7. Coin

“Stand a coin on its edge upon a horizontal surface. Gently spin the coin and investigate the resulting motion as it settles.”

7.1. Basic Concepts

7.1.1. Rotational Inertia

$$I = \int x^2 dm$$

7.1.2. Angular momentum

$$\vec{L} = I \cdot \vec{\omega}$$

7.2. Methodology

- Part A:

  General view of the movement

- Part B:

  Theoretical analyses of the movement

- Part C:

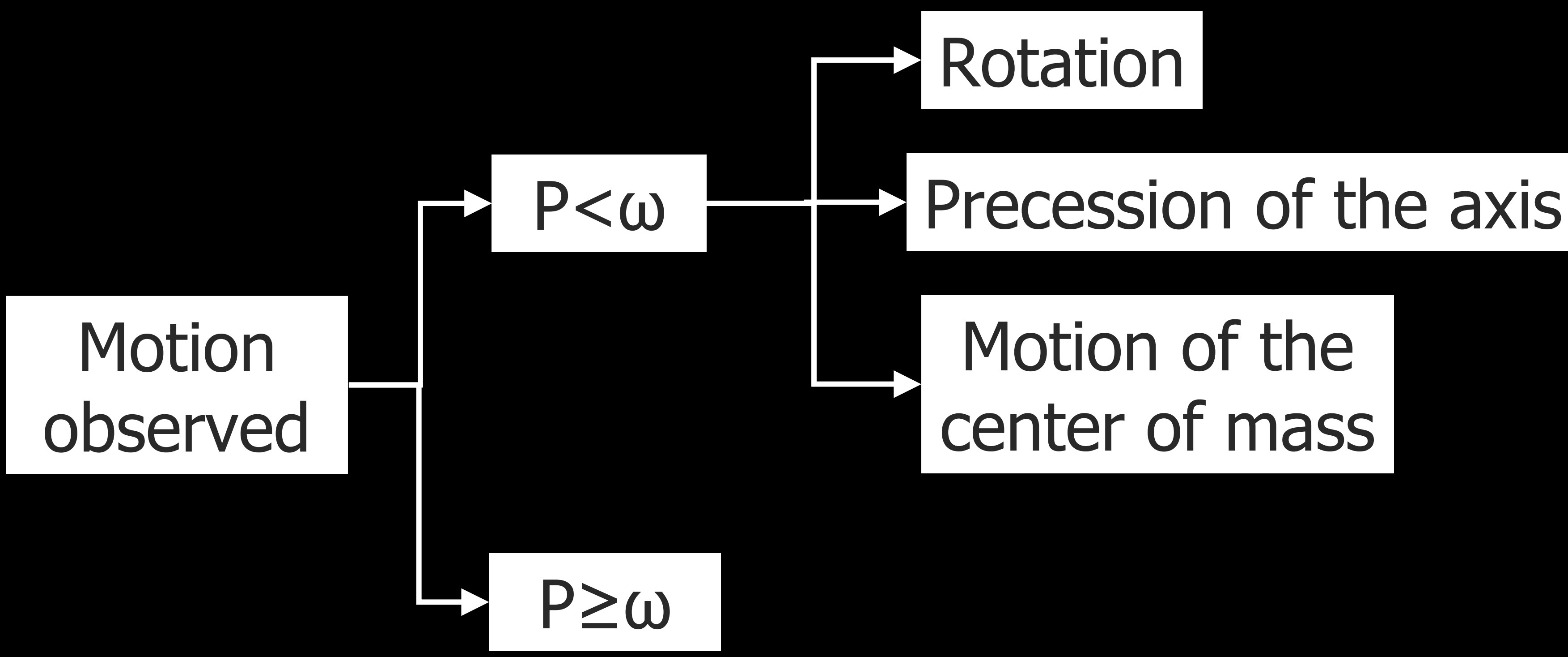
  Experimental verification of the parameters

7.3. General view

7.3. General view

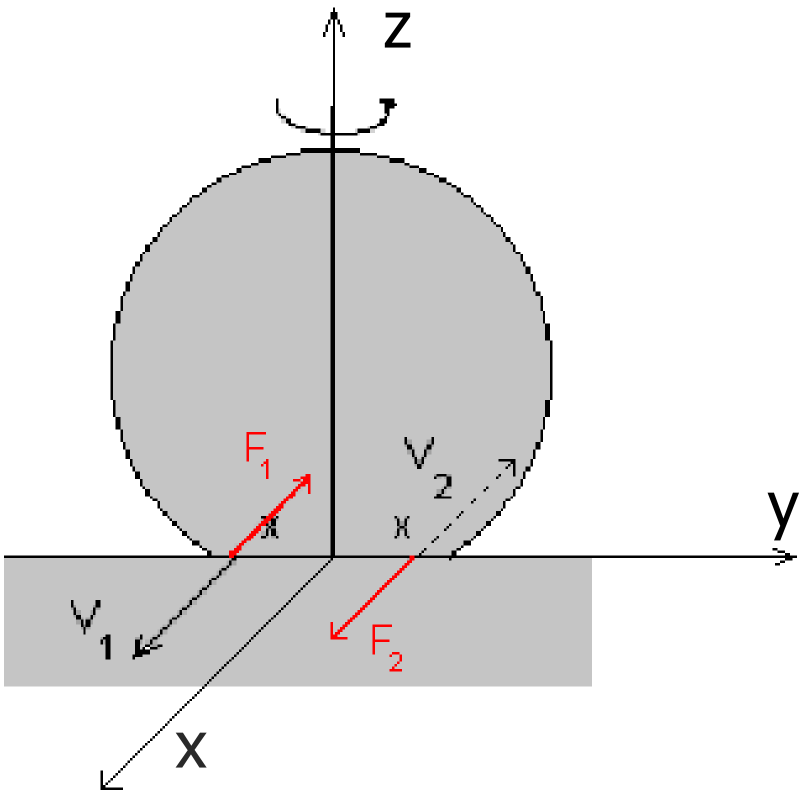
7.4. Theoretical analyses

7.4.1. Strategy



7.4.2. Coin rotation

7.4.2. Coin rotation



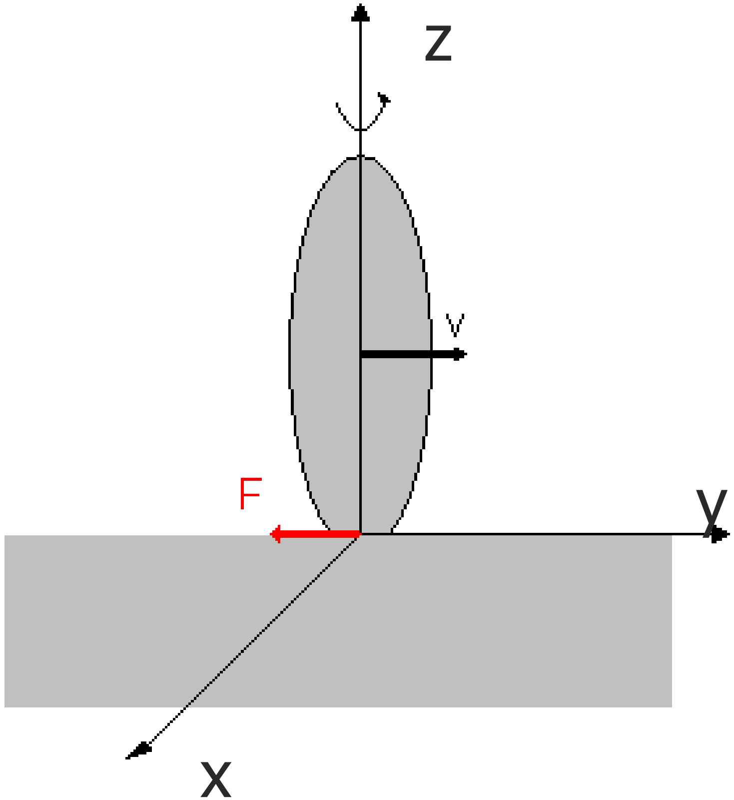
$$L.\hat{k} = I.\omega.\hat{k}$$

$$\vec{\tau} = \vec{r}_1 \wedge \vec{F}_1 + \vec{r}_2 \wedge \vec{F}_2$$

$$\vec{\tau} = -\hat{k}(y_1 F_1 + y_2 F_2)$$

Decreasing angular momentum

7.4.3. Precession of the rotating axis – part I



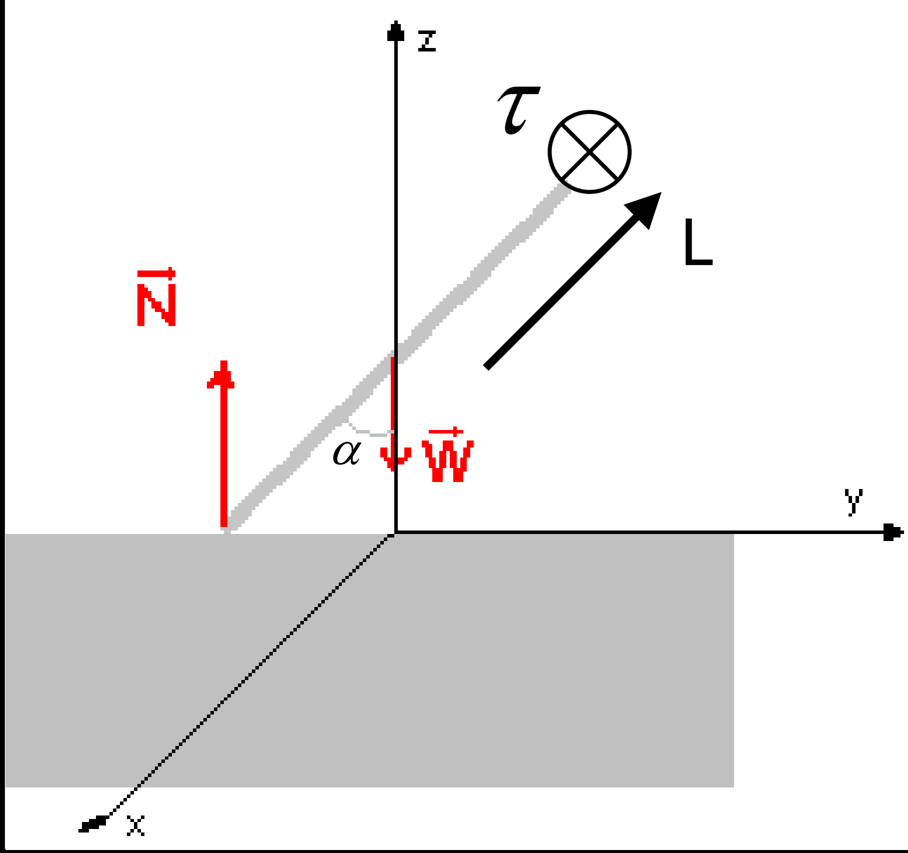
$$F = friction$$

$$\frac{d\vec{L}}{dt} = [r.(-\hat{k})] \wedge [F.(-\hat{j})]$$

$$\frac{d\vec{L}}{dt} = R.F.(-\hat{i})$$

axis

7.4.3. Precession of the rotating axis – part II



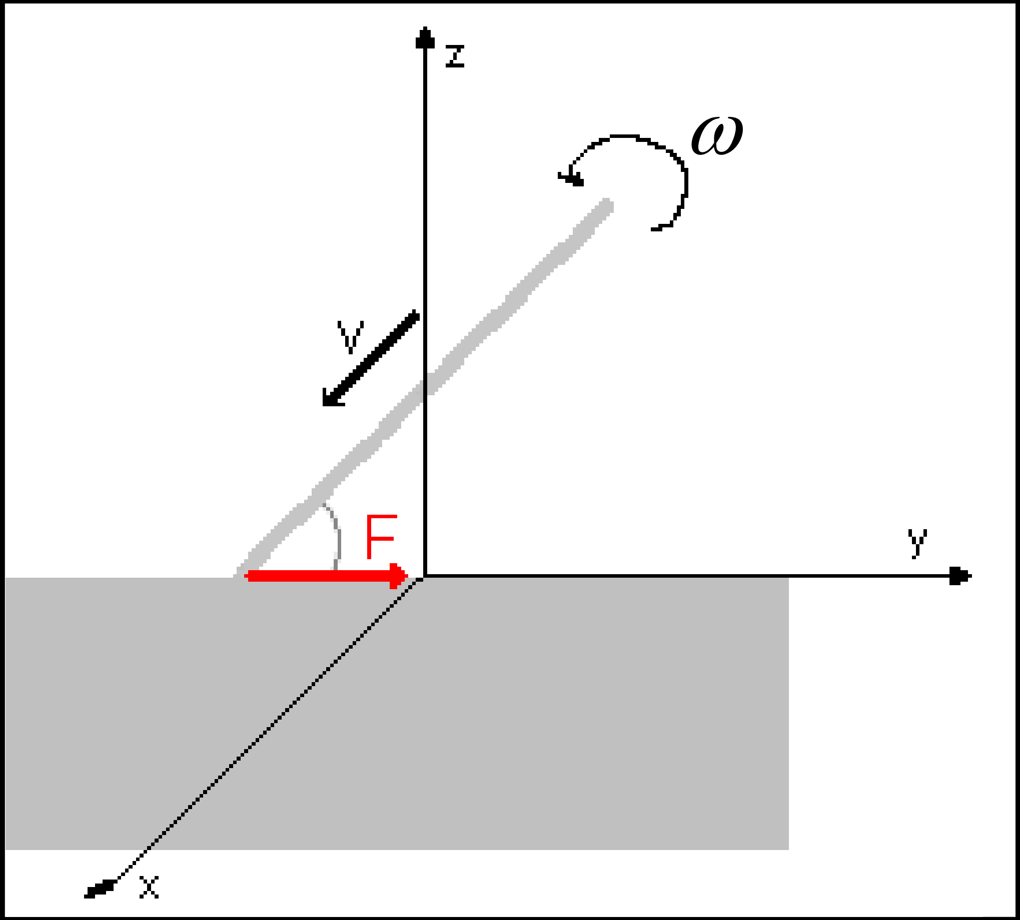
$$\frac{d\vec{L}}{dt} = \vec{r} \wedge \vec{N}$$

$$\frac{d\vec{L}}{dt} = -\hat{i}\,(R.N.\sin\alpha)$$

As the axis inclinates the torque increases.

precession

7.4.4. Motion of the center of mass

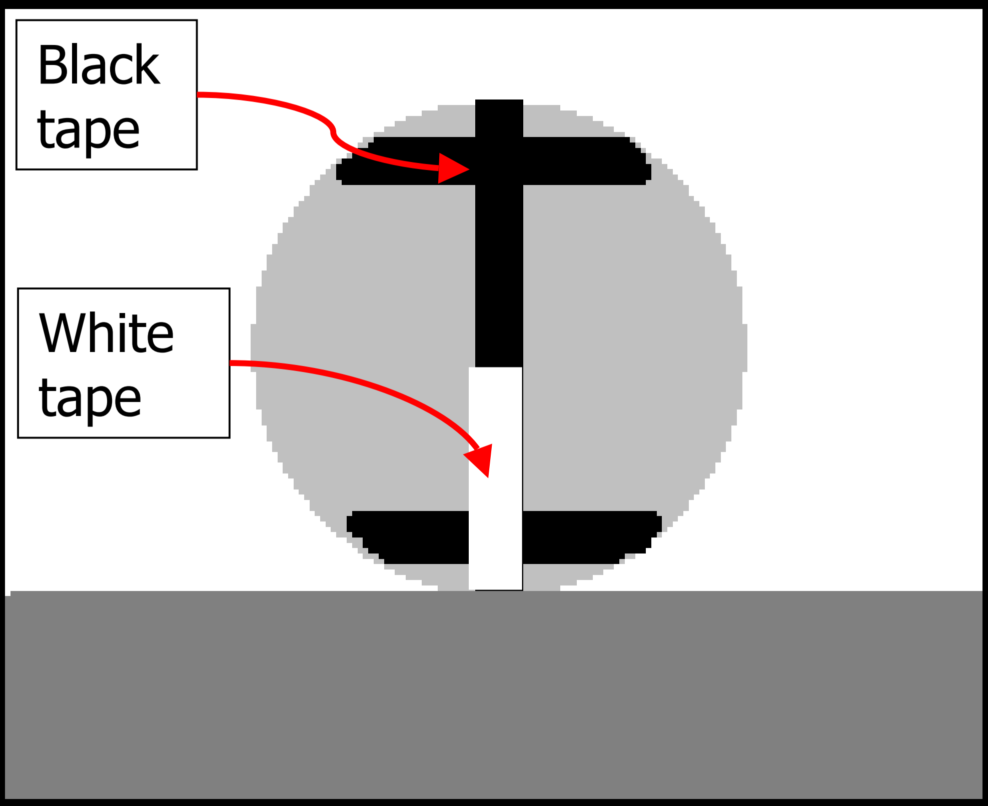


The coin tends to slide. So, a friction force appears as indicated.

The contact point will describe a circle.

trajectory

7.4.5. Special coin – confirming the accuracy of our analysis



In a normal spining coin the axis is changing all the time

By making this distribution of mass we created a fixed rotating axis.

7.4.5. Special coin – confirming the accuracy of our analyses

- The axis will inclinate
- precession of the axis
- Centripetal force in the direction of the axis

7.4.6. Precession predominates

- The angular momentum will decrease
- The torque due to the binary $\vec{W}$ and $\vec{N}$ will increase
- Thus the precession velocity (P) will increase
- Comprovation by the measurement of the frequency of the sound

7.4.6. Precession predominates

7.5 Experimental analyses

7.5.1. Materials used

- Old R$0,05 coin
- new R$0,50 coin
- 30cm-ruler
- chronometer
- sandpaper
- wooden floor
- video camera






7.5. Experimental analyses

7.5.2. Procedure

In order to analyse the influence of the mass and the friction force, it was used:

- Two different coins
- Three different surfaces

7.5. Experimental analyses

7.5.3. Results: Surface A

	R$ 0,05 coin	R$ 0,50 coin
1	15''09	11''78
2	14''78	9''78
3	14''78	10''85
4	13''12	9''28
5	13''40	10''03
6	13''72	10''06
7	14''31	9''35
8	16''93	8''71
9	13''91	10''21
10	13''09	10''47
Average	**14''34**	**10''05**
Error	**1.16 s**	**0.87 s**

7.5. Experimental analyses

7.5.4. Results: Surface B

	R$ 0,05 coin	R$ 0,50 coin
1	10''23	7''41
2	9''69	9''12
3	11''36	7''23
4	10''22	8''23
5	9''41	9''05
6	11''41	8''66
7	9''78	7''62
8	10''04	9''23
9	11''05	9''30
10	9''18	8''85
Average	**10''24**	**8''47**
Error	**0.79 s**	**0.79 s**

7.5. Experimental analyses

7.5.5. Results: Surface C

	R$ 0,05 coin	R$ 0,50 coin
1	2’’72	2’’63
2	3’’74	2’’91
3	3’’19	2’’31
4	3’’21	2’’55
5	3’’24	2’’52
6	2’’60	2’’61
7	3’’06	2’’44
8	3’’14	2’’81
9	3’’04	2’’48
10	3’’16	2’’42
Average	**3’’11**	**2’’57**
Error	**0.31 s**	**0.18 s**

7.5. Experimental analyses

7.5.6. Comparison

	R$0.05 (m')	R$0.50 (m'')
A μ_1	14''34	10''05
B μ_2	10''24	8''47
C μ_3	3''11	2''57

$$\mu_1 < \mu_2 < \mu_3$$

7.6. Error analysis

7.6.1. Main source of error

- Different forces
- Coin width
- Surface inclination
- Chronometer precision

7.7. Conclusion

Therefore we can conclude that there are three important forces in the movement:

- Weight
- Normal Force
- Friction

7.7. Conclusion

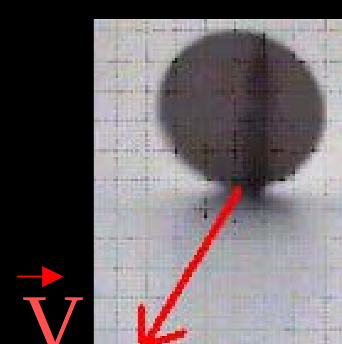
Because of this forces:

- Curve ray gets smaller
- Tangent velocity gets smaller
- Angular velocity gets smaller
- Precession velocity gets bigger

7.4. Results

7.4.1. Analysis of the motion in three stages

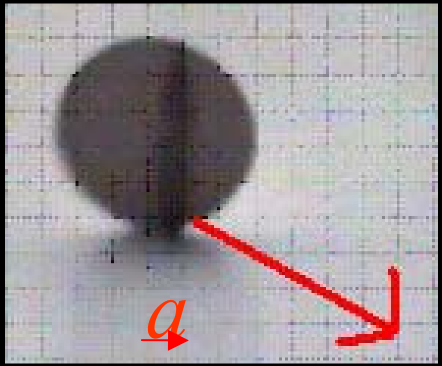
1st Stage: the translation motion predominates, which means that the tangential speed *v* is the main component of the motion.



7.4. Results

<u>7.4.1. Analysis of the motion in three stages</u>

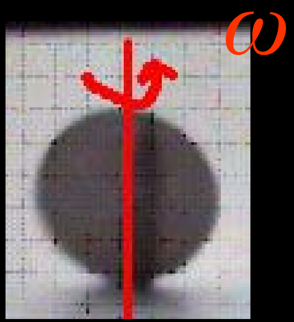
2nd stage: the rotational motion predominates. This means that the centripetal acceleration *a*, caused by the attrition force, is the main responsible for the curvilinear motion of the coin.



7.4. Results

7.4.1. Analysis of the motion in three stages

3rd stage: when the coin is stabilized on a certain position, it is clearly verified the "spinning top" effect, in which the angular speed ω causes the coin to spin around its axle, but also has its axle altered by the weight-force, until the coin falls down.



7.4. Results

7.4.2. Analysis of the influence of the mass

It was verified that the heavier coin (R$0,50) performs its motion in a shorter interval of time. Therefore, since the attrition force is higher, the speed is reduced faster and, because of its weight, the coin falls down faster.

7.4. Results

7.4.3. Analysis of the influence of the surface

It was noticed that the higher the attrition coefficient, the higher the centripetal acceleration on the second stage. Consequently, on the third stage, as the coin will be slower, it will fall down more rapidly.

7.6. Parameters

- Coin mass
- Coin shape
- Coin contact surface
- Coin ray
- Local inclination
- Friction coefficient between materials

7.7. Conclusions

- The linear speed *v* decreases through the trajectory;
- the centripetal acceleration increases
- angular acceleration is distinguished on the last stage of the motion

7.7. Conclusions

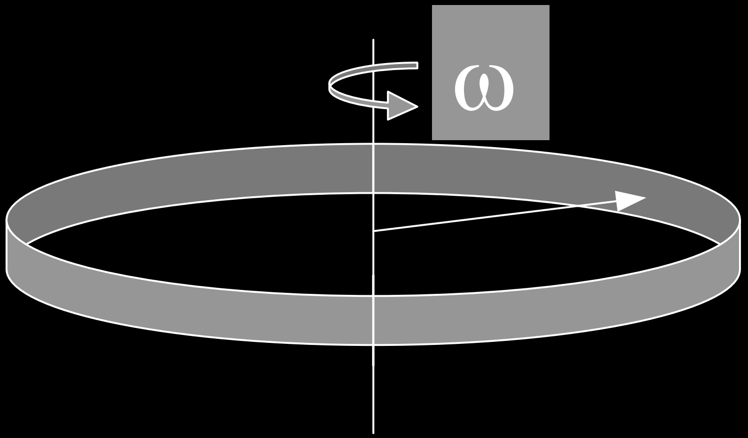
That’s why the coin moves approximately in a spiral form. This awkward phenomenon happens because “the rolling coin squeezes and swirls the air beneath. The flowing air takes up energy, tipping the coin even closer to the surface. At some point, the coin's edge finally loses its grip on the table and falls flat”.

The coin spins longitudinally, transversally and even a slight alteration in the angle of the original impulse alters the resultant trajectory.

7.8. Basic Concepts

Speed of Rotation (ω)

$\omega = \Delta\theta/\Delta t$ **Note similarity to $v = \Delta x/\Delta t$**



Angular Acceleration - Measures how angular velocity is changing (α)

$\alpha = \Delta\omega/\Delta t$ **Note similarity to $a = \Delta v/\Delta t$**

7.8. Basic Concepts

Torque

- Product of Force and Lever Arm
  - Torque = Force X Lever Arm
- Just as unbalanced forces produce acceleration, unbalanced torques produce angular acceleration.

Center of Mass

- Average position of the mass of an object
  - Newton showed that all of the mass of the object acts as if it is located there.

Stability

- In order to balance forces and torques, the center of mass must always be along the vertical line through the base of support.

7.8. Basic Concepts

Centripetal Force

- Any force that causes an object to move in a circle.
- Centripetal forces can be written in different ways:

$$F_{cp} = ma_{cp} \qquad F_{cp} = mv^2/r \qquad F_{cp} = mr\omega^2$$

Centrifugal force

- Fictitious center fleeing force
  - Felt by object in an accelerated reference frame

7.8. Basic Concepts

Angular Momentum

L = (rotational inertia) X (angular velocity)

$L = I\omega$

Compare to linear momentum:

$p = mv$

7.8. Basic Concepts

Linear Momentum and Force
Angular Momentum and Torque

- **Linear** $\Sigma F = \Delta p/\Delta t$
  - **Impulse** $\Delta p = \Sigma F\ \Delta t$

- **Rotational** $\Sigma\tau = \Delta L/\ \Delta t$
  - **Rotational Impulse** $\Delta L = \Sigma\tau\ \Delta t$

Conservation of Momentum

- Linear
  - If $\Sigma F = 0$, then p is constant.
- Angular
  - If $\Sigma \tau = 0$, then L is constant.

7.8. Basic Concepts

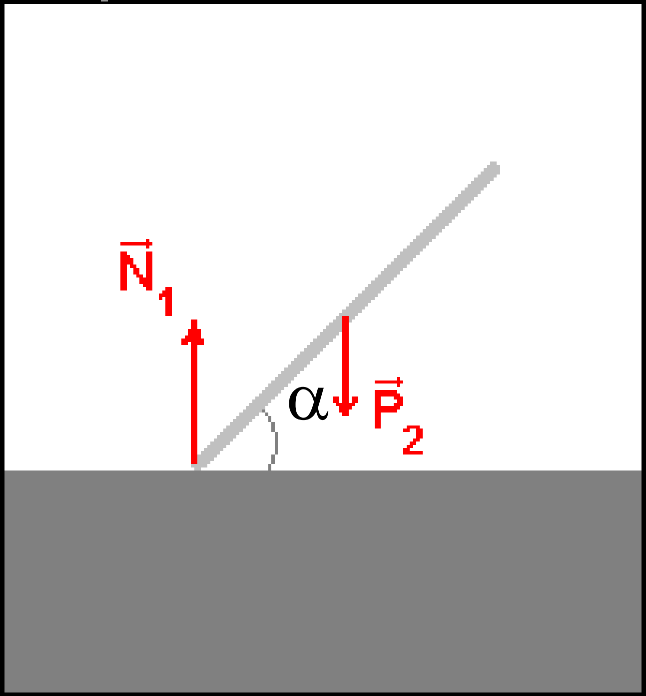
Moment of Inertia

- Property of an object that resists changes in rotation
  - For linear motion mass was a measure of inertia
  - For rotational motion Moment of Inertia (I) is the measure of rotational Inertia
- Depends on:
  - Mass of the Object
  - Axis of Rotation
  - Distribution of Mass in the Object

7.9. Sources

- http://www.findarticles.com/cf_dls/m1200/19_157/62724341/p1/article.jhtml
- http://physicsweb.org/article/news/4/4/12
- http://hyperphysics.phyastr.gsu.edu/hbase/torcon.html
- http://www.brazilnet.net/aboutus/moeda.htm

Video



There is a binary that generates a torque
It is obvious to affirm that the smaller the α, the bigger the torque. As a consequence, the precession movement will be more visible at the end.

Video

It was noticed that at the end of the trajectory, the linear velocity is smaller and F is higher, because:

$\vec{F} = 0$

$\vec{F} = \vec{F}_1$

$\vec{F} = \vec{F}_2$

$\vec{F}_1 < \vec{F}_2$

$$R = \frac{v^2}{F}$$

If v gets smaller,
F gets higher and

R gets smaller

Video

- It should be observed that the coin rotation velocity is reduced simultaneously to the precession velocity when this velocity is higher than the rotation velocity.
- If the precession movement gets higher, it was obtained that the speed of the point in contact with the floor gets higher and, as a consequence the sound frequency gets higher. Experimentally, it was obtained a 300Hz frequency.

7.4. Results

7.4.4. Appendix: What would happen if the experiment was performed in an non-gravitational environment?

If the experiment was performed in an environment without any resistive and gravitational forces, there would be no spendthrift forces ($F_{at}=\mu.N$). Since the attrition force is responsible for the reduction of the linear speed and the centripetal acceleration, the coin wouldn't make curves and wouldn't have its speed *v* modified. Therefore, it is as if the coin remained on the 1^{st} stage of the resulting motion.

7.5. Gyroscope effect

It is a device that:

1. Has almost no dissipative forces
2. Maintain the angular momentum
3. If a force is applied up on the gyroscope, it presents resistance in changing direction
4. A modern version to the top
5. Used in planes and spaceships to keep then in course

- *See live presentation*

7.3. Procedure

7.3.2. calculating the relation between the masses of the coins of R$0,50 and R$0,05.

To calculate the relation between the coin masses, a ruler of 30cm was supported on a high surface and a coin was placed in the tip of the ruler. Along with the coin, the ruler was gradually being dislocated out of the high surface until this system fell. The following values had been written down:

11,5 cm for the R$0,50 coin
13,5 cm for the R$0,05 coin.

7.3. Procedure

Knowing that the torque can be calculated by **M=F.d:**

(i) $M_{50}=m_{50}.g.11,5 = F.3,5$, in which F is the ruler’s weight

(ii) $M_{05}=m_{05}.g.13,5 = F.1,5$, in which F is the ruler’s weight

(i)/(ii):

$$\frac{M_{50}}{M_{05}} = \frac{m_{50}.g}{m_{05}.g} = \frac{13,5 \,.\, 3,5}{11,5 \,.\, 1,5}$$

Then, $\mathbf{m_{50}/m_{05} = 2,739}$

7.3. Procedure

7.3.3. Verification of the trajectory of the coin on different floors

The coin motion after a slight impulse was observed on different surfaces (wooden floor and sandpaper). It was clear that the trajectory on the sandpaper, that has higher attrition coefficient, was more curved yet faster than the coin behaviour on the wooden surface.

7.3.4. Trajectory filming

Through a video camera, the described trajectory was filmed on various angles. **See tape 1.**