- Investigated quantities:
  - Rotational angular velocity
  - Precessional rotational velocity
  - Angle of precession
  - Trajectory of coin

ω_3 – rotational angular velocity

ω_2 – precessional angular velocity

φ – angle of precession

x, y, z – axes of the coin coordinate system

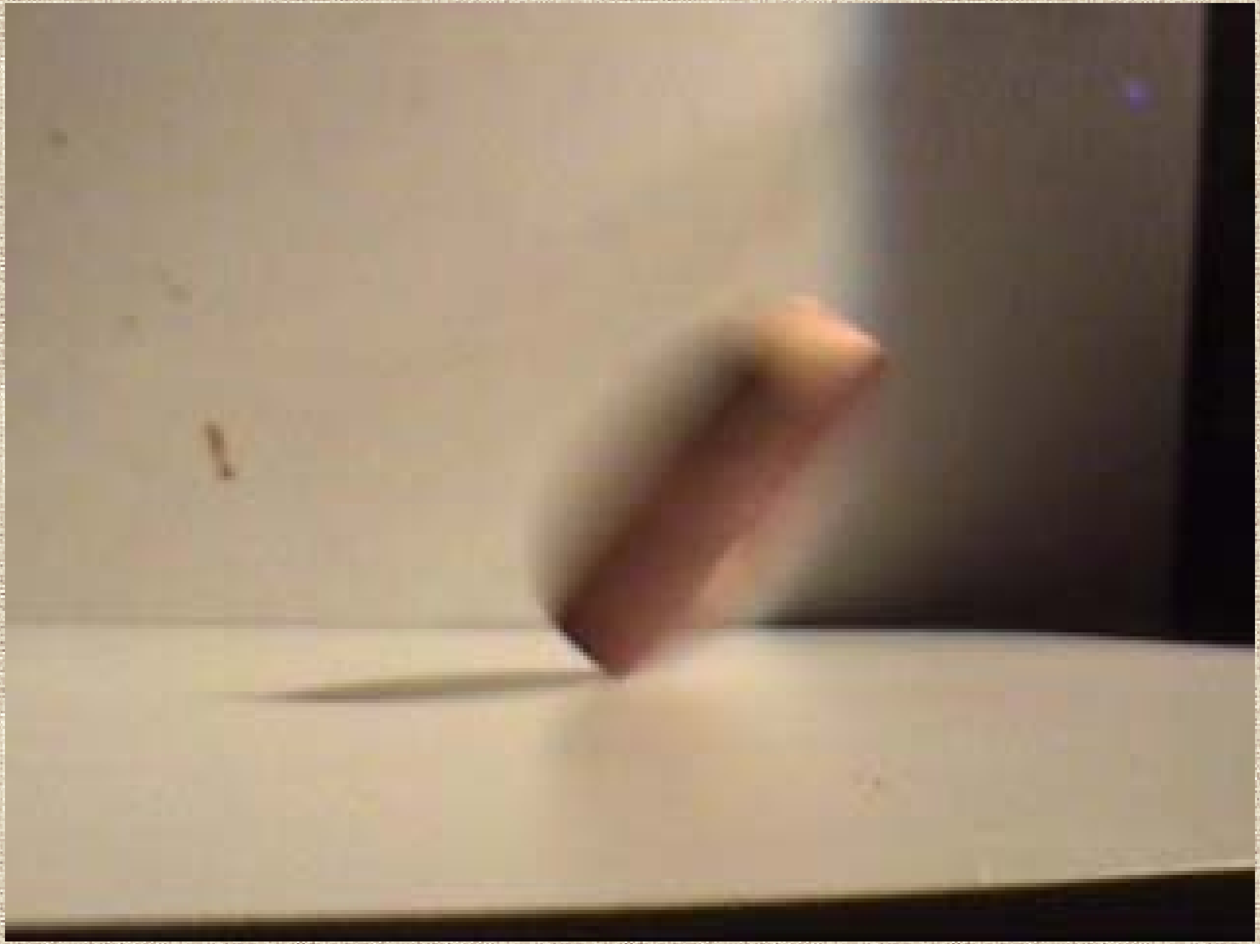
Used coins

mass [g]	radius [cm]	thickness [cm]
7.3	1.22	0.19

mass [g]	radius [cm]	thickness [cm]
3.2	1	0.15

mass [g]	radius [cm]	thickness [cm]
67.3	4.75	0.70

mass [g]	radius [cm]	thickness [cm]
102.5	4.9	2

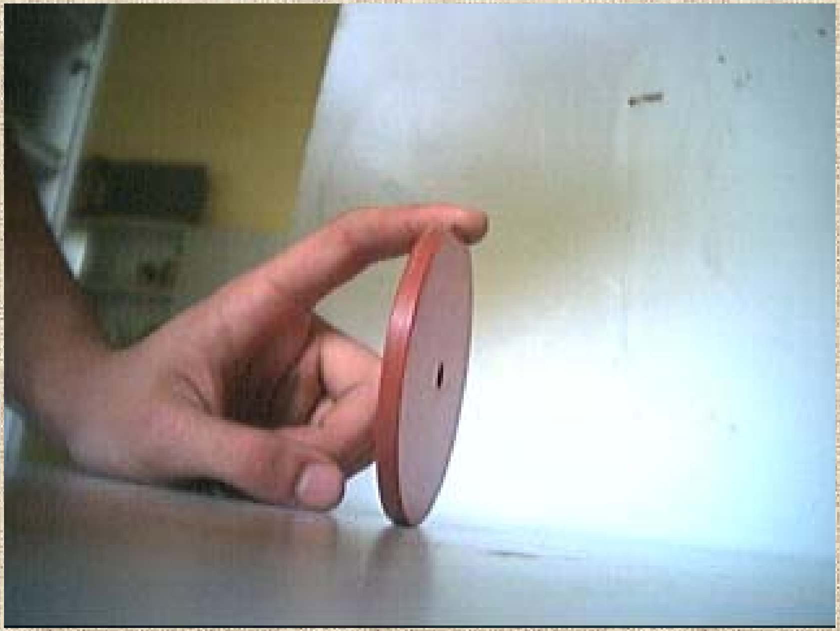


Ways of starting the coin motion

- Two ways of starting the coin motion exist:
    1. No initial horizontal momentum

- The motion is started from above

2. With a significant initial horizontal momentum



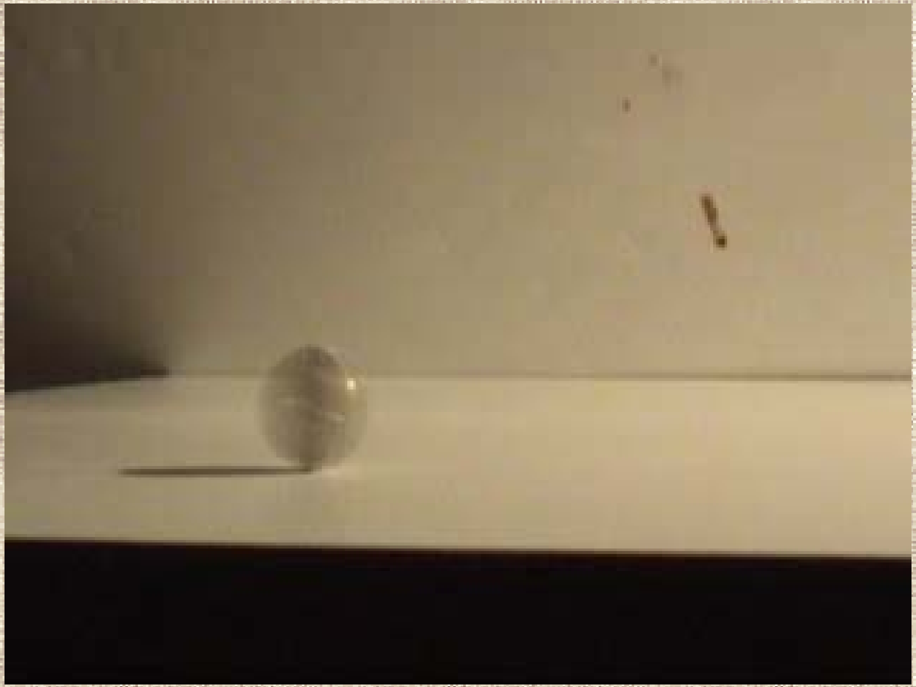
- The motion is started with a side kick

1. The coin has no horizontal velocity

Phases of motion:

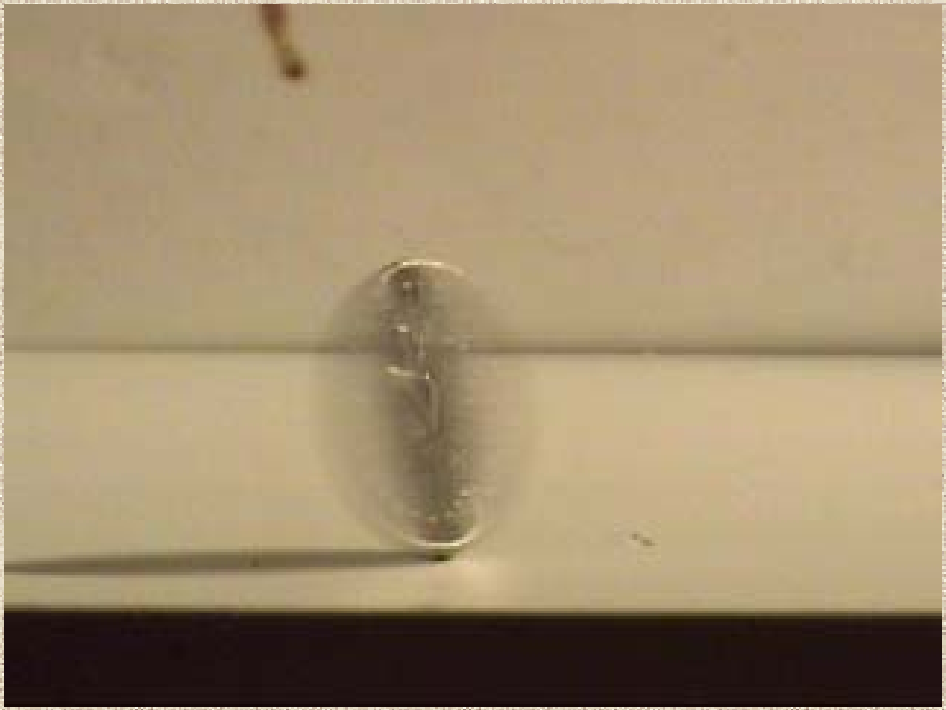
1. Slow precession

- the angle of precession changes slowly with time



2. Slipping

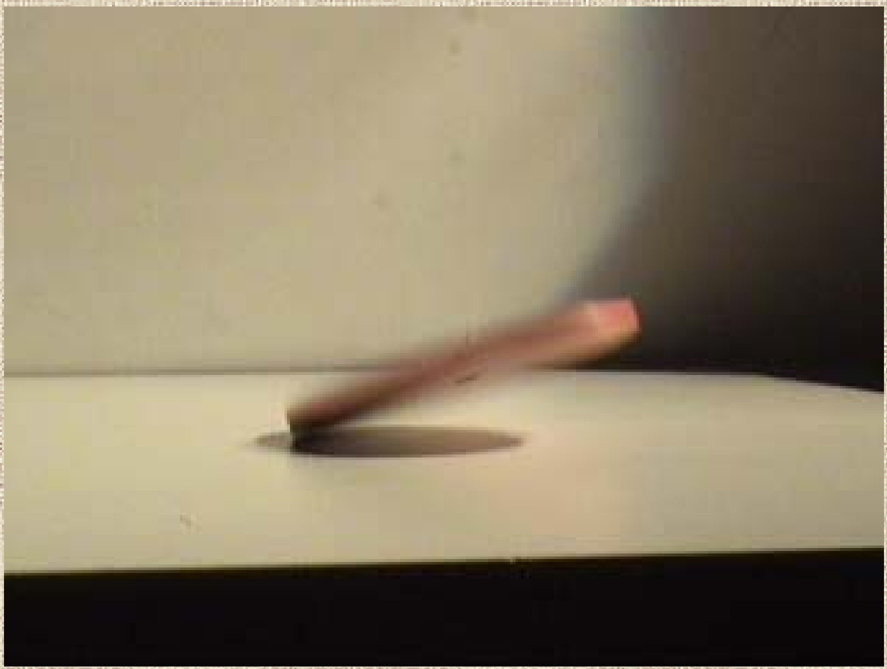
- The precessional angle drops abruptly
- Cause: slipping (the gyroscopic effect and friction cannot hold the coin straight any more)



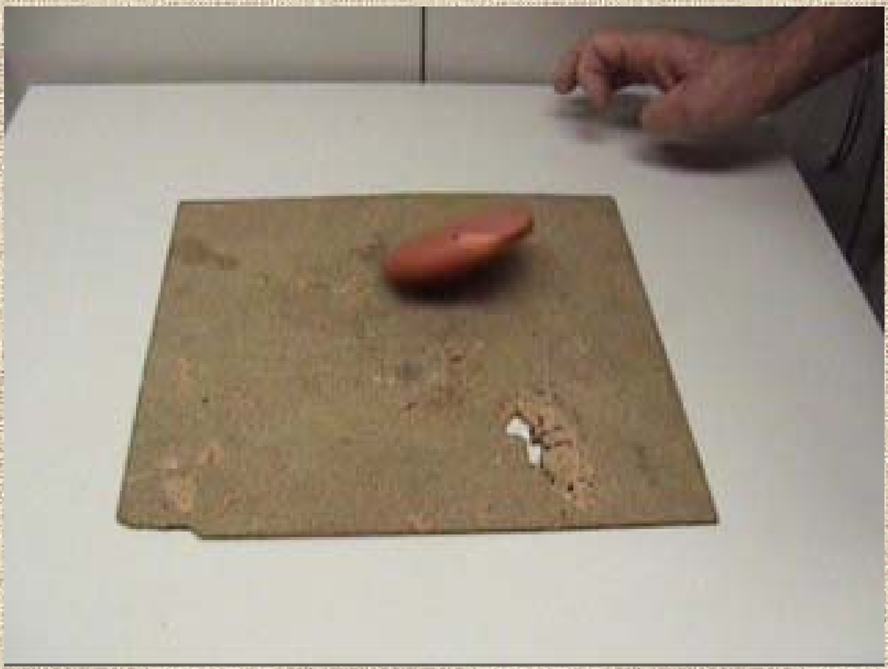
3. Dancing

- The most complex phase
- The coin doesn’t spin any more
- Because of energy losses the edges start to hit the surface
- Thus the characteristical sound is obtained
- The duration of this phase can be quite long depending on the elasticity of the surface and coin:

Dancing on plastic/wood – elastic collisions

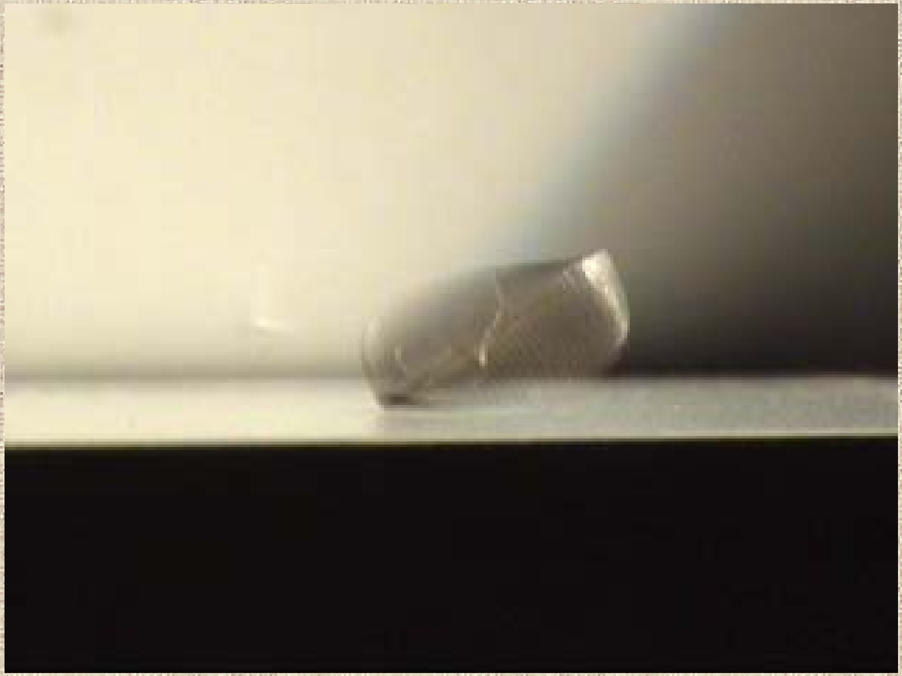


Dancing on cork – great energy losses in collisions



Dancing on cardboard – semielastic collisions

Dancing of elastic metal coin

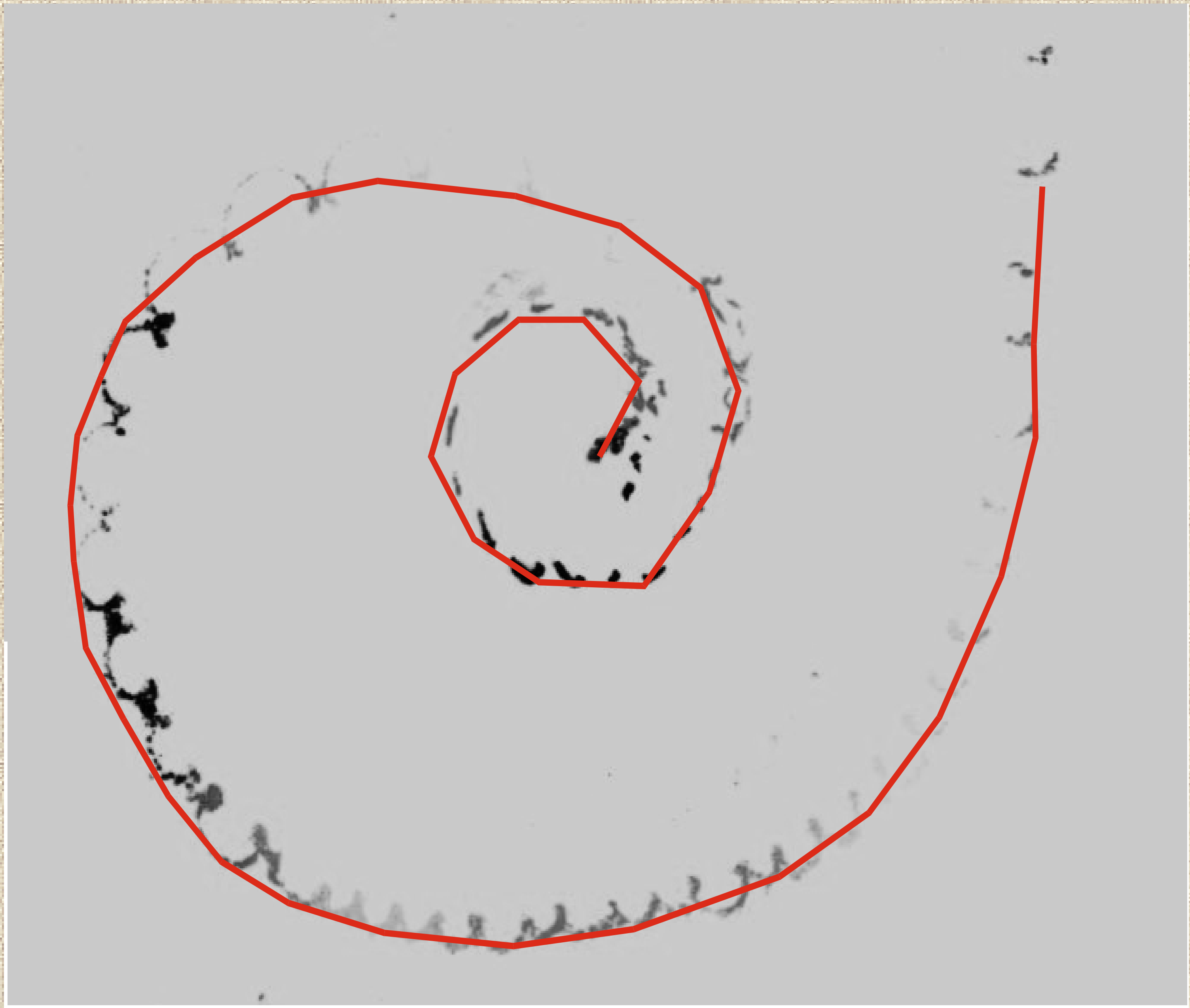


2. The coin is given an initial horizontal velocity

- **Phases of motion**
  - Neglecting the translatory motion, the phases are the same as in the previous case

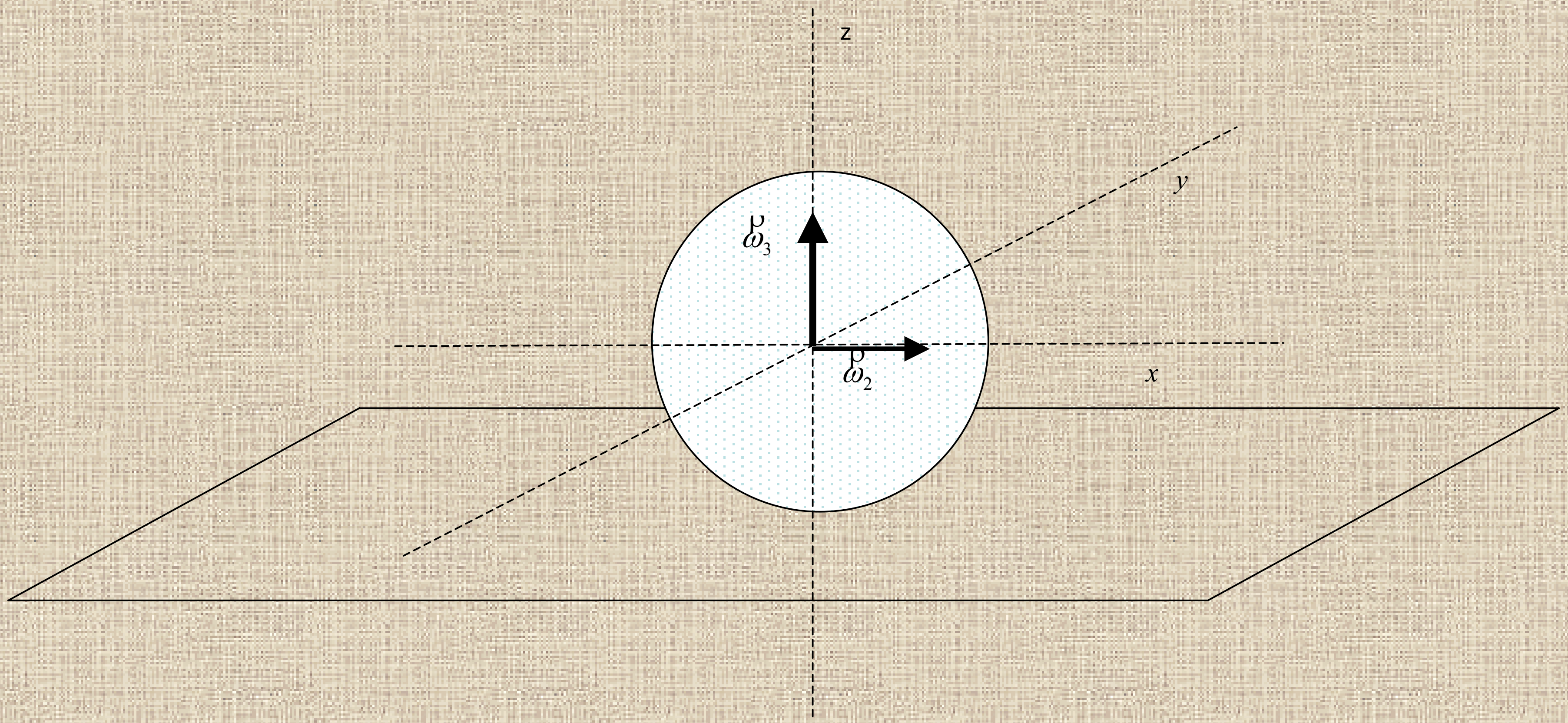
Trajectory

- Because of the precession the initially linear trajectory gets curved
- The length of the linear part depends on the initial momentum of the coin
- The best curve fit for the whole trajectory is a logarithmic spiral
- The trajectory graph was obtained by letting the coin move on a sooty plate and filming the trace it left:



Model of coin motion

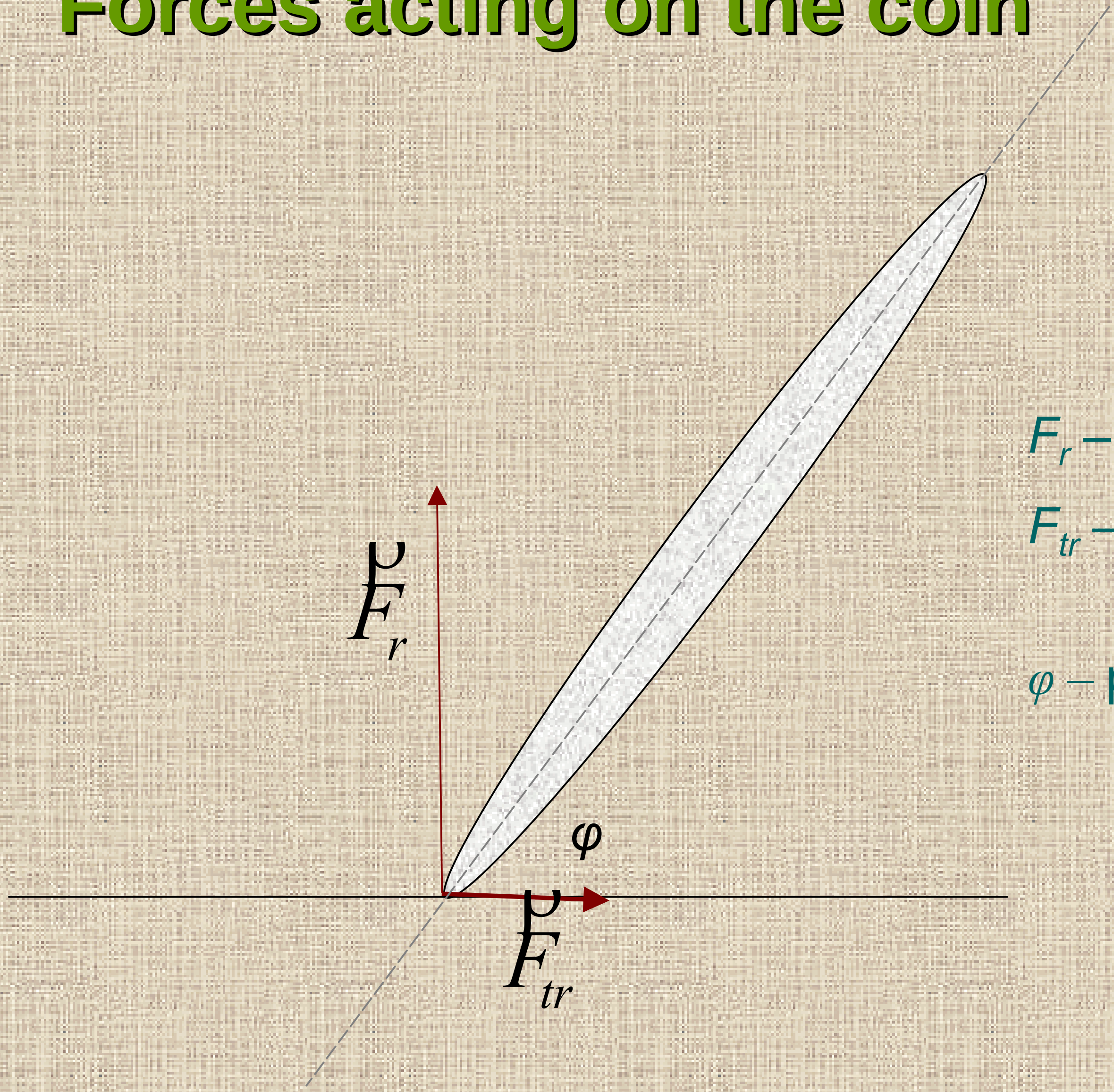
Coin coordinate system



ω_3 – rotational angular velocity

ω_2 – precessional angular velocity

Forces acting on the coin



F_r – reaction force

F_{tr} – force of friction

φ – precessional angle

Euler equations

$$I_1 \frac{d\omega_1}{dt} - (I_2 - I_3)\omega_2\omega_3 = \tau_1$$

$$I_2 \frac{d\omega_2}{dt} - (I_3 - I_1)\omega_3\omega_1 = \tau_2$$

$$I_3 \frac{d\omega_3}{dt} - (I_1 - I_2)\omega_1\omega_2 = \tau_3$$

ω_i – angular velocity components

τ_i – external torque components

I_i – principal moments of inertia

- Neglecting friction the torque components become

$$\tau_1 = \tau_3 = 0$$

$$\tau_2 = \tau_r$$

τ_r – reaction torque

Euler equations cont.

$$I_1 \frac{d\omega_1}{dt} = 0$$

$$\Rightarrow \quad I_2 \frac{d\omega_2}{dt} - (I_2 - I_1)\omega_3\omega_1 = \tau_r$$

$$I_2 \frac{d\omega_3}{dt} - (I_1 - I_2)\omega_1\omega_2 = 0$$

- Initial conditions: $\omega_1(t) = 0$

$$\omega_3 = \omega_0$$

- Reaction torque: $|\tau_r| = -mgr\cos\varphi$

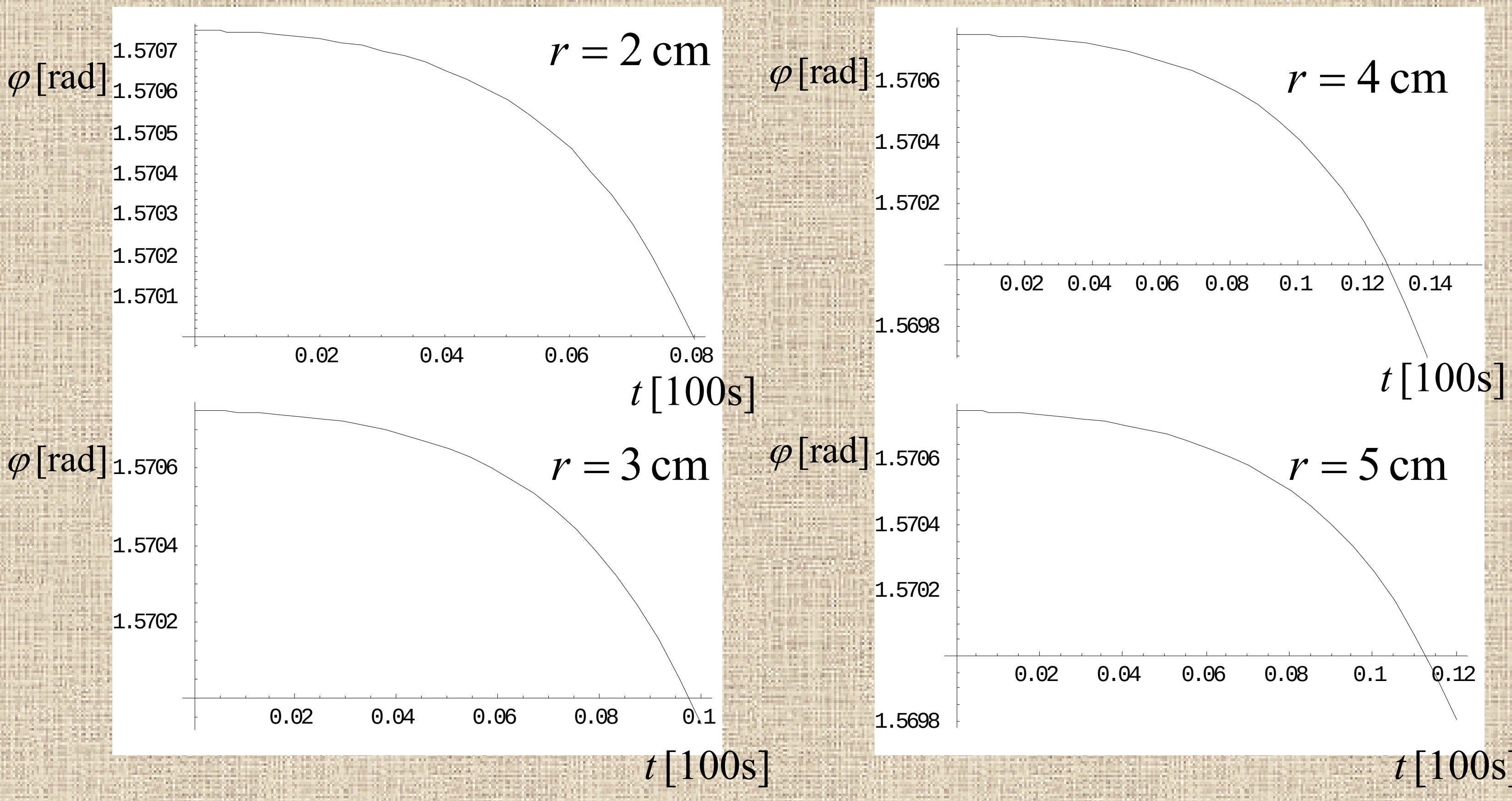
$$\Rightarrow \frac{d^2\varphi}{dt^2} = -\frac{4g}{r}\cos\varphi$$

φ – precessional angle

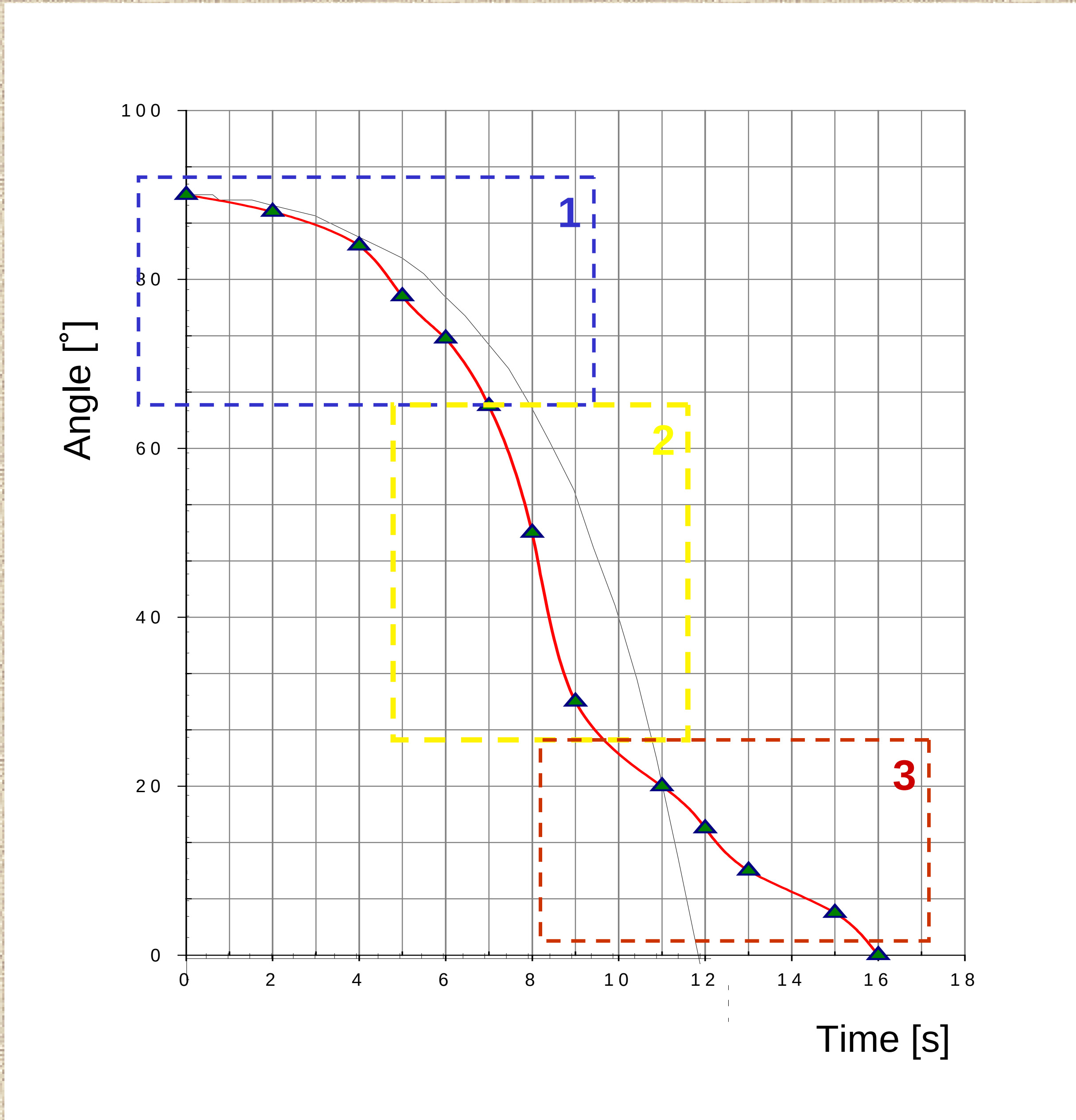
r – coin radius

Solution of Euler equation

- The precessional angle equation isn't soluble in closed form
- Therefore numeric integration was performed:



Fitting the theoretical curve



1. Agreement
2. Slipping (error ~20%)
3. ˝dancing˝ of the coin (feeble agreement)

Conclusion

- Two ways of coin motion were investigated
- A mathematical model of motion was proposed
- The motion can be divided in three parts

1. Large precessional angle – agreement with theory is very good (error ~5%)
2. Slipping – the agreement is not as well because of the coin slipping
3. ˝dancing˝ - agreement is not good because of the additional effects (collisions with the floor)

Add – on: Initial destabilizing torque