• **Investigated quantities:**
  
  • Rotational angular velocity
  
  • Precessional rotational velocity
  
  • Angle of precession
  
  • Trajectory of coin

\[ \omega_3 \text{ – rotational angular velocity} \]

\[ \omega_2 \text{ – precessional angular velocity} \]

\[ \varphi \text{ – angle of precession} \]

\[ x,y,z \text{ – axes of the coin coordinate system} \]
<table>
<thead>
<tr>
<th>mass [g]</th>
<th>radius [cm]</th>
<th>thickness [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.3</td>
<td>1.22</td>
<td>0.19</td>
</tr>
<tr>
<td>mass [g]</td>
<td>radius [cm]</td>
<td>thickness [cm]</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
<td>----------------</td>
</tr>
<tr>
<td>3.2</td>
<td>1</td>
<td>0.15</td>
</tr>
<tr>
<td>mass [g]</td>
<td>radius [cm]</td>
<td>thickness [cm]</td>
</tr>
<tr>
<td>---------</td>
<td>------------</td>
<td>----------------</td>
</tr>
<tr>
<td>67.3</td>
<td>4.75</td>
<td>0.70</td>
</tr>
<tr>
<td>mass [g]</td>
<td>radius [cm]</td>
<td>thickness [cm]</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
<td>---------------</td>
</tr>
<tr>
<td>102.5</td>
<td>4.9</td>
<td>2</td>
</tr>
</tbody>
</table>
Ways of starting the coin motion

- Two ways of starting the coin motion exist:
  1. No initial horizontal momentum
  2. The motion is started from above
2. With a significant initial horizontal momentum

- The motion is started with a side kick
1. The coin has no horizontal velocity

Phases of motion:

1. Slow precession
   - the angle of precession changes slowly with time
2. Slipping

- The precessional angle drops abruptly
- Cause: slipping (the gyroscopic effect and friction cannot hold the coin straight any more)
3. Dancing

- The most complex phase
- The coin doesn’t spin any more
- Because of energy losses the edges start to hit the surface
- Thus the characteristic sound is obtained
- The duration of this phase can be quite long depending on the elasticity of the surface and coin:
Dancing on plastic/wood – elastic collisions

Dancing on cork – great energy losses in collisions
Dancing on cardboard – semielastic collisions

Dancing of elastic metal coin
2. The coin is given an initial horizontal velocity

- **Phases of motion**
  - Neglecting the translatory motion, the phases are the same as in the previous case.
Trajectory

- Because of the precession the initially linear trajectory gets curved.
- The length of the linear part depends on the initial momentum of the coin.
- The best curve fit for the whole trajectory is a logarithmic spiral.
- The trajectory graph was obtained by letting the coin move on a sooty plate and filming the trace it left.
Model of coin motion

Coin coordinate system

\( \omega_3 \) – rotational angular velocity

\( \omega_2 \) – precessional angular velocity
Forces acting on the coin

- $F_r$ – reaction force
- $F_{tr}$ – force of friction
- $\varphi$ – precessional angle
Euler equations

\[ I_1 \frac{d\omega_1}{dt} - (I_2 - I_3)\omega_2 \omega_3 = \tau_1 \]

\[ I_2 \frac{d\omega_2}{dt} - (I_3 - I_1)\omega_3 \omega_1 = \tau_2 \]

\[ I_3 \frac{d\omega_3}{dt} - (I_1 - I_2)\omega_1 \omega_2 = \tau_3 \]

- \( \omega_i \) – angular velocity components
- \( \tau_i \) – external torque components
- \( I_i \) – principal moments of inertia

• Neglecting friction the torque components become

\[ \tau_1 = \tau_3 = 0 \]

\[ \tau_2 = \tau_r \]

\( \tau_r \) – reaction torque
Euler equations cont.

\[ I_1 \frac{d\omega_1}{dt} = 0 \]

\[ \Rightarrow I_2 \frac{d\omega_2}{dt} - (I_2 - I_1) \omega_3 \omega_1 = \tau_r \]

\[ I_2 \frac{d\omega_3}{dt} - (I_1 - I_2) \omega_1 \omega_2 = 0 \]

- **Initial conditions:** \[ \omega_1(t) = 0 \]
  \[ \omega_3 = \omega_0 \]

- **Reaction torque:** \[ |\tau_r| = -mg r \cos \varphi \]
  \[ \varphi \text{ – precessional angle} \]
  \[ r \text{ – coin radius} \]

\[ \Rightarrow \frac{d^2 \varphi}{dt^2} = -\frac{4g}{r} \cos \varphi \]
Solution of Euler equation

- The precessional angle equation isn’t soluble in closed form
- Therefore numeric integration was performed:
Fitting the theoretical curve

1. Agreement
2. Slipping (error ~20%)
3. “dancing” of the coin (feeble agreement)
Conclusion

- Two ways of coin motion were investigated
- A mathematical model of motion was proposed
- The motion can be divided in three parts
  1. Large precessional angle – agreement with theory is very good (error ~5%)
  2. Slipping – the agreement is not as well because of the coin slipping
  3. "dancing" - agreement is not good because of the additional effects (collisions with the floor)
Add – on: Initial destabilizing torque