# 16. Small Fields

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# Problem

Construct a device based upon a compass needle and use your device to measure the Earth's magnetic field.

# Overview

- Our Device
- Absolute measurement
  - Method
  - Results / Error analysis
- Relative Measurement
  - Method
  - Results / Error analysis
- Verification of Results

# Before we get started...

The Earth's magnetic Field is relatively small!

Any metallic or electric objects can strongly influence measurement accuracy!

Try to minimize any influences!

# Our Device

- Compass needle with angle/B-field-scale
- Additional bar magnet
- Non-magnetic materials
- Measurement of horizontal *and* vertical component possible

# Absolute Measurement Method

- Classic method according to Gauß
- Simple devices:
  - Compass needle
  - Bar magnet with dipole moment $m$
- Two measurements:
  - Torsion oscillation of bar magnet: $mB_{\mathrm{e}}$
  - Compass needle deviation: $m/B_{\mathrm{e}}$

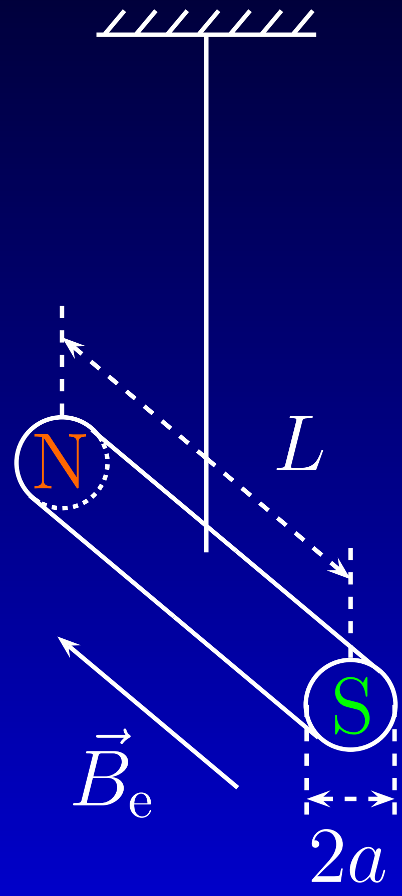
# Torsion Oscillation

- Moment of inertia of cylindric bar:

$$J = M\left(\frac{L^2}{12} + \frac{a^2}{4}\right)$$

- Mechanic torque:

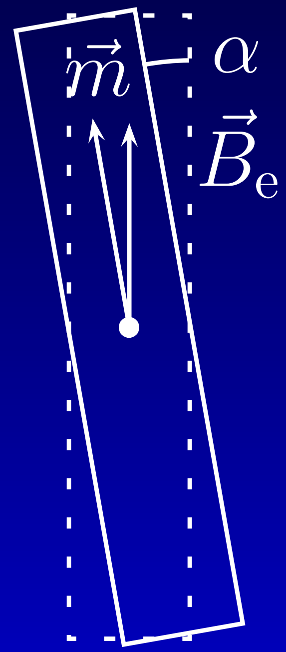
$$T_{\mathrm{mechanic}} = J\ddot{\alpha}$$



# Torsion Oscillation

- $\alpha = \angle(\vec{m}, \vec{B}_\mathrm{e})$
- Magnetic torque:

$$T_\mathrm{magnetic} = mB_\mathrm{e} \sin\alpha \approx mB_\mathrm{e}\alpha$$




- Mechanic torque due to thread negligible:

$$\frac{T_\mathrm{thread}}{T_\mathrm{magnetic}} \ll 1\%$$

# Torsion Oscillation

- Differential equation for harmonic oscillation:

$$\begin{aligned} T_{\mathrm{mechanic}} &= -T_{\mathrm{magnetic}} \\ J\ddot{\alpha} &= -mB_{\mathrm{e}}\alpha \\ \ddot{\alpha} + \frac{mB_{\mathrm{e}}}{J}\alpha &= 0 \end{aligned}$$

$$\Longrightarrow \frac{mB_{\mathrm{e}}}{J} = \omega^2 = \left(\frac{2\pi}{T}\right)^2$$

# Torsion Oscillation Result

- End formula:

$$mB_{\mathrm{e}} = \frac{4\pi^2}{T^2} M \left( \frac{L^2}{12} + \frac{a^2}{4} \right)$$

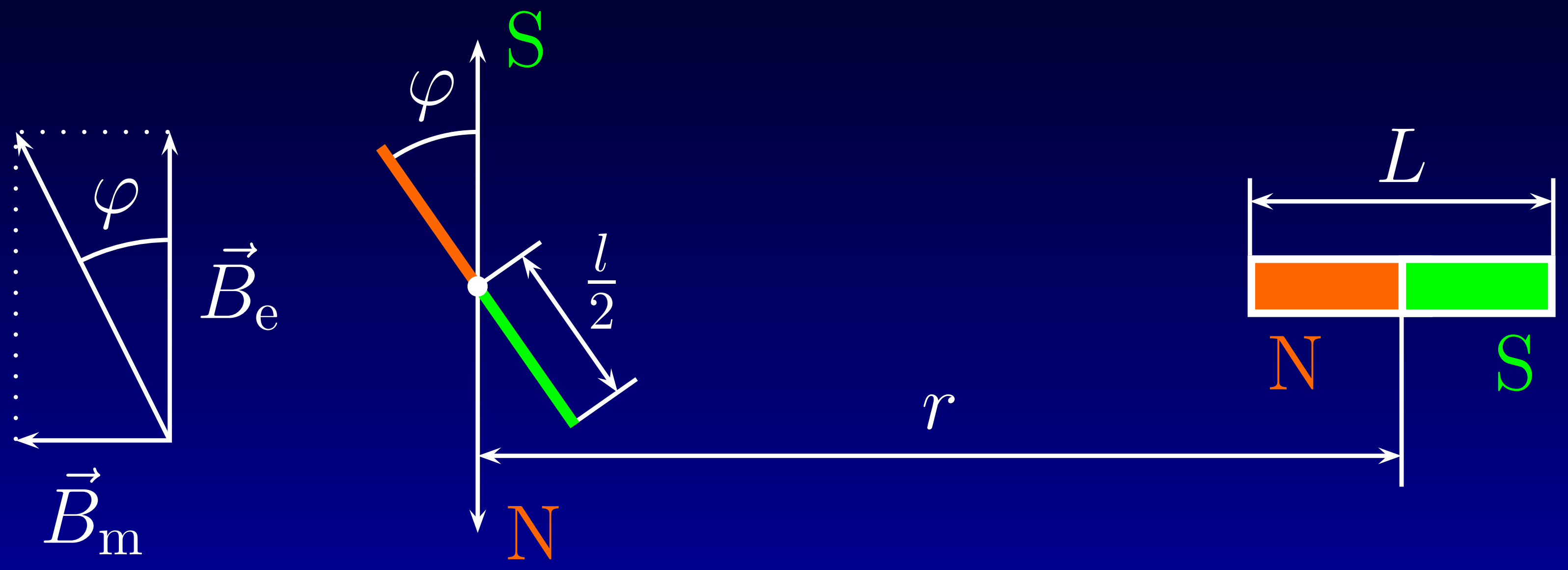
- Measurement:

$$L = 199 \pm 1\,\mathrm{mm}; \qquad 2a = 10 \pm 0.1\,\mathrm{mm}$$

$$M = 101 \pm 0.5\,\mathrm{g}; \qquad 50T = 393 \pm 0.5\,\mathrm{s}$$

$$\begin{aligned} mB_{\mathrm{e}} &= (2.13 \pm 0.02) \cdot 10^{-4}\,\mathrm{Am^2\,T} \\ \varepsilon(mB_{\mathrm{e}}) &= 1.1\,\% \end{aligned}$$

# Needle Deviation




- $\tan\varphi = \frac{B_{\mathrm{m}}}{B_{\mathrm{e}}}$ true for $r \gg \frac{l}{2}, \frac{L}{2}$
- With permanent magnet:

$$\tan\varphi = \frac{1}{B_{\mathrm{e}}} \frac{\mu_0 m}{2\pi r^3} \left(1 - \frac{\zeta}{r^2}\right)$$

# Needle Deviation

$$\frac{m}{B_\mathrm{e}} = \tan\varphi \, \frac{2\pi r^3}{\mu_0} \left( \frac{r^2}{r^2 - \zeta} \right)$$

- Eliminating $\zeta$ by measuring two different $r$
- End Formula:

$$\frac{m}{B_\mathrm{e}} = \frac{2\pi}{\mu_0} \left( \frac{{r_2}^5 \tan\varphi_2 - {r_1}^5 \tan\varphi_1}{{r_2}^2 - {r_1}^2} \right)$$

# Needle Deviation Result

- Measurements:

$$r_1 = 80 \pm 0.1\,\mathrm{cm}; \quad \varphi_1 = 11.5^\circ \pm 0.5^\circ$$

$$r_2 = 60 \pm 0.1\,\mathrm{cm}; \quad \varphi_2 = 27.5^\circ \pm 0.5^\circ$$

- Result:

$$\frac{m}{B_\mathrm{e}} = (4.66 \pm 0.56) \cdot 10^5\,\mathrm{Am^2T^{-1}}$$

$$\varepsilon\left(\frac{m}{B_\mathrm{e}}\right) = 12.0\,\%$$

# Absolute Magnetic Field Value

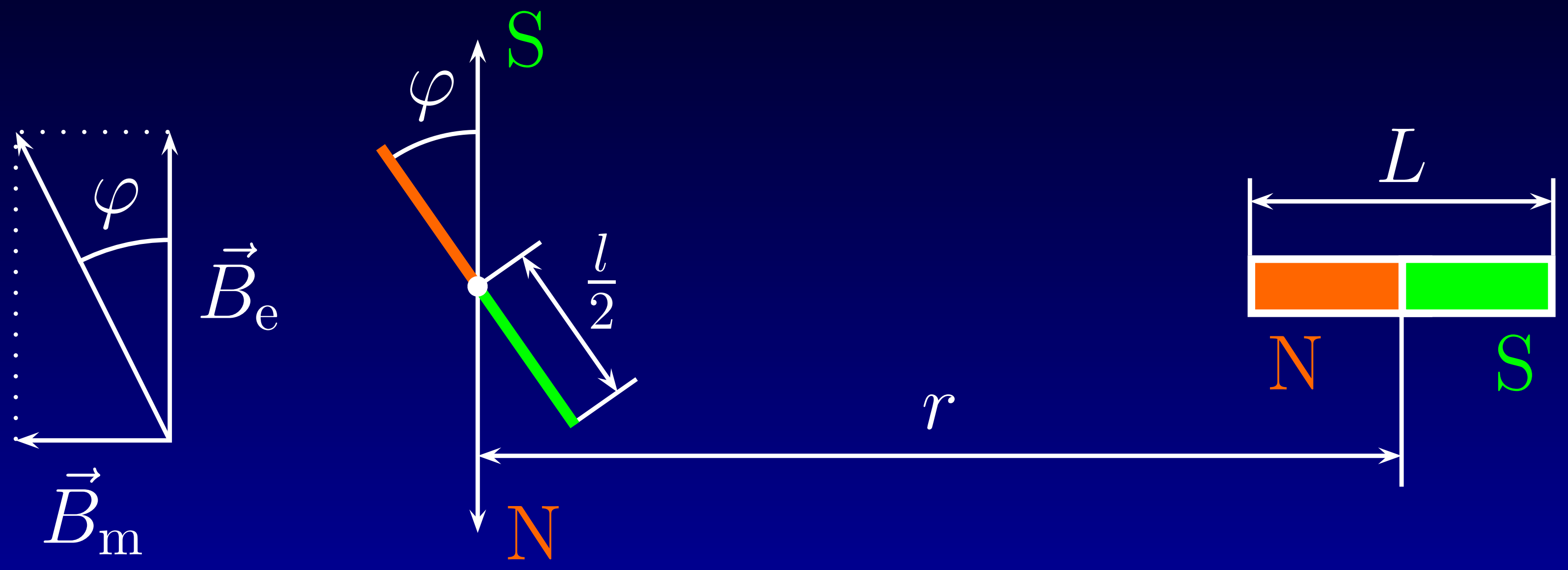
- Merging both measurements:

$$
\begin{aligned}
B_{\mathrm{e}} &= \sqrt{\frac{mB_{\mathrm{e}}}{m/B_{\mathrm{e}}}} \\
\varepsilon(B_{\mathrm{e}}) &= \frac{1}{2}\left(\sqrt{\varepsilon(mB_{\mathrm{e}})^2 + \varepsilon\left(\frac{m}{B_{\mathrm{e}}}\right)^2}\right)
\end{aligned}
$$

- Result:

$$
\begin{aligned}
B_{\mathrm{e}} &= (2.14 \pm 0.13) \cdot 10^{-5}\,\mathrm{T} \\
\varepsilon(B_{\mathrm{e}}) &= 6.0\,\%
\end{aligned}
$$

# Relative Measurement Method




- For $r_0 = 49\,\mathrm{cm}$ we get $\varphi_0 = 45.0^\circ$
- $B_{\mathrm{e}} = B_{\mathrm{m}} = 21.4\,\mu\mathrm{T}$ true for $\varphi_0 = 45.0^\circ$

$$\tan\varphi = \frac{B_{\mathrm{m}}}{B_{\mathrm{e}}} \Rightarrow B_{\mathrm{e}} = \frac{21.4}{\tan\varphi}\,\mu\mathrm{T}$$

# Relative Measurement Method

- Error propagation for relative measurement:

$$\varepsilon(B_{\mathrm{e,rel}}) = \sqrt{\varepsilon(B_{\mathrm{e,abs}})^2 + \varepsilon(\tan\varphi_0)^2 + \varepsilon(\tan\varphi)^2}$$

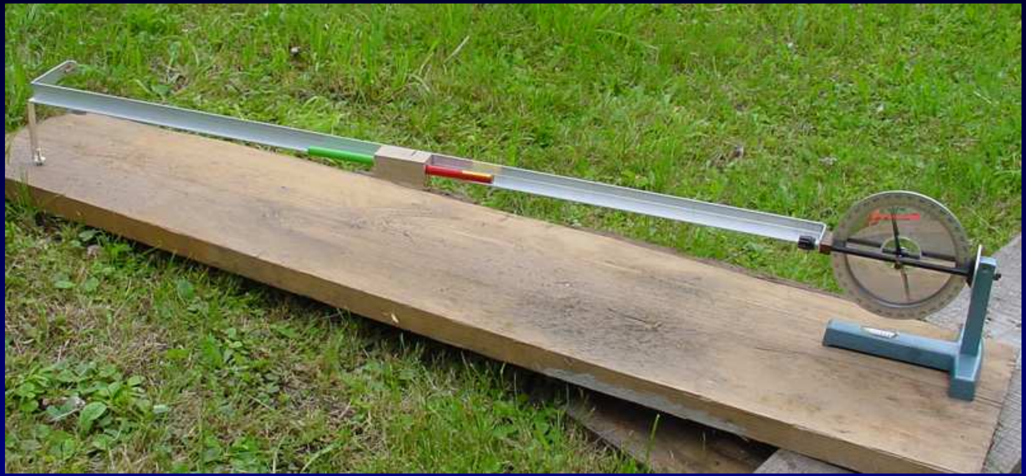
- According to error propagation rules:

$$\varepsilon(\tan\varphi_i) = \frac{1+\tan^2\varphi_i}{\tan\varphi_i}\Delta\varphi_i; \qquad \Delta\varphi_i = \frac{0.5^\circ}{180^\circ}\pi$$

- Minimum error for $\varphi \approx \varphi_0 \approx 45^\circ$:

$$\varepsilon(B_{\mathrm{e,rel}}) = 6.5\,\%$$

# Measuring Vertical Component



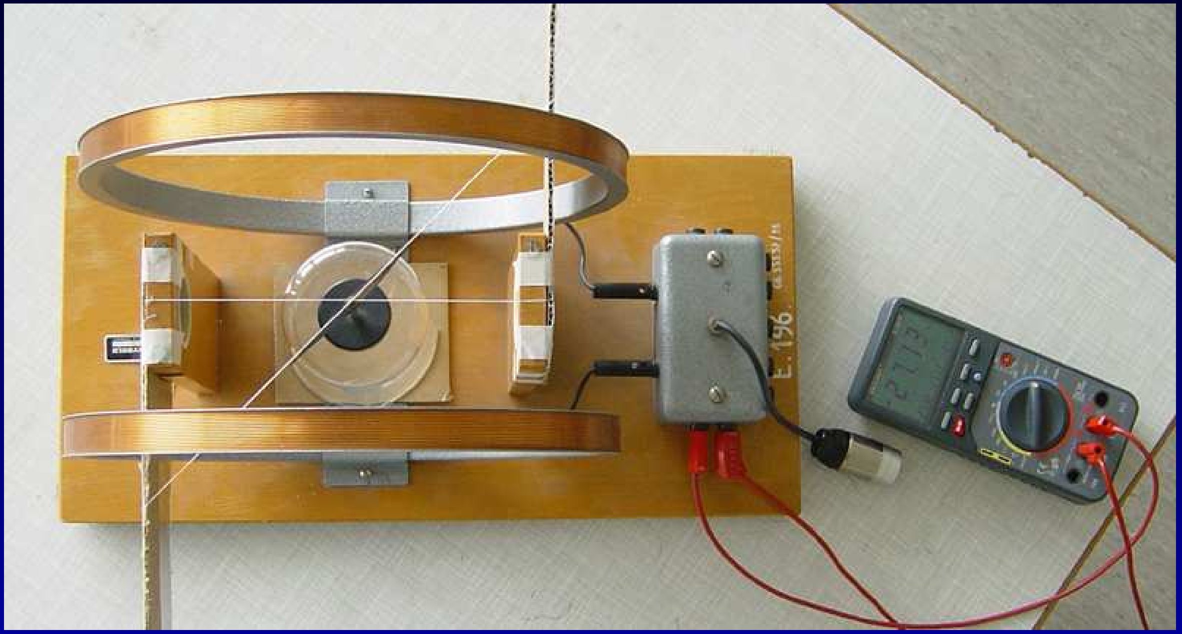
- Measured angle for vertical intensity $26.5^\circ$:

$$B_{\mathrm{e,vertical}} = \frac{21.4}{\tan \varphi_{\mathrm{e,vertical}}}\,\mu\mathrm{T} = 42.9\,\mu\mathrm{T}$$

- Total intensity:

$$B_{\mathrm{e,total}} = \sqrt{B^2_{\mathrm{e,horizontal}} + B^2_{\mathrm{e,vertical}}} = 47.9\,\mu\mathrm{T}$$

# Verification




- Using a helmholtz coil:

$$B = \frac{\mu_0 n I}{r} \left(\frac{4}{5}\right)^{\frac{3}{2}} = 21.2\,\mu\mathrm{T}$$

# Verification with Literature

Measured intensities compared to data of the US National Geophysical Data Center:

| | Own | NGDC |
|---|---|---|
| Horizontal | $21.4\,\mu\mathrm{T}$ | $21.0\,\mu\mathrm{T}$ |
| Vertical | $42.9\,\mu\mathrm{T}$ | $42.5\,\mu\mathrm{T}$ |
| Total | $47.9\,\mu\mathrm{T}$ | $47.4\,\mu\mathrm{T}$ |

# Calibration

- Remember $B_{\mathrm{e}} = \frac{21.4}{\tan\varphi}\,\mu\mathrm{T}$ true for $r = 49\,\mathrm{cm}$

# Summary

- Magnetic field measurement in seconds
- Measurement of horizontal *and* vertical component
- No complicated devices (lasers, coils, etc.)
- Relative accuracy $6.5\,\%$
- Accuracy can hardly be increased with used compass needle

# Summary

- To avoid influences due to environment:

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