11. String Telephone

Matthias Müller

German Team

Task

How do the intensity of sound, transmitted along a string telephone, and the quality of communication between the transmitter and receiver depend upon the distance, tension in the line and other parameters? Design an optimal system.

Overview

- Eperimental Set-up
- Theory – Quality and Intensity Losses in Different Parts of the Telephone
  - Transmission into String and Out of it
  - Damping and Dispersion in String
  - Parametric Optimiziation – Materials, Dimensions
- Experimental Confirmation

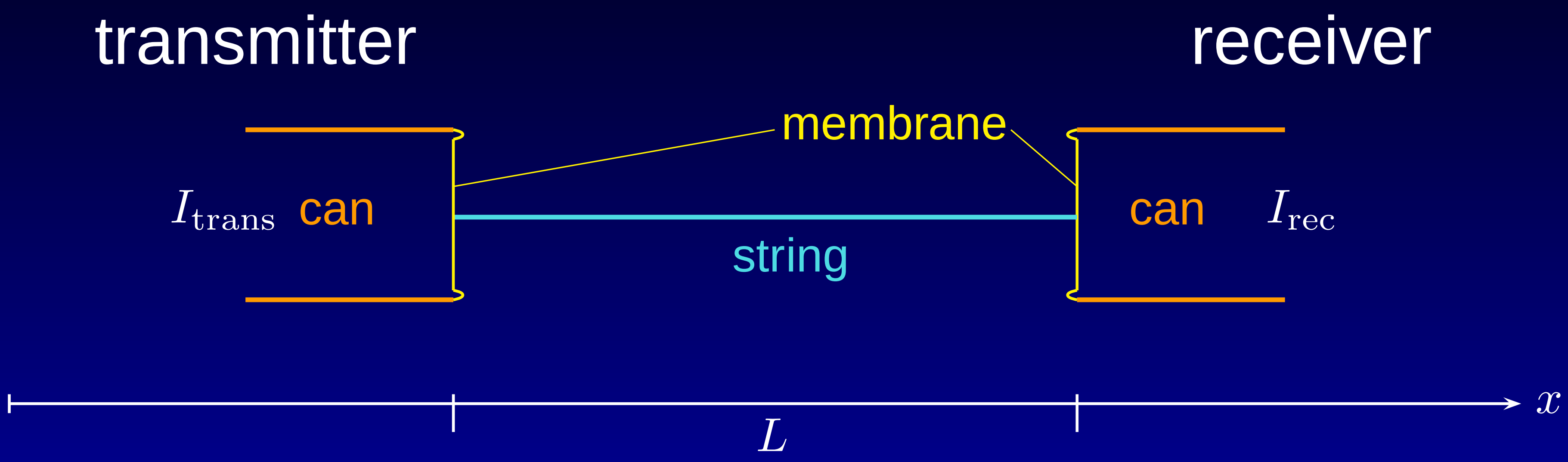
Properties of Optimal System

- High quality of communication
  - low dispersion
  - low frequency-dependence of transmitted intensity (no resonance in speaking range)
- Only of secondary importance:
  - transmitted power as high as possible

Shape of Horns

- Horns with no resonance within range
- Exponential horns (no reflection $\Rightarrow$ no resonance)
- Short cans
  - Eigenfrequency above $1000\,\mathrm{Hz}$ for length smaller $\approx 5\,\mathrm{cm}$

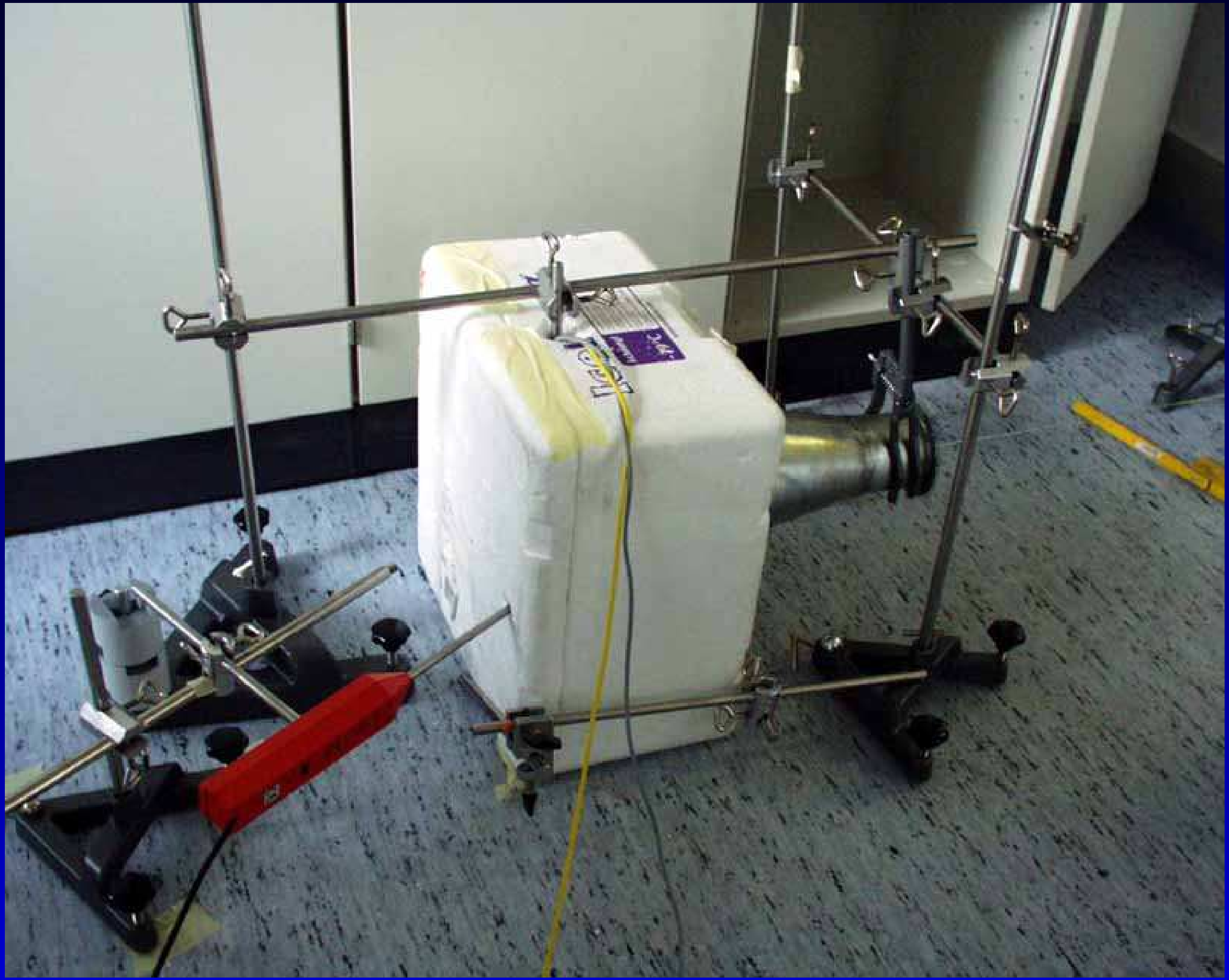
Experimental Set-up



3 important parts:

- Transmission from can into string
- Damping and dispersion in the string
- Transmission from string to can

Experimental Set-up



Intensity of Sound Wave

$P = \frac{1}{2}\rho c A \hat{v}^2$

Abbreviation: $Z \equiv \rho c$, $Z' \equiv ZA$

- Power: $P = \frac{1}{2}Z'\hat{v}^2$
- Intensity: $I = \frac{P}{A} = \frac{1}{2}Z\hat{v}^2$

Transmission through Membrane

Transmission of wave from one media through membrane into other media.

$$P_{\mathrm{trans}} = \alpha_{\mathrm{mb}}(P_{\mathrm{inc}} - P_{\mathrm{refl}})$$
$$v_{\mathrm{trans}} = v_{\mathrm{mb}} = v_{\mathrm{inc}} - v_{\mathrm{refl}}$$

Remember $P = \frac{1}{2} Z' \hat{v}^2$

Therefore

$$\frac{P_{\mathrm{trans}}}{P_{\mathrm{inc}}} = \frac{4\alpha_{\mathrm{mb}}^2 Z'_{\mathrm{trans}} Z'_{\mathrm{inc}}}{(Z'_{\mathrm{trans}} + \alpha_{\mathrm{mb}} Z'_{\mathrm{inc}})^2}$$

Oscillation of Membrane

transmitter-Membrane

Equation of motion for small piece of membrane:

$$\sigma \frac{\partial^2 z}{\partial t^2} + Z_{\mathrm{a}} \frac{\partial z}{\partial t} + T \Delta z + \mathrm{e}^{-kr^{2b}} \left(F_0 + Dz \right) = p(t)$$

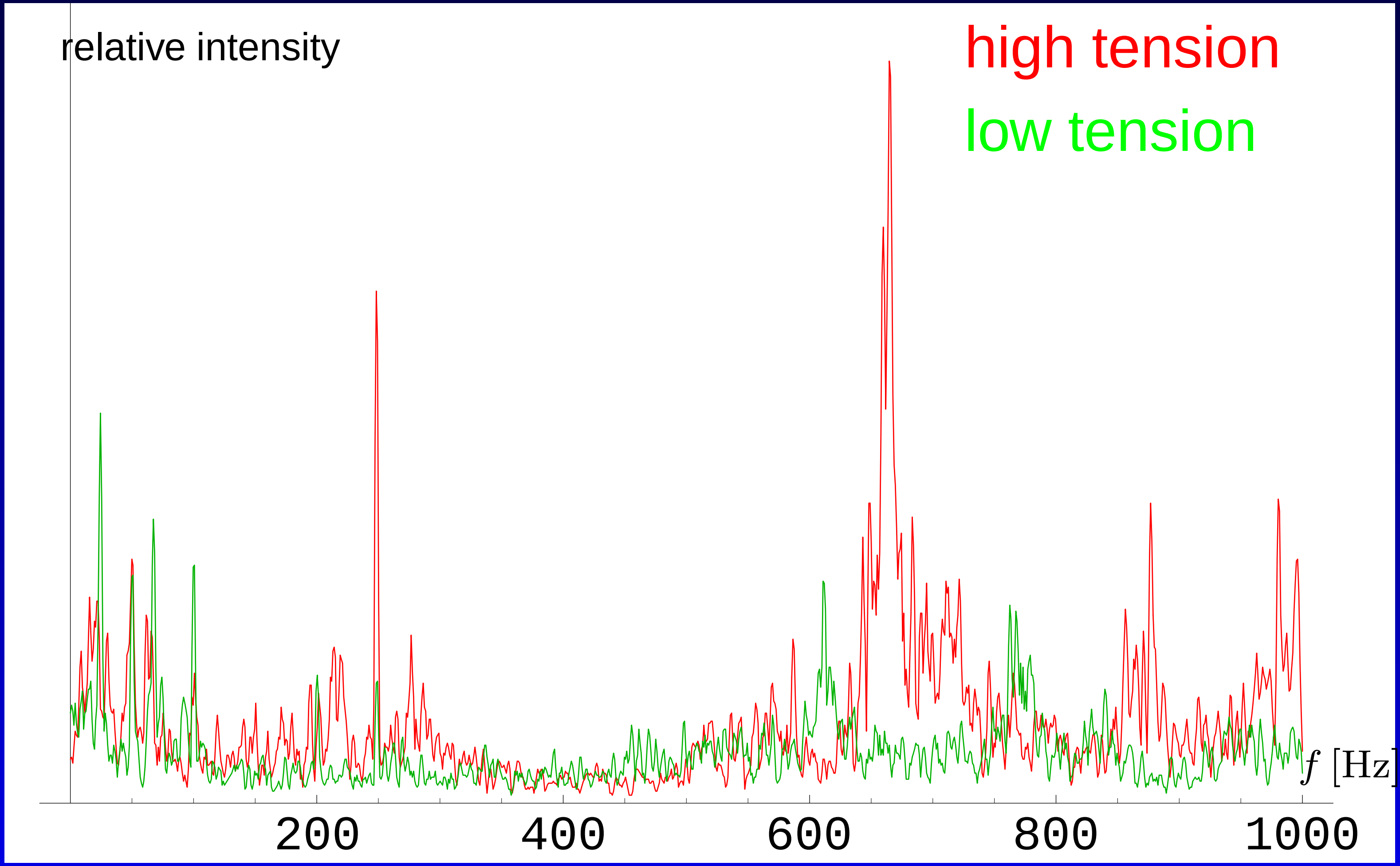
r	Radial Coordinate of membrane
$z(r,\phi,t)$	Elongation
T	Tension of membrane
$F_0 + Dz$	Force excerted by string
$p(t)$	Incoming wave

Identities of Good Membrane

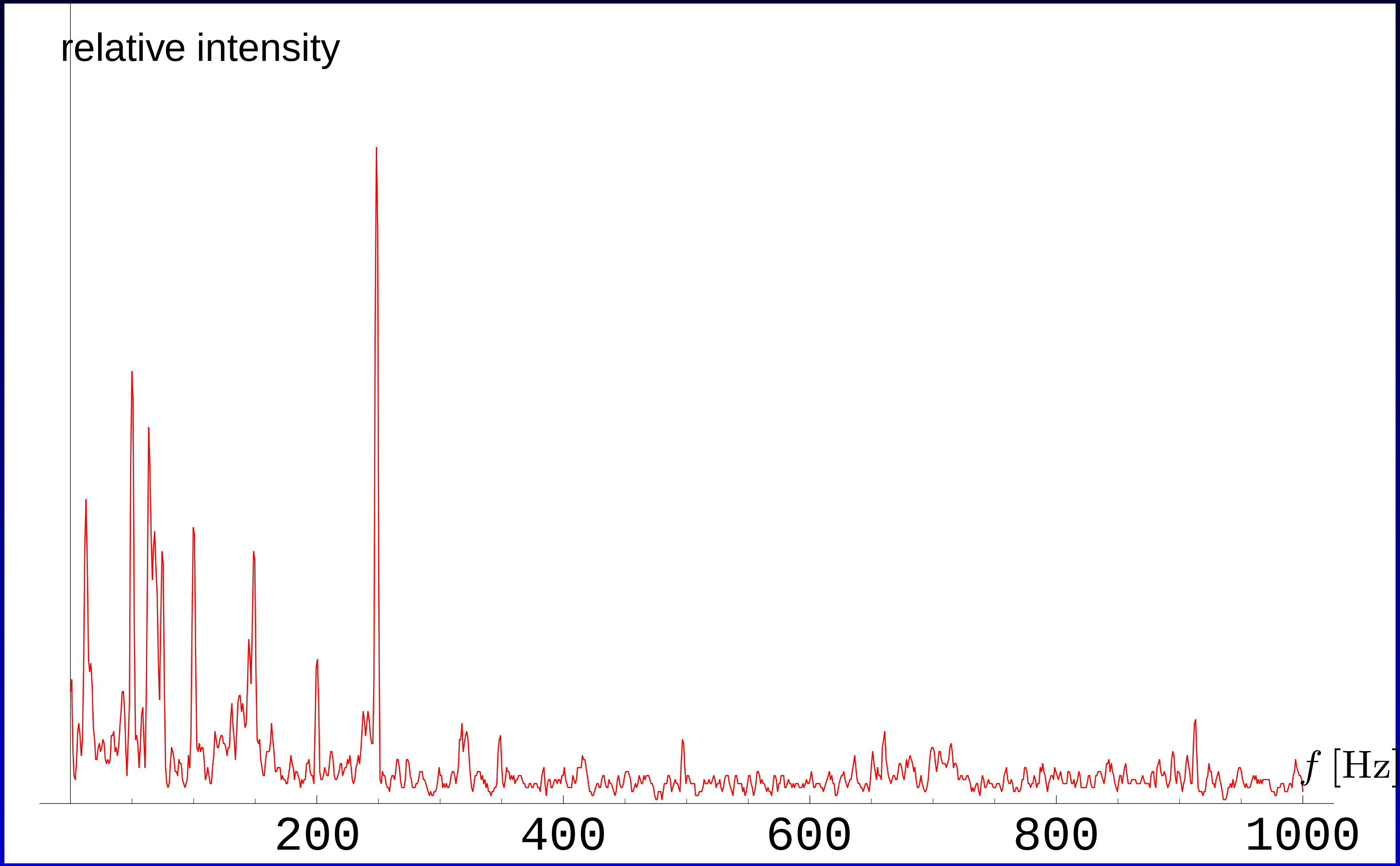
- Good quality for high resonance frequencies
- High frequencies for low mass, high tension, stiffness
- Membrane should be stiff
- Tension in membrane increases with tension in thread

Optimization of Membrane

Input: White Noise; tension: $30\,\mathrm{N}$, $40\,\mathrm{N}$, length: $8.5\,\mathrm{m}$

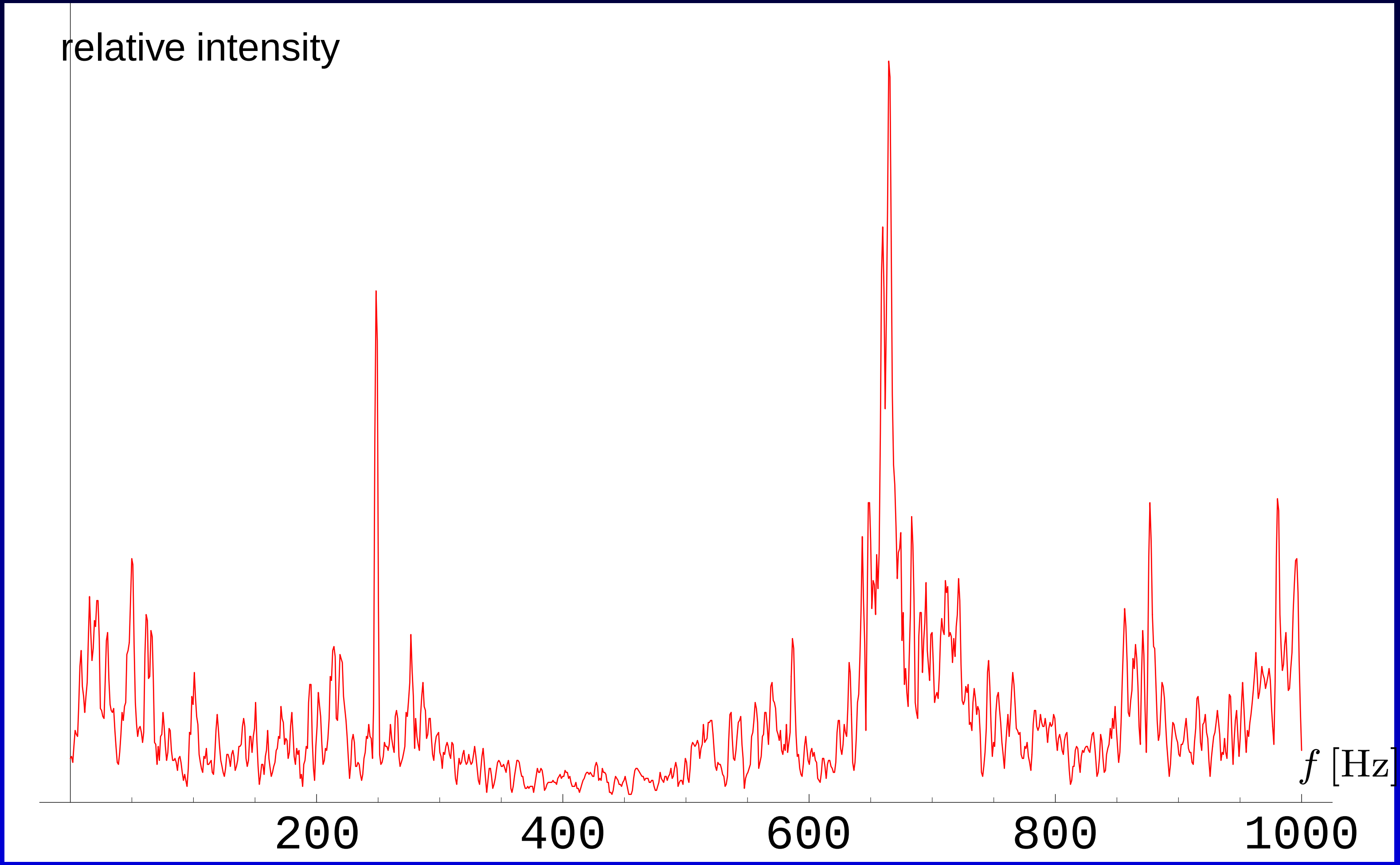


Noise without Rush



Good Membran

Input: White Noise; tension: $40\,\mathrm{N}$, length: $4.5\,\mathrm{m}$



Minimizing of String-Damping

- High tension, thin string to eliminate bends in string
- Intensity of damped wave:

$$I = I_0 \, \mathrm{e}^{-2\alpha \, L}$$

$\alpha = \alpha_0 \omega$ calculated by thermal losses

	rubber	aluminium	iron	titanium
$\alpha_0 \, [10^{-6} s/\mathrm{m}]$	110	11	8.2	5.4

Dispersion in String

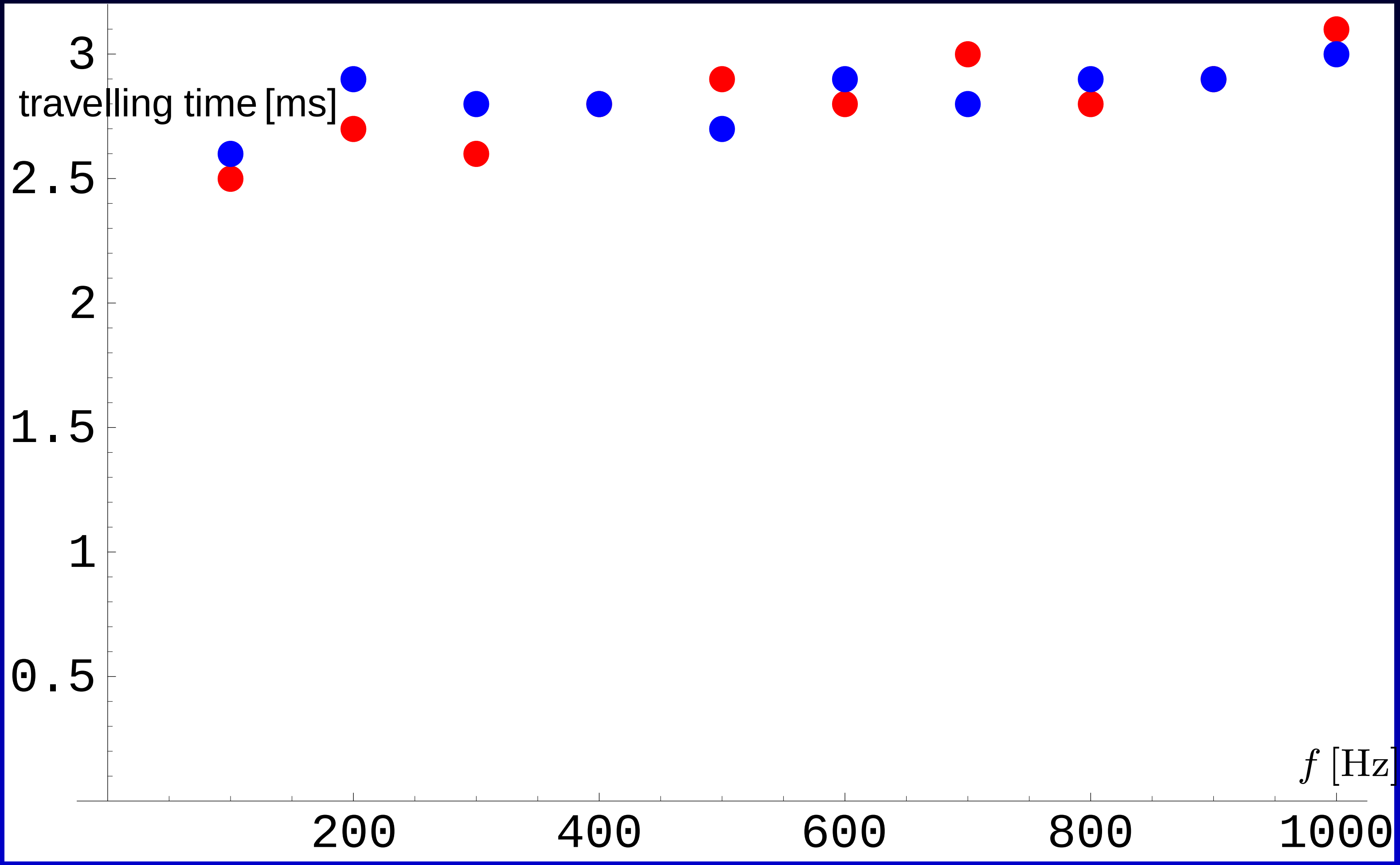
- Dispersion if phase velocity c depends on frequency
- Occurs due to damping:

pressure wave: $p = \hat{p}\mathrm{e}^{\jmath(\omega t - \gamma x)}$

$$\frac{\omega}{c'} = |\gamma(\omega)| = |\beta + \jmath\alpha_0\omega|$$

- Low damping $\rightarrow$ low dispersion

Measurement of Dispersion

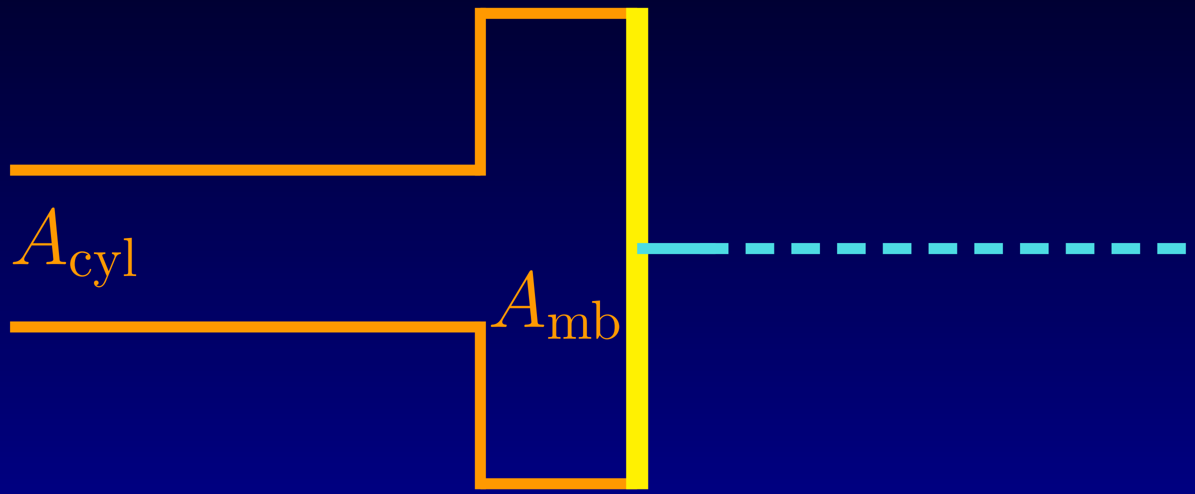


Overall Transmission

$$I_{\mathrm{rec}} \approx P_{\mathrm{trans}} \cdot \frac{1}{A_{\mathrm{cyl}}} \left(\frac{4\alpha_{\mathrm{mb}} A_{\mathrm{mb}} Z_{\mathrm{cyl}}}{A_{\mathrm{str}} Z_{\mathrm{str}}} \right)^2 \cdot \mathrm{e}^{-2\alpha_0 \omega L}$$

- A_{cyl} should be small
- A_{mb} should be large
- problem as A_{mb} at end of can with A_{cyl}

Pressure Chamber



Effect of pressure chamber:

- Larger A_{mb}
  - better transmission from can to string
- Lower A_{cyl}
  - intensity $I = \frac{P}{A}$ highest at opening

New Overall Transmission

Including reflection can $\leftrightarrow$ pressure chamber

$n \equiv \frac{A_{\mathrm{mb}}}{A_{\mathrm{cyl}}}$

$$I_{\mathrm{rec}} \approx P_{\mathrm{trans}} \cdot A_{\mathrm{mb}} \left(\frac{16 \alpha_{\mathrm{mb}} Z_{\mathrm{cyl}}}{A_{\mathrm{str}} Z_{\mathrm{str}}} \right)^2 \frac{n^3}{(n+1)^4} \cdot \mathrm{e}^{-2\alpha_0 \omega L}$$

High receiver-intensity for:

- Large crossectional area of membrane
- Maximum for $n = 3$, so $A_{\mathrm{cyl}} = \frac{1}{3} A_{\mathrm{mb}}$
- Small wave impedance $A_{\mathrm{str}} Z_{\mathrm{str}}$ of the string
- Low damping in the string

Optimal System

- System with pressure chamber
  - diameter of membrane e.g. $15\,\mathrm{cm}$
  - $\Rightarrow$ diameter of can about $8.7\,\mathrm{cm}$
- Membrane of stiff material, high tension in it
- String of suitable metal (aluminium, titanium,. . .)
- Thin string with high tension (for no bends)
- Damping and dispersion increases with length