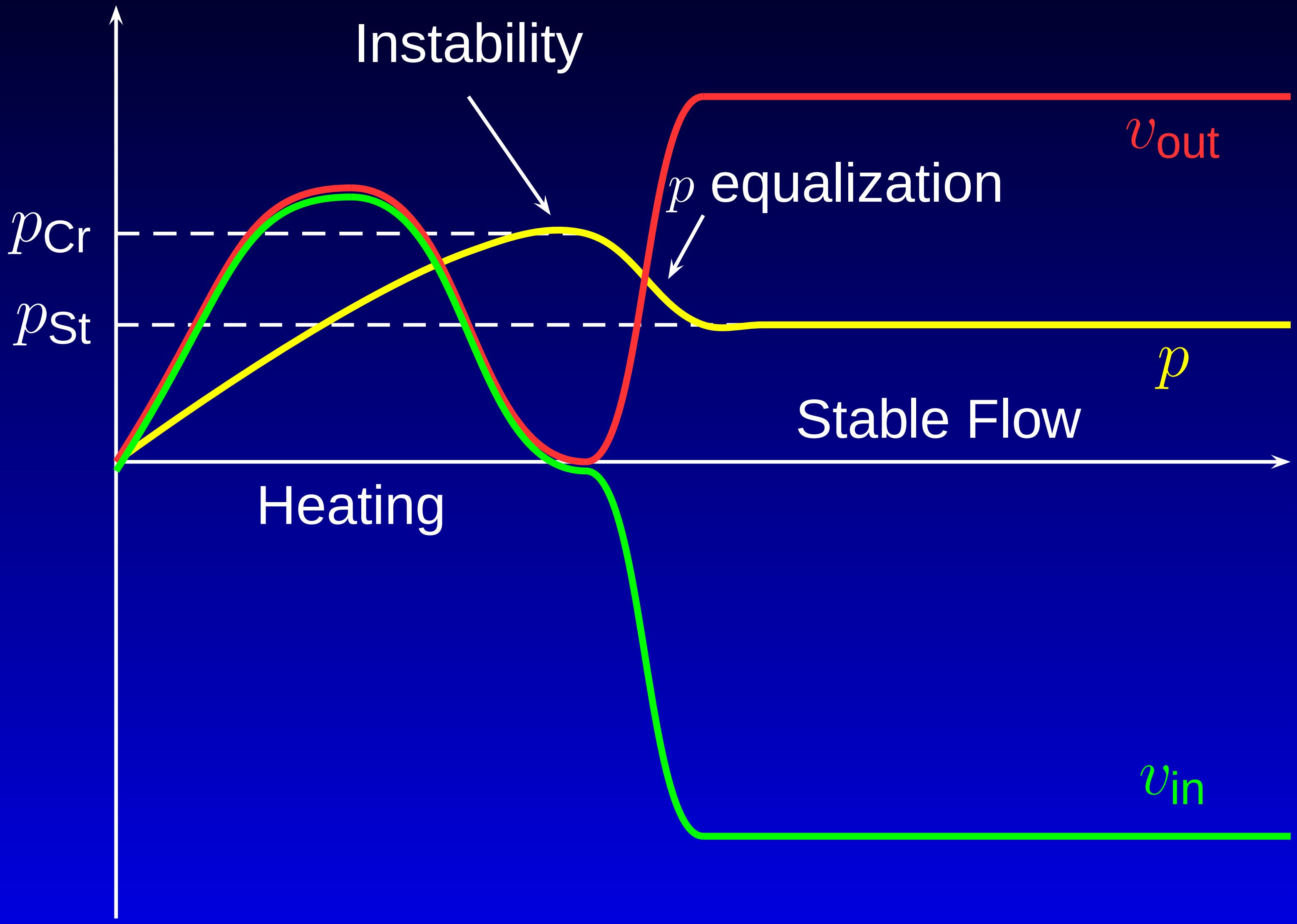
Heating

$$\tau_T \approx \frac{\rho_0 c_p (T_c - T_0)\pi r^2 h}{\mu P}$$

$$\tau_T \sim 1 - 5\,\mathrm{s}$$

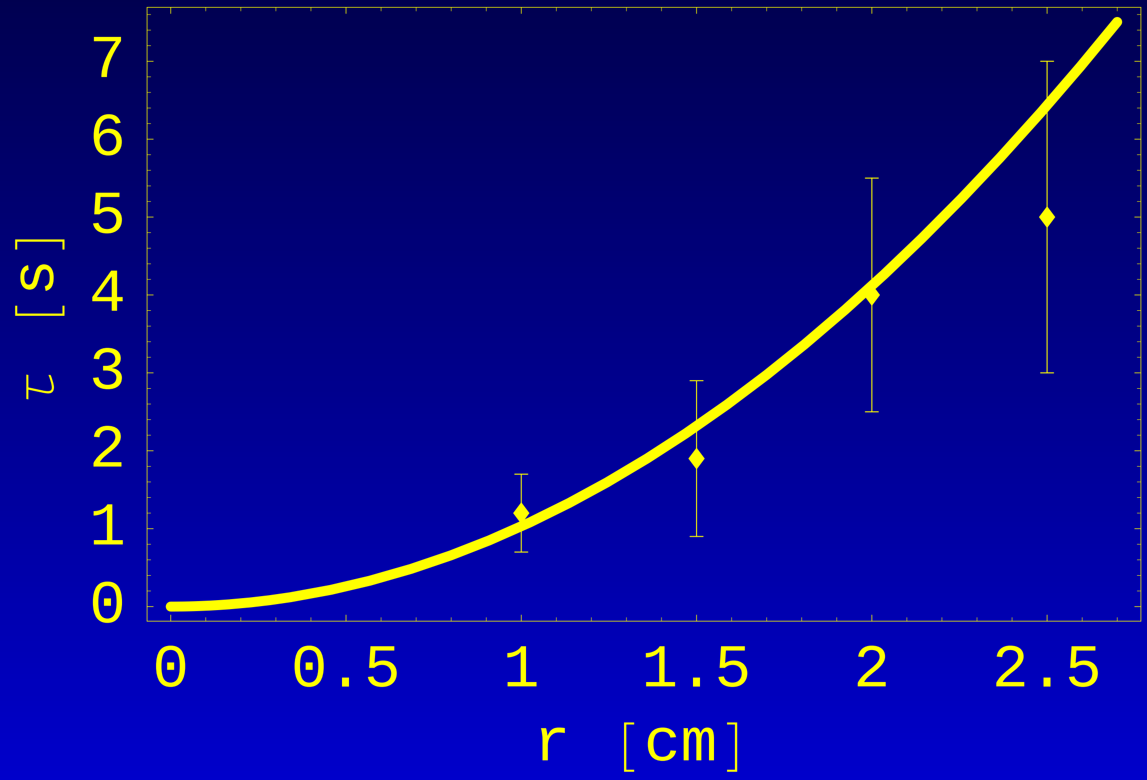
- Heating is the dominating time parameter

Process Flow



Experimental Proof

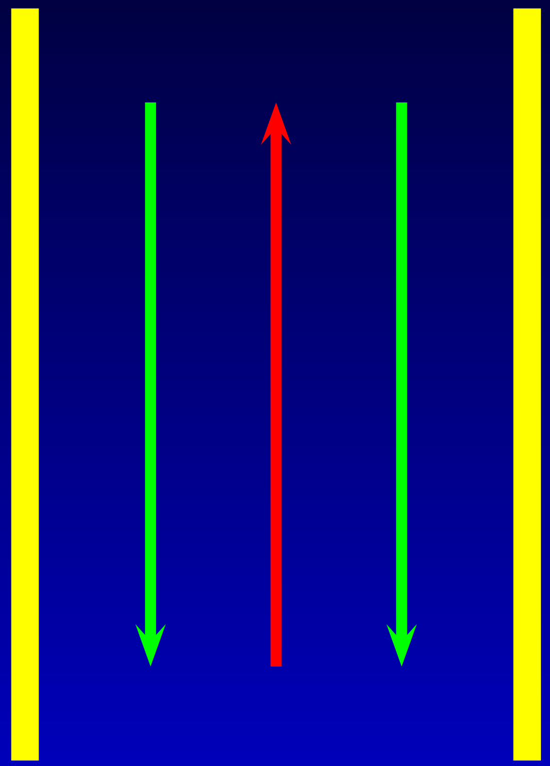
- $r < 0.5\,\mathrm{cm} \Rightarrow \tau \ll 1\,\mathrm{s}$: Chaotic jitter
- $1\,\mathrm{cm} \leq r \leq 2.5\,\mathrm{cm}$: Good agreement



- $r \geq 3.5\,\mathrm{cm}$: Alternating jitter due to reverse convection flow

Flow for Big r

- Reverse flow causes additional instability



- For certain r convection cells stabilize the flow; no jitter

Summary

- Wide range of experimental parameters
- Theoretical explanation
  - Deterministic parameters
  - Indeterministic influences
- Theory vs. Experiment

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